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UNIVARIATE AND MULTIVARIATE HOMOGENEITY TESTS:

A REVIEW AND BIBLOGRAPHY

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ABSTRACT

The review covers recent research into univariate and multivariate homogeneity tests under normality, non–normality and finite population assumptions.

Some key words: Statistic, robust and kurtosis.

1. INTRODUCTION

Tests of equality of variances and covriance matrices are important in contexts:

(1) The assumption of homogeneity "equality of variances" used in Analysis of Variance and assumption of multivariate homogeneity "equality of covariance matrices" used in Multivariate Analysis of Variance.

(2) When it is desired to combine a number of variances or covariance matrices to obtain an estimate of the common variance or covariance matrix.

(3) Much of the multivariate analysis is based on covariance matrices – cf. Morrison, 1976; Mardia et al. 1979. Let, e.g. a problem of multivariate analysis of the attitudes of males and females towards a specific problem. If covariance matrices of the populations are equal, analyze the data using the combined data on males and females; if these are unequal analyze males and females separately.

Therefore it becomes desirable to list the available homogeneity tests under certain assumptions. Placket (1946) provides a survey of literature on testing the equality of variances and covariances in normal populations. After that no attempt is made to provide an exhaustive review.

The aim of this review paper is to mention available literature upto 1986 relevant to the univariate and multivariate homogeneity tests under normality, non–normality and finite population assumptions.

2. EQUALITY OF *k* VARIANCES UNDER NORMALITY

Consider k normal populations with $X_i \xrightarrow{dist.} N(\mu_i, \sigma_i^2)$;

i=1 , . . . , k . Suppose X_{ie} ; e=1 , 2 ,. . . , n_i is a random sample of size n_i from X_i. The hypotheses of interest are:

$$H_{O1}: \sigma_1^2 = \ldots = \sigma_K^2 \ \ vs \ \ H_{H1}: \sigma_1^2 \neq \ldots \neq \sigma_K^2$$

The first approach to the problem of testing the equality of k variances under normality was made by Neyman and Pearson (1931), using the likelihood ratio statistic, which is approximately null distributed as chi–square with (k–1) degrees of freedom. The test was modified by Bartlett (1937) and studied by Box (1949), Korin (1967) and Sugiura & Nagao (1968). For small samples the test has considerably greater sizes (observed significance levels) than the desired nominal levels. Mood (1939) showed that the degree of approximation of the statistic to the χ^2 law with (k–1) degrees of freedom is mainly dependent on n (number of abserrations in sample) and is for all practical purpose independent of k when n is moderately large. Nayer (1936) computed tables of probability levels of the likelihood ratio test of the case of equal samples. Wilks & Thompson (1937) have discussed the general distribution of the criterion when H_O is not true.

The modification by Bartlett (1937) improves the approximation to chi–square. But the investigations carried out by Nair (1938) and Bishop & Nair (1939) showed that the criterion is still not always adequate if some of the degrees of freedom are 1, 2 or 3. The 5% and 1% points of Bartlett's criterion, are given in Table 32 by Pearson & Hartley (1970) permitting degrees of freedom as low as 2. Welch (1935, 1936) generalized the idea of Neyman & Pearson (1931) to cover the case when residuals from a fitted regression equation are tested for homoscedascity. Abedi (1974) found that under normal conditions for reasonable size of samples Bartlett's test exercises very good control over Type–I error rates at the three nominal levels (i.e. 0.10, 0.05, and 0.01). Under such conditions the test also demonstrated very high power. He discussed six other tests and concluded that if the normality of the distribution is assumed, the Bartlett's test is the best choice for testing homogeneity of variances. The further properties of the Bartlett's test are studied by Nair (1938), Bishop & Nair (1939), Brown (1939) and Pitman (1939); and agreed that it is unbiased in the sence defined by Neyman & Pearson (1936, 1938). The further refinements were made by Hartley (1940) and Box (1949). The F–test is also available when only the equality of two population variances is to be tested. Placket (1960, Chapter 5), Tiku (1964, p.93), Kendall & Stuart ((1967, p.465–69); (1968, p.97–103)), and Tiku (1982, p. 2550) are worth seeing.

Cochran (1941) introduced a statistic for quick assessment, which is best in the situation when just one of the several populations is suspected to have a larger variance. Darling (1952) contributed some additional results concerning Cochran's statistic. Hartley (1950a) derived a statistic which compares the largest and the smallest of the sample variances under consideration. David (1952) have given some accurate percentage points for this statistic. Cadwell (1953) introduced a statistic by comparing the largest and the smallest sample ranges. The percentage points of the test statistic are obtained by Leslie & Brown (1966). Hartley (1950b) compared the power of some of these tests empirically. Gratiside (1972) compared the size as well.

The testing criteria discussed above, for the equality of k variances are derived under the assumption of normality for the random variables. A desirable characteristic of a test is that the significance level and the power of the test should be insensitive or "robust" to departures from this assumption since many random variables are not normally distributed. This was realized by Pearson & Adyanthaya (1929), and Pearson (1932). There are various studies "on robustness" on the effects of non-normality upon univariate normal–theory procedures. Preason (1931) pointed out the sensitivity to non–normality of the tests for comparing two variances.

Geary (1947), Finch (1950), and Gayen (1950) confirmed the findings of Pearson (1931). These authors agreed that this test is particularly sensitive to changes of kurtosis co–efficient from the normal–theory value of zero. Box (1953) showed that this sensitivity is even greater when the number of variances to be compared exceeds two. In general terms, the main findings is that the tests are non–robust. Gayen (1950) and Tiku (1964, 1975) empirically suggested that the F–test is affected seriously by the departures from normality. It is not at all robust even asymptotically; its Type I error is very sensitive particular to symmetric distributions – (ef. Conover 1980, p.247). These results confirm the early findings by Box (1953), Geary (1947) and Pearson (1932)among others. A good discussion was produced by Kendall & Stuart (1979, Vol.II, Chapter 31). Thompson (1937), Wilks (1937, 1946), Thompson & Merington (1946), Pearson (1966), Nagao (1970, 1972) and Tan (1982) are relevant references.

The one way analysis of variance can be used to test the hypothesis of equal variances. This test of course enjoys the well known robustness of the F–test in a fixed effect model – cf. Scheffe (1959). Bartlett & Kendall (1946), and Plackett (1947) compared the homogeneity of variances of k populations by using analysis of variance.

3. EQUALITY OF K VARIANCES UNDER NON–NORMALITY

Suppose $X_{ie}\,; e = 1,2, \dots n_i\,; i = 1,2, \dots$ k is a random sample of size n_i from k non–normal populations. The hypotheses of interest are:

$$H_{O2}: \sigma_1^2 = \dots = \sigma_k^2 \text{ vs } H_{A2}: \sigma_1^2 \neq \dots \neq \sigma_K^2 .$$

Miller (1964, 1968) used jackknife test for the equality of two variances. Brillinger (1966), McGrathy (1966) and Mellor (1973) have suggested the application of jackknife with complex sample design. Extensive discussion of jackknife method is given in Gray and Schucany (1972), Efron (1982) and Wolter (1985). Box (1953) introduced the grouping test. The grouping test is similar to the method of random groups used by Wolter (1985). The only difference is that, after drawing random samples without replacement from finite population the grouping test procedure divide the parent sample into a number of groups at random, while Wolter (1985) preferred to draw random groups from the parent samples by random sampling without replacement. These methods are suffering from the nonuniqueness of the results and the loss of power caused by subdividing the samples – cf. Miller, 1968. Tiku and Balakrishnan (1984) have given a test for k=2. The test is based on symmetrically censored samples along the lines of Tiku (1982) and showed that this test has good robustness properties particularly for symmetric distribution, and is considerably powerful. Pervaiz (1990a) has given X_{SE}, X_G and X_J tests (k=2) for finite populations under cluster sampling and stratified cluster sampling designs. Pervaiz (1990b) modified these tests when finite populations cut across the clusters, i.e. the clusters consist of units from both finite populations. Levene (1960)proposed a statistic for equal sample sizes which was subsequently generalized to unequal sample sizes – cf. Draper & Hunter (1969). Gartside (1972), Layard (1973), and Fliger & Killeen (1976) proposed several statistics for the problem. Brown & Forsythe (1974) and Conover et al. (1981) studied the problem as well.

4. EQUALITY OF K–COVARIANCE MATHRICES UNDER NORMALITY

Consider k multivariate normal populations with $\underline{X}_i \xrightarrow{dist} N_P(\underline{\mu}_i, \underline{\Sigma}_i)$ Suppose $X_{ie} = (X_{ie1}, X_{ie2}, \ldots\ldots, X_{iep})^T$; $e = 1,2,\ldots,n_i$ is a random sample of size n_i form $\underline{X}_i$, $i = 1,2,\ldots,k$. The hypotheses of interest are:

$$H_{o3} : \underline{\Sigma}_1 = \ldots = \underline{\Sigma}_K \quad vs \quad H_{A3} : \underline{\Sigma}_1 \neq \ldots \neq \underline{\Sigma}_K$$

The test for the equality of k covariance matrices under normality was derived by Wilks (1932), using the likelihood ratio statistic, approximately null distributed as chi-square with (k–1)p (p+1)/2 degrees of freedom. A modification is given by Box (1949), which is a generalization of the Bartlett's (1954) test "for homogeneity of variances". Korin (1969), has proposed Tables of the upper 5% critical values of the Box (1949) criterion for the case of equal sample sizes. These have been reproduced by Pearson (1969). Hopkins and Clay

(1963), Ito (1969), Layard (1972, 1974), Mardia (1974) and Davis (1980) studied the effect of non–normality on the test and found it to be sensitive to non–normality. Ito (1969) and Mardia (1974) proved that these normal theory tests are affected by the kurtosis co–efficient of the parent distribution. Khatri & Srivastava (1971) and Nagao (1972) obtained the non–null distribution of the likelihook ratio test. Brown (1939), Pitman (1939), Ramachandran (1958), Sugiura & Nagao (1968, 1969), Cohen & Strawderman (1971), Carter & Srivastava (1977), srivastava, Khatri & Carter (1978), and Perlman (1980) have discussed unbiasedness of the test. Muirhead (1982) discussed the central/non–central moments of the test. He obtained the asymptotic null and non–null distribution of the test as well.

Giri (1973) and Hsieh (1979) extended the test for multivariate given by Sukhatme (1935). Bishop (1939), Anderson (1958), Mardia (1971), Gupta & Jain (1973) and Lee et al. (1977) are also worth seeing. There are a few other tests for k=2 proposed by Pillai (1955), Bagai (1962), and Pillai & Jayachandran (1967, 1968) which are based on the covariance matrices, but these are also non robust. These are named as:

(1) Roy's largest root (cf. Roy, 1945)
(2) Hotelling's trace (cf. Pillai, 1955)
(3) Pillai's trace (cf. Pillai, 1955)
(4) Wilk's criterion (cf. Wilks, 1932)

Pillai (1960) produced perentage points for the criterion given by Pillai (1955). Pillai & Sudjana (1975) have obtained the distribution of the tests. They used these distributions to look at the robustness of the tests and found that the tests are not robust against non–normality. Subrahmaniam (1975), obtained the non–null distributions of the four statistics. Das Gupta & Giri (1973) studied the unbiasedness and monotonicity property of the power functions of a class of tests for the equality of covariance matrices. Sugiura & Nagao (1968), Das Gupta (1969), and Anderson & Das Gupta (1964) has provided some ground for Das Gupta & Giri (1973). Pillai and Jayachandran (1968) studied the power function of four test criteria for testing the equality of two covriance matrices. Chattopadhyay (1977) extended this study that allows for departures from the null hypothesis even in the form of the parent distribution. Chu & Pillai (1979) made a power comparison of above four criteria as well

(1) Roy's largest–smallest roots (cf. Roy, 1953)

(2) Modified likelihood ratio (cf. Anderson, 1958 and Bartlett, 1937).

Nagao (1974) obtained the non-null distributions of two test criteria for equality of covariance matrices under local alternatives. Tukey & Wilks (1946), Nagao (1967), Giri (1968), Khatri & Pillai (1968), Davis (1971), Nishida (1972), Khatri & Srivastava (1971), Pillai & Nagarsenker (1972), Greenstreet & Connor (1974), Nagao (1973, 1974), Chang, Krishnaiah & Lee (1977), Krishnaiah & Lee (1979) and Perllman (1980) are excellent contributions.

Pillai & Young (1969) proposed max. U–ratio test. Pillai & Young (1974) investigated the max. U–ratio test, studied its exact distribution and tabulated its percentage points. The non–null distribution of the test criterian is found as well.

5. EQUALITY OF TWO COVARIANCE MATRICES UNDER NON–NORMALITY

Layard (1974) defined the null hypothesis under possible non–normality. Suppose two independent samples of size n_1 and n_2 from populations with cdf's F and G, covariance matrices $\underline{\Sigma}_1$ *and* $\underline{\Sigma}_2$ and finite fourth moments. The problem is to test:

$$H_{o4} : F(x_1, ..., x_p) = G(x_1 + \xi_1, ..., x_p + \xi_p) \quad vs$$

$$H_{A4} : \underline{\Sigma}_1 \neq \underline{\Sigma}_2$$

Where $\xi_1, ..., \xi_p$ are unspecified constants.

Layard (1972) described and Layard (1974) empirically evaluated standard error, grouping and jackknife tests which are asymptotically robust. Under the saymmetrical distributions with finite means and variances Tiku and Balakrishnan (1985) suggested T^2 and T_R^2 test statistics for equality of two covariance matrices. The T^2 test is the modification of the test for equality of mean vectors by Tiku and Singh

(1982). The T_R^2 test is the multivariate generalization of the "robust" test for testing the equality of variances by Tiku and Balakrishnan (1984). Muirhead & Waternaux (1980), Muirhead (1982) proposed a test for elliptical distributions. Browne (1984) is a relevant reference. Pervaiz and Skinner (1988) has given one and two moments approximation of likelihood ratio test under elliptical distributions, i.e. X^2_{OMALR} and X^2_{TMALR} tests. Pervaiz (1989a) evaluated the tests empirically for bivariate populations and found that the tests are asymptotically robust but fairly large samples are needed to achieve the true nominal levels. On the basis of samples from multivariate populations Pervaiz (1989b) concluded that the tests are affected by the increase in number of variables and higher size of samples are needed for better sizes for higher dimensional distributions.

6. EQUALITY OF TWO COVARIANCE MATRICES FOR FINITE POPULATIONS

The finite populations with covariance matrices, $C_i's$ may be assumed to be random samples from infinite populations called superpopulations with covariance matrices $\underline{\Sigma}_i s$. It may be assumed that $\underline{C}_i$ converge to $\underline{\Sigma}_i$ as the population size go to infinity. Thus the hypotheses of interest are:

$$H_{o5}: \underline{\Sigma}_1 = \underline{\Sigma}_2 \quad vs \quad H_{A5}: \underline{\Sigma}_1 \neq \underline{\Sigma}_2$$

Pervaiz (1988a) has given the asymptotically robust tests, standard error, grouping and jackknife under cluster and stratified cluster sampling designs, when finite populations consist on separate clusters. Pervaiz (1988b) modified these tests when the finite populations cut across the clusters.

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