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A NOTE ON SAMPLE MEDIANS AND QUASIMEDIANS OF LOGISTIC DISTRIBUTION

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1. INTRODUCTION

Birbaum and Dudman [1] obtain the moments of sample medians and quasimedians but for the reasons as indicated by Memon and Daghel [2] their computations cannot be numerically made. Memon and Daghel consider the moments of

$$Z = aY_i + bY_j, \qquad 1 \leq i < j \leq n \qquad (1.1)$$

Where $Y_1, Y_2, \ldots, Y_n$ are the order statistics based on a random sample of n observations from the logistic distribution defined in its reduced form by the equation

$$f(x) = \frac{e^{-x}}{(1+e^{-x})^2}, \qquad -\infty < x < \infty \qquad (1.2)$$

which has the mean zero and variance $\frac{\Pi^2}{3}$. Giving relevant values to the constants a and b, they derive the first four moments of order statistics, sample medians and quasimedians. These moments are expressed in terms of $\psi^i(x)$ functions used by shah [3], the evaluation of which is not difficult by a computer.

This paper uses the above mentioned expressions to enumerate the coefficients of kurtosis for the distributions of medians and quasimedians based on sample sizes n = 1, 2,....., 44, Information so computed is useful in comparing these distributions and assessing their closeness to normal distribution. For instance we discover that a quasimedian generally performs better for normal approximation.

These tables afford a clear picture regarding closeness of sample statistics under consideration to a normal random variable with mean zero and variance unity. Although for practical applications, it may be left for the reader to make particular decisions, we may make the following conclusion of general interest.

i) As n advances over odd (or even) numbers there occurs a gradual improvement in the normal approximation of median and each quasimedian.

ii) For the same sample size n, in general, the γ_2 value undergoes a steady fall and then a rise as we proceed across the quasimedians; the highest value being at the last quasimedian. The minimum value of γ_2 occurs for the $[x]^{th}$ quasimedian; [x] being the largest integer not larger than x, where $x = \frac{1}{6}(n+1)$ for $n \geq 5$, the minimum occurs at the first quasimedian otherwise. It is thus clear that for $n \geq 5$ this quasimedian provides a better normal approximation as compared to median.

REFERENCES

[1] **BRINBAUM, ALLEN AND DUDMAN, JACK (1963):** LOGISTIC ORDER STATISTICS. ANN. MATH. STATIST. *34, 658-663.*

[2] **MEMON, A.Z. AND DAGHEL, MOHAMMED (1995).** JOURNAL OF STATISTICS, GOVERNMENT COLLEGE, LAHORE *VOL 3 NO. 1 37-51.*

[3] **SHAH, B.K. (1970) :** NOTE ON MOMENTS OF A LOGISTIC ORDER STATISTICS. ANN. MATH. STATIST. *41, 2150-2152.*

TABLE NO. 1

Median		Qiaso-Median No																				
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
1	1.200																					
3	0.594	0.593																				
5	0.381	0.287	0.737																			
7	0.279	0.212	0.274	0.841																		
9	0.219	0.173	0.173	0.304	0.910																	
11	0.180	0.148	0.136	0.173	0.336	0.957																
13	0.153	0.129	0.115	0.125	0.183	0.365	0.991															
15	0.133	0.114	0.102	0.102	0.125	0.197	0.388	1.017														
17	0.117	0.103	0.092	0.088	0.098	0.130	0.210	0.408	1.037													
19	0.105	0.093	0.084	0.079	0.082	0.098	0.137	0.222	0.424	1.054												
21	0.095	0.085	0.077	0.072	0.072	0.080	0.101	0.144	0.233	0.438	1.067											
23	0.087	0.079	0.072	0.067	0.065	0.069	0.080	0.105	0.151	0.243	0.450	1.078										
25	0.080	0.073	0.067	0.062	0.060	0.061	0.068	0.082	0.109	0.158	0.251	0.460	1.087									
27	0.074	0.068	0.063	0.058	0.056	0.056	0.060	0.068	0.085	0.113	0.164	0.259	0.469	1.095								
29	0.069	0.064	0.059	0.055	0.053	0.052	0.054	0.059	0.069	0.087	0.118	0.170	0.266	0.477	1.102							
31	0.064	0.060	0.056	0.052	0.050	0.048	0.049	0.052	0.059	0.071	0.090	0.122	0.175	0.272	0.484	1.109						
33	0.061	0.057	0.053	0.050	0.047	0.046	0.046	0.048	0.052	0.060	0.073	0.093	0.126	0.180	0.278	0.490	0.114					
35	0.057	0.054	0.050	0.047	0.045	0.043	0.043	0.044	0.047	0.052	0.061	0.075	0.096	0.129	0.184	0.283	0.495	1.119				
37	0.054	0.051	0.048	0.045	0.043	0.041	0.041	0.041	0.043	0.047	0.053	0.062	0.077	0.099	0.133	0.188	0.288	0.500	1.123			
39	0.051	0.048	0.046	0.043	0.041	0.040	0.039	0.039	0.040	0.042	0.047	0.054	0.064	0.079	0.102	0.136	0.192	0.292	0.505	1.127		
41	0.049	0.046	0.044	0.041	0.039	0.038	0.037	0.037	0.037	0.039	0.042	0.047	0.055	0.066	0.081	0.104	0.139	0.196	0.296	0.509	1.130	
43	0.047	0.044	0.042	0.040	0.038	0.036	0.035	0.035	0.035	0.036	0.039	0.042	0.048	0.056	0.067	0.083	0.107	0.142	0.199	0.299	0.513	1.134

TABLE NO. 2

	Median	Qiaso-Median No																				
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
2	0.596																					
4	0.364	0.666																				
6	0.269	0.269	0.795																			
8	0.213	0.185	0.288	0.879																		
10	0.177	0.149	0.171	0.320	0.935																	
12	0.151	0.128	0.128	0.177	0.351	0.975																
14	0.131	0.113	0.107	0.124	0.190	0.377	1.005															
16	0.116	0.101	0.094	0.099	0.127	0.203	0.398	1.028														
18	0.104	0.092	0.084	0.084	0.097	0.133	0.216	0.416	1.046													
20	0.094	0.084	0.077	0.075	0.081	0.099	0.140	0.228	0.431	1.061												
22	0.086	0.078	0.071	0.068	0.070	0.080	0.103	0.148	0.238	0.444	1.073											
24	0.079	0.072	0.067	0.063	0.063	0.068	0.081	0.107	0.154	0.247	0.455	1.083										
26	0.074	0.068	0.062	0.059	0.058	0.060	0.068	0.083	0.111	0.161	0.255	0.465	1.092									
28	0.069	0.063	0.059	0.055	0.054	0.055	0.059	0.069	0.086	0.115	0.167	0.263	0.473	1.099								
30	0.064	0.060	0.055	0.052	0.050	0.050	0.053	0.059	0.070	0.089	0.120	0.172	0.269	0.480	1.106							
32	0.060	0.056	0.053	0.049	0.047	0.047	0.048	0.052	0.060	0.072	0.092	0.124	0.177	0.275	0.487	1.111						
34	0.057	0.053	0.050	0.047	0.045	0.044	0.045	0.047	0.052	0.061	0.074	0.095	0.128	0.182	0.281	0.493	1.116					
36	0.054	0.051	0.048	0.045	0.043	0.042	0.042	0.043	0.047	0.053	0.062	0.076	0.098	0.131	0.186	0.286	0.498	1.121				
38	0.051	0.048	0.045	0.043	0.041	0.040	0.040	0.040	0.043	0.047	0.053	0.063	0.078	0.100	0.135	0.190	0.290	0.503	1.125			
40	0.049	0.046	0.044	0.041	0.039	0.038	0.038	0.038	0.039	0.042	0.047	0.054	0.065	0.080	0.103	0.138	0.194	0.294	0.507	1.129		
42	0.046	0.044	0.042	0.040	0.038	0.037	0.036	0.036	0.037	0.039	0.042	0.048	0.055	0.066	0.082	0.106	0.141	0.197	0.298	0.511	1.132	
44	0.044	0.042	0.040	0.038	0.036	0.035	0.034	0.034	0.035	0.036	0.039	0.042	0.048	0.056	0.068	0.084	0.108	0.144	0.200	0.301	0.514	1.135