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Catalogue of Efficient Minimal Circular Generalized RMDs in Periods of Two Different Sizes

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Abstract

Repeated measurements design (RMDs) is economical, therefore, often used in several areas like, psychology, medicine, animal sciences, and pharmacology. In RMDs carry over effects arise which become the source of bias to estimate the treatment effects. Minimal strongly balanced RMDs and Generalized strongly balanced RMDs are used to control the carry over effects and to estimate the direct effects and carry over effects. Catalogues of the designs are always useful for the experimenters and practitioners because these provide them the readymade solution. Catalogue of efficient minimal circular generalized strongly balanced RMDs for $v = ip_1+2p_2-2$, i odd, p_1 odd and p_2 integer is not available in the literature. In this article, a catalogue of these efficient designs is presented for $v \leq 99$, $5 \leq p_1$ (odd) ≤ 11, $3 \leq p_2 \leq 10$.

Keywords

Residual effects, Cyclic shifts, Minimal design, Efficiency of separability.

1. Introduction

The designs in which two or more treatments are applied in an equal time interval on the same experimental unit (subject), in a specific sequence is known as repeated measurements design (RMD). RMDs are cost-effective, therefore, often used in several areas like psychology, medicine, animal sciences, and pharmacology. But major disadvantage of RMDs is that the carry over effects may arise which may become major source of bias to estimate treatment effects, where effect which lasts over up to the next period is known as carryover effect. BRMDs and SBRMDs handle carry over effects at design stage. BRMDs control the carry over effects while SBRMDs control the carry over effects as well as estimate the direct effects and carry over effects independently. RMD is said to be BRMD if every treatment is immediately preceded λ' times by each other treatment, excluding itself. RMD is said to be SBRMD if every treatment is immediately preceded λ' times by each other treatment, including itself. In other words, pairs of same treatment (0,0), (1,1), …, (v-1, v-1) do not appear in BRMDs while appear in SBRMDs. In a circular SBRMD, if $\lambda' = 1$ then it is MCSBRMD. MCSBRMDs can easily be constructed through method of cyclic shifts (Rule I) for v odd. Rule II produces the MCNSBRMDs for some of the remaining cases. MCNSBRMDs are the designs in which

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each treatment is immediately preceded once by all other treatments (including itself) except the treatment labelled as v-1 which is not preceded with itself. For remaining cases of v odd, either MCSPBRMDs or MCGSBRMDs are constructed through Rule II. MCSPBRMDs are designs in which (i) some ordered pairs of treatments do not appear as their preceded values while the remaining pairs appear once and (ii) pairs (0,0), (1,1), …, (v-1, v-1) appear once. MCGSBRMDs are designs in which (i) some ordered pairs of treatments appear twice as their preceded values while the remaining pairs appear once and (ii) pairs (0,0), (1,1), …, (v-1, v-1) appear once.

Catalogues of the designs are always useful for the experimenters and practitioners because these provide them the readymade solutions. Catalogue of efficient MCGSBRMDs for $v = ip_1+2p_2-2$, i odd, p_1 odd and p_2 integer is not available in the literature. The presentation of such a catalogue will be a novel wok to control the carry over effects. Considering its novelty, therefore, a catalogue of efficient MCGSBRMDs for $v = ip_1+2p_2-2$, $5 \leq p_1$ (odd) ≤ 11, $3 \leq p_2 \leq 10$ with i odd, $v \leq 99$ and $p_1 > p_2$ is presented in the periods of two different sizes. These efficient designs are generated through i sets of shifts for p_1 and two for p_2.

2. Method of cyclic shifts

Method of cyclic shifts introduced by Iqbal (1991) is explained here for the construction of MCGSBRMDs through Rule II. Let q_{ji} and $q_{(i+1)u}$ be values of set(s) of shifts, where $j = 1, 2, \ldots, i$, $i = 1, 2, \ldots, p$-1 and $u = 1, 2, \ldots, p$-2. If $0 \leq q_{ji}, q_{(i+1)u} \leq v$-2 and each of 0, 1, 2, …, v-2 appears exactly once in S* for v odd, except (v+1)/2 which appear twice then it will be a MCGSBRMD, where S* contains (i) elements of S_j and $S_{(i+1)}$, and (ii) (v-1)-[sum of elements (mod (v-1)) each of S_j].

Under this logic, procedure can be simplified as: For $v = ip_1+2p_2-2$, Divide **A** = [0,1,2,…, v-2, (v+1)/2] into i (odd) groups of size p_1 (odd) and one of size p_2 such that their sum is divisible by (v-1). Delete anyone value from these (i+1) groups, last group containing p_2-2 values will remain same. These resultant (i+2) sets produce MCGSBRMD for v (odd) = ip_1+2p_2-2.

2.1 Example

Consider [0, 1, 2, 3, 4, 5, 6, 7, 5] for $v = 9$, $p_1 = 5$ and $p_2 = 3$.
Group-I: (2, 4, 5, 6, 7), Group-II: (0, 3, 5), Group-III: (1)
Hence S_1 = [2, 4, 5, 6], S_2 = [3, 5], S_3 = [1]t produce following MCGSBRMD in Table 1.

In Table 2, take v more subjects for S_2 = [2, 10, 9]. Get the design in the similar way as taken through S_1.

Table 1: Array developed from S_1 = [2, 4, 5, 6].

Subjects							
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
0	1	2	3	4	5	6	7
2	3	4	5	6	7	0	1
6	7	0	1	2	3	4	5
3	4	5	6	7	0	1	2
1	2	3	4	5	6	7	0

Table 2: Array developed from S_2 = [3, 5].

Subjects							
9	**10**	**11**	**12**	**13**	**14**	**15**	**16**
0	1	2	3	4	5	6	7
3	4	5	6	7	0	1	2
0	1	2	3	4	5	6	7

Tables 1, 2 and 3 jointly produce MCGSBRMD in $p_1 = 5$, $p_2 = 3$ for $v = 9$, through Rule II, using 24 experimental subjects.

3. Efficiency of separability

Following is the modification of Divecha and Gondaliya (2014) for the efficiency of Separability (ES) for CGSBRMDs and CNSBRMDs. Design will be efficient to control the carry over effects if it possesses the high value of ES.

$ES = \left[\frac{v\sqrt{v-1}-1}{v\sqrt{v-1}}\right] \times 100\%$, where, v is the number of treatments.

4. Catalogue of MCGSBRMDs in two different period sizes with two sets for p_2

Table 3: MCGSBRMDs for $v = ip_1+2p_2-2,\ 5 \leq p_1\ (odd) \leq 11,\ 3 \leq p_2 \leq 10$ with i odd, $v \leq 99$ and $p_1 > p_2$.

v	p_1	p_2	Sets of shifts	Es
9	5	3	[7, 2, 5, 4] + [5, 3] + [1]t	0.751
19	5	3	[17, 10, 2, 12] + [5, 6, 7, 8] + [9, 16, 4, 14] + [3, 15] + [1]t	0.852
29	5		[24, 27, 3, 4] + [6, 7, 8, 12] + [10, 11, 22, 25] + [20, 15, 14, 17] + [2, 21, 9, 5] + [13, 15] + [1]t	0.888
39	5	3	[34, 2, 3, 4] + [5, 6, 7, 8] + [10, 11, 13, 14] + [15, 16, 17, 35] + [37, 21, 32, 26] + [25, 23, 27, 9] + [24, 19, 22, 20] + [18, 20] + [1]t	0.907
49	5	3	[44, 2, 3, 4] + [6, 7, 8, 9] + [46, 11, 12, 13] + [15, 16, 17, 29] + [32, 21, 22, 45] + [36, 25, 26, 27] + [42, 28, 40, 47] + [34, 33, 31, 5] + [39, 38, 37, 10] + [23, 25] + [1]t	0.910

v	p_1	p_2	Sets of shifts	Es
59	5	3	[54, 2, 3, 4] + [5, 6, 7, 8] + [10, 9, 12, 13] + [15, 16, 17, 18] + [20, 21, 22, 23] + [37, 26, 27, 55] + [56, 43, 31, 11] + [34, 35, 42, 25] + [39, 40, 44, 52] + [41, 45, 51, 47] + [49, 19, 46, 36] + [28, 30] + [1]t	0.936
69	5	3	[64, 2, 3, 4] + [6, 7, 8, 9] + [18, 11, 12, 13] + [41, 16, 17, 43] + [46, 21, 22, 23] + [25, 26, 27, 28] + [42, 31, 32, 65] + [66, 60, 36, 37] + [47, 35, 15, 29] + [44, 45, 20, 39] + [49, 67, 51, 52] + [54, 55, 48, 57] + [59, 40, 61, 62] + [33, 35] + [1]t	0.935
79	5	3	[74, 2, 3, 4] + [5, 6, 7, 8] + [28, 11, 12, 13] + [15, 16, 10, 18] + [66, 21, 22, 23] + [75, 48, 33, 49] + [77, 55, 35, 30] + [32, 26, 34, 25] + [68, 40, 41, 42] + [56, 45, 46, 51] + [17, 50, 47, 67] + [71, 58, 20, 54] + [59, 65, 61, 64] + [62, 9, 27, 60] + [69, 70, 57, 72] + [38, 40] + [1]t	0.937
89	5	3	[87, 2, 3, 4] + [58, 6, 7, 8] + [38, 11, 12, 13] + [15, 16, 17, 18] + [20, 21, 19, 23] + [25, 26, 27, 28] + [46, 31, 32, 33] + [35, 36, 37, 29] + [52, 41, 42, 85] + [86, 45, 30, 55] + [49, 50, 51, 61] + [64, 47, 82, 57] + [59, 68, 79, 84] + [83, 65, 66, 67] + [69, 77, 60, 72] + [63, 75, 76, 40] + [78, 73, 81, 56] + [43, 45] + [1]t	0.942
99	5	3	[90, 97, 2, 3] + [68, 6, 7, 8] + [10, 11, 12, 13] + [15, 16, 17, 18] + [84, 20, 22, 24] + [25, 26, 27, 89] + [35, 73, 19, 33] + [96, 63, 45, 40] + [38, 41, 28, 42] + [44, 39, 34, 30] + [95, 37, 53, 55] + [51, 67, 56, 61] + [23, 83, 60, 62] + [91, 65, 64, 69] + [14, 58, 76, 72] + [86, 78, 71, 77] + [79, 31, 81, 82] + [57, 85, 75, 87] + [43, 94, 70, 92] + [48, 50] + [1]t	0.946
11	7	3	[6, 9, 7, 3, 4, 5] + [2, 8] + [1]t	0.837
25	7	3	[20, 23, 2, 3, 4, 5] + [14, 8, 9, 10, 21, 12] + [13, 7, 6, 16, 17, 18] + [11, 13] + [1]t	0.886
39	7	3	[34, 2, 3, 4, 5, 6] + [7, 8, 9, 10, 11, 12] + [14, 15, 16, 23, 35, 13] + [20, 21, 37, 33, 24, 25] + [27, 28, 29, 26, 31, 32] + [18, 20] + [1]t	0.887
53	7	3	[48, 2, 3, 4, 5, 6] + [8, 9, 10, 11, 12, 13] + [47, 15, 16, 17, 18, 19] + [21, 22, 23, 20, 46, 26] + [49, 45, 29, 40, 31, 32] + [33, 35, 51, 37, 38, 39] + [7, 42, 43, 44, 28, 30] + [25, 27] + [1]t	0.927
29	5		[24, 27, 3, 4] + [6, 7, 8, 12] + [10, 11, 22, 25] + [20, 15, 14, 17] + [2, 21, 9, 5] + [13, 15] + [1]t	0.888
39	5	3	[34, 2, 3, 4] + [5, 6, 7, 8] + [10, 11, 13, 14] + [15, 16, 17, 35] + [37, 21, 32, 26] + [25, 23, 27, 9] + [24, 19, 22, 20] + [18, 20] + [1]t	0.907
49	5	3	[44, 2, 3, 4] + [6, 7, 8, 9] + [46, 11, 12, 13] + [15, 16, 17, 29] + [32, 21, 22, 45] + [36, 25, 26, 27] + [42, 28, 40, 47] + [34, 33, 31, 5] + [39, 38, 37, 10] + [23, 25] + [1]t	0.910
59	5	3	[54, 2, 3, 4] + [5, 6, 7, 8] + [10, 9, 12, 13] + [15, 16, 17, 18] + [20, 21, 22, 23] + [37, 26, 27, 55] + [56, 43, 31, 11] + [34, 35, 42, 25] + [39, 40, 44, 52] + [41, 45, 51, 47] + [49, 19, 46, 36] + [28, 30] + [1]t	0.936

v	p_1	p_2	**Sets of shifts**	**Es**
69	5	3	[64, 2, 3, 4] + [6, 7, 8, 9] + [18, 11, 12, 13] + [41, 16, 17, 43] + [46, 21, 22, 23] + [25, 26, 27, 28] + [42, 31, 32, 65] + [66, 60, 36, 37] + [47, 35, 15, 29] + [44, 45, 20, 39] + [49, 67, 51, 52] + [54, 55, 48, 57] + [59, 40, 61, 62] + [33, 35] + [1]t	0.935
79	5	3	[74, 2, 3, 4] + [5, 6, 7, 8] + [28, 11, 12, 13] + [15, 16, 10, 18] + [66, 21, 22, 23] + [75, 48, 33, 49] + [77, 55, 35, 30] + [32, 26, 34, 25] + [68, 40, 41, 42] + [56, 45, 46, 51] + [17, 50, 47, 67] + [71, 58, 20, 54] + [59, 65, 61, 64] + [62, 9, 27, 60] + [69, 70, 57, 72] + [38, 40] + [1]t	0.937
89	5	3	[87, 2, 3, 4] + [58, 6, 7, 8] + [38, 11, 12, 13] + [15, 16, 17, 18] + [20, 21, 19, 23] + [25, 26, 27, 28] + [46, 31, 32, 33] + [35, 36, 37, 29] + [52, 41, 42, 85] + [86, 45, 30, 55] + [49, 50, 51, 61] + [64, 47, 82, 57] + [59, 68, 79, 84] + [83, 65, 66, 67] + [69, 77, 60, 72] + [63, 75, 76, 40] + [78, 73, 81, 56] + [43, 45] + [1]t	0.942
99	5	3	[90, 97, 2, 3] + [68, 6, 7, 8] + [10, 11, 12, 13] + [15, 16, 17, 18] + [84, 20, 22, 24] + [25, 26, 27, 89] + [35, 73, 19, 33] + [96, 63, 45, 40] + [38, 41, 28, 42] + [44, 39, 34, 30] + [95, 37, 53, 55] + [51, 67, 56, 61] + [23, 83, 60, 62] + [91, 65, 64, 69] + [14, 58, 76, 72] + [86, 78, 71, 77] + [79, 31, 81, 82] + [57, 85, 75, 87] + [43, 94, 70, 92] + [48, 50] + [1]t	0.946
11	7	3	[6, 9, 7, 3, 4, 5] + [2, 8] + [1]t	0.837
25	7	3	[20, 23, 2, 3, 4, 5] + [14, 8, 9, 10, 21, 12] + [13, 7, 6, 16, 17, 18] + [11, 13] + [1]t	0.886
39	7	3	[34, 2, 3, 4, 5, 6] + [7, 8, 9, 10, 11, 12] + [14, 15, 16, 23, 35, 13] + [20, 21, 37, 33, 24, 25] + [27, 28, 29, 26, 31, 32] + [18, 20] + [1]t	0.887
53	7	3	[48, 2, 3, 4, 5, 6] + [8, 9, 10, 11, 12, 13] + [47, 15, 16, 17, 18, 19] + [21, 22, 23, 20, 46, 26] + [49, 45, 29, 40, 31, 32] + [33, 35, 51, 37, 38, 39] + [7, 42, 43, 44, 28, 30] + [25, 27] + [1]t	0.927
29	5		[24, 27, 3, 4] + [6, 7, 8, 12] + [10, 11, 22, 25] + [20, 15, 14, 17] + [2, 21, 9, 5] + [13, 15] + [1]t	0.888
39	5	3	[34, 2, 3, 4] + [5, 6, 7, 8] + [10, 11, 13, 14] + [15, 16, 17, 35] + [37, 21, 32, 26] + [25, 23, 27, 9] + [24, 19, 22, 20] + [18, 20] + [1]t	0.907
67	7	3	[50, 2, 3, 4, 5, 6] + [64, 8, 9, 10, 11, 12] + [14, 15, 16, 17, 31, 19] + [51, 22, 23, 24, 25, 26] + [28, 29, 30, 13, 58, 33] + [34, 35, 36, 37, 38, 39] + [40, 42, 43, 44, 61, 46] + [48, 49, 60, 21, 52, 47] + [55, 56, 57, 63, 59, 65] + [32, 34] + [1]t	0.935
81	7	3	[76, 79, 2, 3, 4, 5] + [7, 8, 9, 20, 11, 12] + [15, 16, 17, 18, 19, 61] + [21, 22, 23, 24, 25, 47] + [28, 29, 30, 31, 32, 33] + [36, 37, 38, 77, 70, 27] + [41, 42, 43, 44, 45, 46] + [10, 49, 50, 51, 52, 53] + [54, 56, 34, 73, 75, 60] + [74, 72, 64, 69, 66, 67] + [65, 40, 6, 63, 58, 62] + [39, 41] + [1]t	0.932

v	p_1	p_2	Sets of shifts	Es
95	7	3	[75, 93, 2, 3, 4, 5] + [7, 8, 9, 10, 11, 12] + [83, 15, 16, 17, 18, 19] + [21, 22, 23, 24, 25, 26] + [82, 40, 30, 31, 32, 33] + [35, 36, 13, 38, 39, 80] + [42, 43, 44, 45, 91, 27] + [48, 49, 69, 51, 52, 53] + [55, 56, 57, 58, 29, 60] + [77, 63, 64, 65, 66, 67] + [81, 89, 71, 72, 74, 87] + [76, 62, 78, 79, 59, 88] + [14, 92, 85, 86, 50, 73] + [46, 48] + [1]t	0.937
13	9	3	[8, 11, 2, 10, 4, 5, 6, 7] + [9, 3] + [1]t	0.860
31	9	3	[26, 29, 2, 3, 4, 5, 6, 7] + [9, 10, 11, 12, 13, 27, 15, 28] + [17, 18, 19, 20, 21, 22, 23, 24] + [14, 16] + [1]t	0.912
49	9	3	[44, 47, 2, 3, 4, 5, 6, 7] + [9, 37, 11, 12, 13, 14, 15, 16] + [18, 19, 20, 21, 22, 45, 24, 46] + [8, 43, 38, 39, 30, 31, 32, 33] + [35, 36, 10, 28, 29, 40, 41, 42] + [23, 25] + [1]t	0.905
67	9	3	[62, 65, 2, 3, 4, 5, 6, 7] + [9, 10, 11, 12, 13, 14, 15, 31] + [18, 19, 20, 21, 22, 23, 24, 25] + [27, 28, 29, 30, 16, 63, 33, 64] + [35, 60, 37, 38, 39, 61, 41, 42] + [8, 45, 46, 47, 48, 49, 50, 51] + [53, 54, 55, 56, 57, 58, 59, 36] + [32, 34] + [1]t	0.931
85	9	3	[80, 83, 2, 3, 4, 5, 6, 7] + [55, 10, 11, 12, 13, 14, 15, 16] + [72, 19, 20, 21, 17, 23, 24, 25] + [79, 28, 29, 30, 26, 32, 33, 34]+ [36, 37, 38, 39, 35, 68, 42, 82] + [78, 70, 59, 47, 48, 49, 50, 51] + [53, 54, 60, 56, 57, 58, 46, 9] + [8, 44, 64, 65, 66, 67, 81, 69] + [71, 18, 73, 74, 75, 76, 77, 63] + [41, 43] + [1]t	0.942
15	11	3	[10, 13, 2, 3, 4, 5, 11, 7, 12, 8] + [6, 8] + [1]t	0.885
37	11	3	[32, 35, 2, 3, 4, 5, 6, 7, 31, 9] + [11, 12, 13, 14, 15, 16, 33, 18, 34, 30] + [21, 22, 23, 24, 25, 26, 27, 28, 29, 19] + [17, 19] + [1]t	0.911
81	11	3	[76, 79, 2, 3, 4, 5, 6, 7, 8, 9] + [32, 12, 13, 14, 15, 16, 17, 18, 19, 20] + [22, 23, 24, 25, 26, 27, 28, 29, 30, 31] + [53, 34, 35, 36, 37, 38, 77, 40, 78, 10] + [43, 44, 45, 46, 47, 48, 49, 50, 51, 62] + [54, 11, 56, 57, 58, 59, 60, 61, 52, 71] + [65, 66, 67, 68, 69, 70, 63, 72, 73, 74] + [39, 41] + [1]t	0.942
11	5	4	[4, 2, 3, 6] + [9, 6, 7] + [0, 1]t	0.847
21	5	4	[19, 2, 17, 4] + [6, 7, 13, 9] + [11, 11, 12, 16] + [15, 8, 3] + [0, 1]t	0.881
31	5	4	[23, 2, 3, 4] + [6, 7, 8, 15] + [11, 12, 13, 14] + [16, 16, 17, 19] +[25, 21, 27, 29] + [20, 26, 9] + [0, 1]t	0.942
41	5	4	[39, 36, 3, 4] + [6, 7, 8, 14] + [11, 12, 33, 9] + [16, 17, 18, 19] + [20, 34, 21, 22] + [37, 35, 31, 27] + [28, 26, 24, 13] + [25, 2, 32] + [0, 1]t	0.925
51	5	4	[42, 49, 2, 3] + [6, 7, 8, 9] + [11, 12, 13, 24] + [16, 17, 18, 19] + [5, 21, 25, 23] + [22, 26, 47, 27] + [15, 32, 31, 33] + [44, 35, 36, 37] + [10, 43, 38, 14] + [29, 46, 41] + [0, 1]t	0.944
61	5	4	[59, 2, 3, 4] + [6, 7, 8, 9] + [11, 12, 13, 14] + [16, 17, 18, 19] + [20, 31, 22, 23] + [42, 26, 27, 28] + [21, 29, 32, 33] + [34, 35, 36, 37] + [40, 41, 25, 43] + [45, 46, 47, 58] +	0.927

v	p_1	p_2	**Sets of shifts**	**Es**
			[49, 39, 51, 48] + [54, 55, 56] + [0, 1]t	
71	5	4	[65, 2, 3, 8] + [4, 5, 6, 7] + [15, 12, 13, 10] + [26, 17, 18, 19] + [50, 21, 22, 23] + [25, 31, 27, 28] + [53, 32, 61, 34] + [35, 36, 36, 37] + [40, 45, 42, 44] + [41, 46, 51, 63] + [59, 14, 47, 52] + [55, 56, 57, 58] + [16, 33, 69, 43] + [67, 11, 68] + [0, 1]t	0.941
81	5	4	[79, 2, 3, 4] + [6, 7, 8, 9] + [11, 12, 13, 34] + [16, 17, 18, 14] + [70, 21, 22, 23] + [25, 26, 27, 28] + [31, 32, 33, 19] + [77, 36, 37, 48] + [41, 41, 75, 43] + [65, 46, 47, 38] + [49, 35, 51, 52] + [78, 71, 56, 57] + [60, 76, 62, 63] + [64, 55, 66, 67] + [69, 39, 30, 29] + [20, 61, 5] + [0, 1]t	0.940
91	5	4	[89, 2, 3, 4] + [6, 7, 8, 9] + [11, 12, 13, 14] + [16, 17, 18, 24] + [19, 21, 22, 33] + [70, 26, 27, 28] + [31, 23, 32, 35] + [30, 36, 37, 38] + [41, 42, 43, 44] + [46, 46, 47, 48] + [49, 65, 51, 52] + [54, 45, 56, 57] + [50, 61, 62, 63] + [64, 5, 66, 67] + [75, 71, 72, 73] + [55, 76, 77, 78] + [79, 87, 81, 88] + [84, 20, 86] + [0,1]t	0.939
13	7	4	[11, 9, 3, 4, 5, 6] + [7, 8, 2] + [0, 1]t	0.901
27	7	4	[24, 25, 2, 3, 4, 5] + [7, 8, 9, 10, 11, 12] + [14, 6, 16, 17, 18, 19] + [13, 22, 23] + [0, 1]t	0.861
41	7	4	[28, 29, 35, 3, 4, 5] + [7, 8, 9, 10, 11, 12] + [15, 6, 27, 18, 19, 21] + [21, 22, 13, 24, 34, 26] + [37, 38, 39, 30, 31, 32] + [2, 25, 36] + [0, 1]t	0.925
55	7	4	[53, 2, 3, 4, 5, 6] + [8, 9, 10, 11, 12, 13] + [15, 16, 17, 7, 19, 20] + [22, 23, 18, 25, 26, 27] + [28, 29, 30, 31, 32, 38] + [51, 36, 37, 33, 39, 40] + [42, 43, 44, 52, 46, 47] + [48, 49, 41] + [0, 1]t	0.929
69	7	4	[50, 2, 3, 4, 5, 6] + [8, 9, 10, 11, 20, 13] + [7, 16, 17, 18, 27, 37] + [22, 23, 24, 25, 34, 19] + [29, 30, 31, 32, 33, 21] + [35, 36, 49, 38, 39, 40] + [42, 43, 44, 45, 46, 59] + [12, 67, 51, 52, 56, 54] + [53, 26, 58, 47, 60, 41] + [63, 64, 15] + [0, 1]t	0.939
83	7	4	[81, 2, 3, 4, 5, 71] + [7, 20, 9, 10, 11, 12] + [15, 16, 17, 18, 23, 61] + [22, 19, 24, 25, 26, 27] + [29, 30, 31, 32, 62, 34] + [54, 37, 38, 39, 40, 41] + [42, 43, 44, 45, 46, 66] + [49, 50, 51, 64, 53, 75] + [56, 57, 58, 59, 60, 65] + [73, 52, 8, 47, 67, 48] + [70, 6, 77, 78, 74, 36] + [72, 63, 35] + [0, 1]t	0.935
97	7	4	[95, 2, 3, 4, 5, 6] + [8, 9, 10, 11, 12, 13] + [15, 16, 17, 18, 19, 20] + [22, 23, 48, 25, 26, 27] + [29, 30, 31, 32, 7, 35] + [36, 34, 38, 42, 40, 41] + [43, 44, 45, 46, 47, 24] + [75, 50, 51, 52, 53, 54] + [56, 93, 58, 59, 68, 61] + [63, 64, 65, 66, 73, 60] + [70, 71, 80, 67, 74, 49] + [94, 78, 79, 86, 81, 82] + [84, 55, 92, 14, 90, 62] + [91, 72, 37] + [0, 1]t	0.946
15	9	4	[8, 13, 2, 3, 4, 5, 6, 7] + [9, 10, 11] + [0, 1]t	0.869
33	9	4	[31, 2, 3, 4, 5, 6, 7, 8] + [10, 11, 12, 13, 14, 15, 16, 17] +	0.891

v	p_1	p_2	Sets of shifts	Es
			[18, 19, 9, 21, 22, 23, 24, 29] + [17, 28, 25] + [0, 1]t	
51	9	4	[49, 2, 3, 4, 5, 6, 7, 36] + [10, 11, 12, 13, 14, 15, 16, 17] + [19, 20, 21, 22, 23, 24, 25, 28] + [27, 8, 29, 30, 31, 32, 33, 34] + [46, 37, 48, 39, 40, 41, 9, 43] + [45, 26, 35] + [0, 1]t	0.932
69	9	4	[67, 2, 3, 4, 5, 6, 7, 53] + [9, 10, 11, 12, 13, 14, 15, 16] + [19, 20, 21, 22, 23, 24, 26, 32] + [28, 29, 30, 31, 25, 33, 34, 35] + [18, 37, 38, 39, 40, 41, 42, 50] + [8, 46, 47, 48, 49, 62, 51, 52] + [64, 55, 56, 66, 58, 59, 60, 61] + [63, 54, 44] + [0, 1]t	0.933
87	9	4	[85, 2, 3, 4, 5, 6, 7, 62] + [9, 11, 12, 13, 14, 15, 16, 17] + [19, 20, 21, 22, 23, 24, 25, 26] + [28, 29, 30, 10, 32, 33, 34, 35]+ [37, 38, 39, 40, 41, 69, 43, 44] + [45, 46, 47, 48, 49, 50, 51, 52]+ [54, 55, 56, 57, 58, 59, 63, 61] + [8, 60, 64, 31, 66, 67, 68, 72] + [82, 83, 74, 75, 76, 77, 18, 36] + [71, 44, 73] + [0, 1]t	0.937
17	11	4	[12, 15, 2, 3, 4, 5, 6, 7, 8, 9] + [11, 14, 13] + [0, 1]t	0.866
39	11	4	[37, 2, 3, 4, 5, 6, 32, 8, 9, 10] + [12, 13, 14, 15, 16, 17, 18, 19, 20, 35] + [22, 23, 24, 25, 26, 27, 33, 29, 30, 31] + [7, 21, 20] + [0, 1]t	0.925
61	11	4	[58, 59, 2, 3, 4, 5, 6, 7, 8, 9] + [12, 13, 14, 15, 16, 17, 18, 23, 20, 21] + [10, 24, 25, 26, 27, 28, 29, 30, 31, 31] + [33, 34, 35, 36, 37, 38, 52, 40, 41, 42] + [44, 45, 46, 47, 48, 56, 50, 51, 57, 53] + [55, 49, 22] + [0, 1]t	0.928
83	11	4	[80, 81, 2, 3, 4, 5, 6, 7, 8, 9] + [12, 13, 14, 15, 16, 17, 18, 19, 20, 27] + [23, 24, 25, 26, 58, 28, 29, 30, 31, 32] + [34, 35, 36, 37, 38, 39, 40, 10, 45, 42] + [44, 42, 46, 47, 48, 49, 50, 51, 52, 66] + [55, 74, 57, 78, 59, 60, 61, 62, 53, 64] + [63, 70, 68, 69, 76, 71, 72, 73, 56, 43] + [65, 21, 11] + [0, 1]t	0.936
15	7	5	[11, 1, 2, 13, 4, 5] + [7, 8, 8, 9] + [0, 12, 3]t	0.815
29	7	5	[25, 1, 2, 3, 4, 5] + [7, 8, 9, 10, 11, 12] + [22, 15, 13, 6, 17, 18] + [20, 19, 26, 23] + [0, 14, 15]t	0.906
43	7	5	[39, 1, 26, 3, 4, 5] + [7, 22, 9, 10, 11, 12] + [14, 15, 16, 17, 18, 19] + [40, 41, 8, 23, 24, 30] + [20, 38, 29, 25, 31, 32] + [34, 33, 36, 37] + [0, 21, 22]t	0.925
57	7	5	[53, 1, 2, 3, 4, 5] + [7, 8, 9, 10, 11, 12] + [14, 15, 16, 17, 18, 19] + [21, 22, 23, 24, 25, 26] + [54, 52, 29, 30, 50,32] + [47, 35, 36, 37, 38, 39] + [41, 42, 49,6, 45, 46] + [43, 31, 34, 20] + [0, 28, 29]t	0.914
71	7	5	[67, 1, 2, 3, 4, 5] + [7, 8, 9, 10, 11, 12] + [14, 36, 16, 17, 18, 19] + [63, 22, 23, 24, 25, 26] + [21, 29, 30, 31, 32, 33] + [68, 69, 59, 37, 38, 39] + [66, 42, 60, 44, 45, 46] + [48, 49, 43, 51, 52, 53] + [55, 56, 57, 6, 15, 50] + [62, 28, 64, 65] + [0, 35, 36]t	0.929
85	7	5	[81, 1, 68, 3, 4, 5] + [21, 8, 9, 10, 11, 12] + [63, 15, 16, 17, 18, 19] + [7, 36, 23, 24, 25, 26] + [28, 29, 65, 31, 32, 33] + [35, 22, 37, 38, 39, 40] +	0.933

v	p_1	p_2	**Sets of shifts**	**Es**
			[82, 83, 43, 44, 45, 46] + [48, 49, 50, 30, 52, 53] + [55, 56, 57, 58, 59, 60] + [62, 14, 64, 51, 76, 67] + [79, 70, 71, 72, 73, 78] + [47, 74, 69, 80] + [0, 42, 43]t	
17	9	5	[13, 1, 11, 3, 4, 5, 6, 7] + [15, 9, 10, 2] + [0, 8, 9]t	0.855
35	9	5	[31, 1, 2, 3, 4, 5, 6, 7] + [8, 10, 11, 12, 13, 14, 15, 21] + [33, 18, 19, 20, 16, 22, 23, 24] + [26, 27, 28, 25] + [0, 17, 18]t	0.891
53	9	5	[49, 1, 2, 3, 4, 5, 6, 7] + [9, 10, 11, 12, 13, 14, 15, 22] + [18, 19, 20, 21, 41, 23, 24, 25] + [51, 8, 28, 29, 30, 31, 16, 33] + [35, 36, 37, 38, 39, 46, 48, 42] + [44, 45, 40, 47] + [0, 26, 27]t	0.943
71	9	5	[67, 1, 2, 3, 4, 5, 6, 7] + [9, 10, 11, 12, 13, 14, 15, 16] + [18, 19, 20, 21, 22, 23, 24, 25] + [65, 28, 29, 30, 31, 32, 33, 34] + [69, 36, 37, 26, 39, 17, 41, 42] + [66, 8, 46, 47, 48, 49, 50, 51] + [27, 54, 52, 56, 57, 64, 59, 60] + [62, 63, 58, 53] + [0, 35, 36]t	0.930
89	9	5	[85, 1, 2, 3, 4, 5, 6, 7] + [9, 69, 11, 12, 13, 14, 15, 16] + [18, 19, 28, 21, 22, 23, 24, 25] + [27, 20, 29, 30, 31, 32, 26, 34] + [36, 37, 76, 39, 40, 41, 42, 43] + [87, 10, 46, 47, 48, 49, 50, 51] + [53, 54, 70, 56, 57, 58, 59, 60] + [62, 33, 64, 65, 66, 67, 71, 45] + [82, 81, 73, 74, 75, 83, 77, 80] + [72, 68, 38, 8] + [0, 44, 45]t	0.936
19	11	5	[15, 1, 2, 3, 4, 5, 6, 7, 14, 16] + [11, 12, 13, 8] + [0, 9, 10]t	0.860
41	11	5	[37, 1, 2, 31, 4, 5, 6, 7, 8, 9] + [11, 12, 13, 14, 15, 16, 17, 18, 3, 20] + [21, 38, 23, 24, 25, 30, 27, 28, 29, 36] + [33, 34, 35, 26] + [0, 22, 19]t	0.897
63	11	5	[56, 1, 2, 3, 4, 5, 6, 7, 8, 9] + [11, 12, 13, 14, 15, 16, 17, 18, 19, 30] + [22, 20, 24, 25, 26, 27, 28, 29, 41, 33] + [61, 60, 53, 32, 36, 37, 38, 39, 49, 43] + [10, 44, 42, 46, 58, 45, 55, 50, 40, 52] + [51, 59, 57, 47] + [0, 31, 32]t	0.921
85	11	5	[81, 1, 2, 3, 4, 5, 6, 7, 8, 41] + [11, 12, 13, 14, 15, 16, 9, 18, 19, 20] + [22, 23, 24, 25, 26, 27, 28, 29, 69, 31] + [33, 34, 35, 36, 37, 38, 39, 70, 17, 82] + [80, 67, 45, 46, 47, 48, 49, 50, 51, 52] + [77, 55, 56, 57, 58, 54, 60, 61, 62, 68] + [65, 66, 78, 63, 79, 40, 71, 72, 73, 74] + [59, 44, 30, 43] + [0, 42, 43]t	0.938
17	7	6	[11, 1, 2, 3, 4, 5] + [15, 8, 9, 10, 13] + [0, 12, 14, 7]t	0.827
31	7	6	[26, 1, 2, 3, 4, 5] + [8, 9, 10, 11, 12, 13] + [15, 16, 16, 17, 18, 24] + [21, 22, 23, 6, 28] + [0, 7, 25, 29]t	0.918
45	7	6	[1, 2, 3, 4, 5, 33] + [7, 8, 9, 10, 11, 12] + [15, 16, 17, 18, 19, 20] + [34, 23, 23, 24, 25, 26] + [28, 29, 30, 37, 32, 6] + [42, 36, 43, 38, 39] + [0, 41, 35, 13]t	0.924

v	p_1	p_2	Sets of shifts	Es
59	7	6	[54, 1, 2, 3, 4, 5] + [7, 14, 9, 10, 11, 12] + [8, 15, 16, 17, 18, 19] + [22, 29, 24, 25, 26, 27] + [48, 30, 30, 31, 32, 33] + [35, 36, 37, 51, 39, 40] + [42, 43, 44, 45, 46, 57] + [49, 50, 38, 34, 41] + [0, 55, 56, 6]t	0.934
73	7	6	[1, 2, 3, 4, 5, 61] + [7, 8, 9, 10, 11, 12] + [13, 16, 17, 18, 19, 20] + [21, 23, 24, 25, 26, 28] + [29, 30, 31, 32, 33, 34] + [36, 37, 37, 38, 39, 66] + [42, 43, 44, 45, 46, 54] + [49, 50, 51, 62, 53, 47] + [65, 57, 58, 59, 60, 6] + [63, 64, 56, 40, 67] + [0, 22, 52, 71]t	0.940
87	7	6	[65, 1, 2, 3, 4, 5] + [8, 9, 10, 11, 12, 13] + [15, 16, 17, 18, 19, 20] + [22, 7, 24, 25, 26, 47] + [29, 30, 31, 32, 33, 34] + [28, 37, 38, 39, 40, 41] + [43, 44, 44, 45, 46, 80] + [36, 50, 51, 52, 53, 54] + [56, 57, 58, 59, 84, 61] + [63, 75, 82, 66, 14, 68] + [70, 60, 83, 73, 85, 64] + [77, 78, 79, 71, 49] + [0, 72, 27, 74]t	0.949
19	9	6	[14, 1, 2, 3, 4, 5, 6, 7] + [9, 10, 11, 17, 15] + [0, 13, 16, 8]t	0.879
37	9	6	[32, 1, 2, 3, 4, 5, 6, 7] + [9, 11, 25, 13, 14, 15, 16, 17] + [19, 19, 20, 21, 22, 23, 30, 8] + [27, 28, 33, 35, 31] + [0, 29, 34, 10]t	0.894
55	9	6	[50, 1, 2, 3, 4, 5, 6, 7] + [10, 11, 12, 13, 14, 15, 16, 45] + [19, 20, 21, 22, 23, 24, 25, 28] + [27, 9, 29, 8, 31, 32, 35, 17] + [36, 37, 38, 39, 49, 41, 42, 43] + [33, 46, 47, 48, 52] + [0, 18, 40, 51]t	0.922
73	9	6	[68, 1, 2, 3, 4, 5, 6, 7] + [36, 10, 11, 12, 13, 14, 15, 16] + [37, 20, 21, 22, 23, 24, 25, 26] + [28, 29, 30, 31, 32, 33, 43, 35] + [19, 37, 38, 39, 40, 41, 42, 34] + [45, 46, 47, 8, 49, 50, 51, 65] + [54, 55, 56, 57, 58, 59, 60, 52] + [63, 64, 61, 66, 44] + [0, 69, 9, 67]t	
91	9	6	[86, 1, 2, 3, 4, 5, 6, 7] + [10, 11, 12, 13, 14, 15, 16, 80] + [19, 20, 21, 22, 23, 24, 44, 79] + [28, 29, 30, 31, 32, 33, 25, 35] + [37, 38, 39, 50, 41, 42, 43, 34] + [46, 46, 47, 48, 49, 40, 51, 52] + [81, 55, 56, 57, 58, 54, 65, 61] + [63, 64, 60, 8, 67, 73, 69, 74] + [72, 68, 70, 75, 76, 82, 78, 26] + [59, 88, 45, 84, 85] + [0, 87, 77, 17]t	0.960
21	11	6	[16, 1, 2, 3, 4, 5, 15, 7, 8, 9] + [11, 12, 13, 14, 19] + [0, 17, 18, 6]t	0.899
43	11	6	[29, 1, 2, 3, 4, 5, 6, 7, 8, 9] + [12, 13, 28, 15, 16, 17, 18, 19, 20, 21] + [22, 23, 24, 25, 26, 27, 14, 38, 36, 37] + [33, 34, 40, 30, 41] + [0, 39, 35, 11]t	0.907

v	p_1	p_2	**Sets of shifts**	**Es**
65	11	6	[60, 1, 2, 3, 4, 5, 6, 7, 21, 9] + [12, 13, 14, 15, 16, 17, 18, 19, 20, 37] + [23, 61, 25, 26, 27, 28, 29, 30, 31, 59] + [33, 34, 35, 36, 8, 38, 39, 40, 41, 47] + [44, 22, 46, 42, 48, 49, 50, 51, 52, 53] + [63, 56, 57, 58, 32] + [0, 24, 62, 43]t	0.938
87	11	6	[82, 1, 2, 3, 4, 5, 6, 7, 8, 9] + [12, 13, 10, 15, 16, 17, 18, 19, 20, 21] + [70, 24, 25, 26, 27, 28, 29, 30, 31, 32] + [34, 35, 36, 23, 38, 39, 54, 53, 42, 43] + [44, 14, 46, 47, 48, 49, 80, 51, 52, 41] + [55, 56, 57, 58, 59, 60, 61, 62, 63, 72] + [66, 67, 68, 69, 81, 71, 64, 73, 74, 76] + [77, 78, 79, 84, 37] + [0, 83, 50, 40]t	0.955
21	9	7	[15, 1, 2, 3, 4, 5, 6, 7] + [10, 11, 11, 12, 13, 14] + [0, 16, 8, 18, 19]t	0.922
39	9	7	[33, 1, 2, 3, 4, 5, 6, 7] + [9, 10, 11, 12, 13, 14, 8, 16] + [18, 19, 20, 20, 17, 22, 23, 24] + [37, 28, 29, 35, 31, 32] + [0, 34, 30, 26, 25]t	0.914
57	9	7	[51, 1, 2, 3, 4, 5, 6, 7] + [9, 10, 11, 12, 8, 14, 15, 16] + [44, 19, 20, 21, 22, 23, 24, 25] + [54, 28, 29, 29, 30, 31, 32, 13]+ [35, 36, 37, 38, 39, 40, 41, 27] + [55, 45, 42, 47, 48, 49] + [0, 52, 53, 46, 18]t	0.922
75	9	7	[70, 1, 2, 3, 4, 5, 6, 7] + [9, 10, 41, 12, 13, 14, 15, 16] + [17, 19, 20, 21, 22, 23, 24, 25] + [43, 28, 29, 30, 31, 32, 33, 34]+ [35, 37, 38, 38, 39, 40, 42, 57] + [27, 67, 46, 47, 48, 49, 26, 8] + [53, 54, 55, 56, 58, 73, 59, 65] + [62, 63, 64, 66, 60, 61] + [0, 69, 71, 72, 11]t	0.946
93	9	7	[87, 1, 2, 3, 4, 5, 6, 7] + [9, 10, 11, 12, 13, 14, 15, 16] + [18, 19, 20, 21, 8, 23, 24, 25] + [91, 28, 29, 30, 31, 32, 33, 34] + [36, 37, 38, 39, 40, 41, 42, 43] + [45, 46, 47, 47, 48, 49, 50, 51] + [53, 54, 55, 56, 70, 58, 59, 86] + [62, 63, 64, 65, 66, 67, 68, 22] + [88, 72, 73, 74, 57, 76, 44, 78] + [80, 81, 79, 83, 17, 85] + [0, 71, 89, 90, 27]t	0.930
23	11	7	[17, 1, 18, 3, 4, 5, 6, 7, 8, 9] + [12, 12, 20, 14, 15, 16] + [0, 2, 19, 13, 11]t	0.928
45	11	7	[39, 1, 40, 3, 4, 5, 6, 7, 8, 9] + [12, 13, 14, 15, 16, 17, 18, 19, 20, 21] + [42, 23, 23, 24, 25, 26, 27, 28, 29, 30] + [41, 34, 35, 36, 37, 38] + [0, 2, 33, 22, 32]t	0.945
67	11	7	[61, 1, 2, 3, 4, 5, 6, 7, 8, 9] + [33, 12, 13, 14, 15, 16, 17, 18, 19, 20] + [60, 23, 24, 25, 10, 27, 28, 29, 41, 31] + [11, 34, 34, 35, 36, 37, 38, 39, 40, 50] + [63, 44, 45, 46, 47, 48, 49, 30, 51, 52] + [64, 55, 56, 57, 62, 59] + [0, 58, 22, 54, 65]t	0.943
89	11	7	[83, 1, 2, 3, 4, 5, 6, 7, 8, 9] + [11, 12, 13, 14, 15, 16, 17, 18, 19, 20] +	0.950

v	p_1	p_2	**Sets of shifts**	**Es**
			[77, 23, 24, 25, 26, 27, 28, 29, 30, 31] + [55, 34, 35, 36, 37, 38, 39, 40, 41, 42] + [80, 45, 45, 46, 47, 10, 49, 50, 51, 52] + [54, 87, 56, 57, 58, 82, 60, 61, 62, 63] + [85, 66, 67, 68, 69, 70, 71, 72, 73, 76] + [74, 22, 78, 79, 44, 84] + [0, 81, 65, 86, 33]t	
23	9	8	[15, 2, 3, 4, 5, 6, 7, 8] + [9, 10, 11, 12, 12, 13, 20] + [0, 17, 18, 19, 14, 21]t	0.903
41	9	8	[34, 1, 2, 3, 4, 5, 6, 7] + [9, 10, 11, 12, 13, 14, 15, 16] + [39, 17, 21, 21, 22, 23, 24, 25] + [27, 28, 29, 30, 31, 32, 37] + [0, 35, 36, 33, 38, 19]t	0.940
59	9	8	[52, 1, 2, 3, 4, 5, 7, 8] + [10, 11, 12, 13, 14, 15, 16, 26] + [19, 20, 21, 22, 23, 24, 25, 43] + [28, 29, 30, 30, 31, 32, 33, 50] + [36, 37, 38, 39, 40, 41, 46, 53] + [45, 42, 47, 48, 49, 6, 56] + [0, 17, 54, 44, 51, 9]t	0.920
77	9	8	[70, 1, 2, 3, 4, 5, 6, 7] + [10, 11, 12, 13, 14, 15, 16, 17] + [19, 50, 21, 22, 23, 24, 25, 26] + [28, 29, 30, 31, 32, 33, 34, 60]+ [37, 38, 39, 39, 65, 41, 42, 43] + [58, 46, 47, 48, 49, 20, 51, 52] + [8, 55, 56, 57, 45, 59, 60, 61] + [73, 74, 40, 66, 67, 68, 69] + [0, 71, 72, 63, 64, 35]t	0.940
95	9	8	[88, 1, 2, 3, 4, 5, 6, 7] + [10, 11, 12, 13, 14, 15, 16, 17] + [9, 20, 21, 22, 23, 24, 25, 26] + [28, 29, 30, 31, 32, 33, 34, 38] + [37, 35, 39, 59, 41, 42, 43, 44] + [46, 47, 48, 48, 49, 84, 51, 52]+ [54, 55, 56, 57, 58, 87, 60, 61] + [63, 64, 65, 66, 67, 68, 69, 40]+ + [8, 73, 50, 75, 53, 77, 78, 79] + [81, 82, 83, 74, 89, 86, 70] + [0, 85, 90, 91, 92, 19]t	0.950
25	11	8	[18, 19, 2, 3, 4, 5, 6, 7, 8, 9] + [11, 12, 13, 14, 20, 16, 21] + [0, 1, 10, 17, 22, 23]t	0.886
47	11	8	[40, 1, 2, 3, 4, 5, 6, 21, 8, 9] + [12, 13, 14, 15, 16, 17, 18, 19, 20, 29] + [23, 24, 25, 26, 24, 27, 37, 7, 30, 31] + [43, 44, 35, 36, 28, 38, 10] + [0, 41, 32, 33, 34, 45]t	0.913
69	11	8	[20, 2, 3, 4, 5, 6, 7, 8, 9, 10] + [12, 13, 14, 15, 16, 17, 18, 19, 48, 21] + [23, 24, 25, 1, 27, 28, 29, 30, 31, 32] + [53, 35, 35, 36, 37, 38, 39, 43, 41, 52] + [44, 45, 46, 47, 26, 49, 50, 54, 42, 33] + [55, 56, 57, 58, 59, 60, 65] + [0, 63, 64, 61, 51, 34]t	0.940
91	11	8	[32, 2, 3, 4, 5, 6, 7, 8, 9, 20] + [12, 13, 14, 15, 16, 17, 18, 19, 24, 21] + [23, 10, 25, 26, 27, 28, 29, 30, 31, 1] + [34, 35, 36, 37, 38, 39, 72, 41, 42, 43] + [45, 46, 46, 47, 48, 49, 50, 51, 52, 62] + [55, 56, 57, 58, 59, 60, 76, 83, 63, 64] + [66, 67, 68, 69, 70, 71, 22, 73, 74, 75] + [86, 78, 79, 80, 81, 85, 53] + [0, 82, 77, 87, 61, 54]t	0.952
27	11	9	[19, 1, 2, 3, 4, 5, 6, 7, 8, 9] + [11, 12, 13, 14, 10, 15, 16, 17] + [0, 20, 21, 18, 23, 24, 25]t	0.897

v	p_1	p_2	**Sets of shifts**	**Es**
49	11	9	[41, 1, 2, 3, 4, 5, 6, 7, 8, 9] + [12, 13, 14, 15, 16, 17, 18, 19, 20, 21] + [23, 24, 25, 25, 26, 11, 28, 29, 30, 31] + [33, 34, 35, 46, 37, 38, 39, 42] + [0, 40, 43, 44, 45, 22, 47]t	0.934
71	11	9	[63, 1, 2, 3, 4, 5, 6, 7, 8, 9] + [12, 13, 14, 15, 16, 17, 18, 19, 20, 21] + [23, 24, 25, 26, 27, 28, 29, 30, 36, 10] + [34, 35, 36, 31, 37, 38, 39, 40, 41, 42] + [44, 11, 46, 33, 48, 49, 60, 51, 52, 53] + [67, 56, 57, 58, 59, 69, 61, 65] + [0, 64, 62, 66, 55, 54, 50]t	0.947
93	11	9	[85, 1, 2, 3, 4, 5, 6, 7, 8, 9] + [12, 13, 14, 15, 16, 17, 18, 19, 28, 21] + [23, 24, 25, 26, 27, 90, 29, 30, 31, 41] + [34, 35, 36, 37, 38, 39, 40, 83, 42, 43] + [45, 46, 47, 47, 48, 49, 50, 51, 52, 53] + [55, 56, 57, 58, 59, 60, 61, 62, 63, 44] + [91, 67, 68, 10, 70, 71, 72, 73, 74, 75] + [77, 78, 79, 80, 81, 82, 86, 84] + [0, 32, 87, 88, 76, 20, 66]t	0.939
29	11	10	[20, 1, 2, 3, 4, 5, 6, 7, 8, 9] + [12, 13, 14, 15, 15, 26, 17, 18, 27] + [0, 21, 22, 23, 24, 25, 16, 10]t	0.888
51	11	10	[42, 1, 2, 3, 4, 5, 6, 7, 8, 9] + [12, 37, 14, 15, 16, 17, 18, 19, 20, 21] + [23, 24, 25, 26, 26, 27, 28, 29, 30, 40] + [33, 34, 35, 36, 10, 38, 39, 49, 44] + [0, 43, 41, 45, 46, 47, 48, 31]t	0.938
47	11	8	[40, 1, 2, 3, 4, 5, 6, 21, 8, 9] + [12, 13, 14, 15, 16, 17, 18, 19, 20, 29] + [23, 24, 25, 26, 24, 27, 37, 7, 30, 31] + [43, 44, 35, 36, 28, 38, 10] + [0, 41, 32, 33, 34, 45]t	0.913
69	11	8	[20, 2, 3, 4, 5, 6, 7, 8, 9, 10] + [12, 13, 14, 15, 16, 17, 18, 19, 48, 21] + [23, 24, 25, 1, 27, 28, 29, 30, 31, 32] + [53, 35, 35, 36, 37, 38, 39, 43, 41, 52] + [44, 45, 46, 47, 26, 49, 50, 54, 42, 33] + [55, 56, 57, 58, 59, 60, 65] + [0, 63, 64, 61, 51, 34]t	0.940
91	11	8	[32, 2, 3, 4, 5, 6, 7, 8, 9, 20] + [12, 13, 14, 15, 16, 17, 18, 19, 24, 21] + [23, 10, 25, 26, 27, 28, 29, 30, 31, 1] + [34, 35, 36, 37, 38, 39, 72, 41, 42, 43] + [45, 46, 46, 47, 48, 49, 50, 51, 52, 62] + [55, 56, 57, 58, 59, 60, 76, 83, 63, 64] + [66, 67, 68, 69, 70, 71, 22, 73, 74, 75] + [86, 78, 79, 80, 81, 85, 53] + [0, 82, 77, 87, 61, 54]t	0.952
27	11	9	[19, 1, 2, 3, 4, 5, 6, 7, 8, 9] + [11, 12, 13, 14, 10, 15, 16, 17] + [0, 20, 21, 18, 23, 24, 25]t	0.897
49	11	9	[41, 1, 2, 3, 4, 5, 6, 7, 8, 9] + [12, 13, 14, 15, 16, 17, 18, 19, 20, 21] +	0.934

v	p_1	p_2	Sets of shifts	Es
			[23, 24, 25, 25, 26, 11, 28, 29, 30, 31] + [33, 34, 35, 46, 37, 38, 39, 42] + [0, 40, 43, 44, 45, 22, 47]t	
71	11	9	[63, 1, 2, 3, 4, 5, 6, 7, 8, 9] + [12, 13, 14, 15, 16, 17, 18, 19, 20, 21] + [23, 24, 25, 26, 27, 28, 29, 30, 36, 10] + [34, 35, 36, 31, 37, 38, 39, 40, 41, 42] + [44, 11, 46, 33, 48, 49, 60, 51, 52, 53] + [67, 56, 57, 58, 59, 69, 61, 65] + [0, 64, 62, 66, 55, 54, 50]t	0.947
93	11	9	[85, 1, 2, 3, 4, 5, 6, 7, 8, 9] + [12, 13, 14, 15, 16, 17, 18, 19, 28, 21] + [23, 24, 25, 26, 27, 90, 29, 30, 31, 41] + [34, 35, 36, 37, 38, 39, 40, 83, 42, 43] + [45, 46, 47, 47, 48, 49, 50, 51, 52, 53] + [55, 56, 57, 58, 59, 60, 61, 62, 63, 44] + [91, 67, 68, 10, 70, 71, 72, 73, 74, 75] + [77, 78, 79, 80, 81, 82, 86, 84] + [0, 32, 87, 88, 76, 20, 66]t	0.939
29	11	10	[20, 1, 2, 3, 4, 5, 6, 7, 8, 9] + [12, 13, 14, 15, 15, 26, 17, 18, 27] + [0, 21, 22, 23, 24, 25, 16, 10]t	0.888
51	11	10	[42, 1, 2, 3, 4, 5, 6, 7, 8, 9] + [12, 37, 14, 15, 16, 17, 18, 19, 20, 21] + [23, 24, 25, 26, 26, 27, 28, 29, 30, 40] + [33, 34, 35, 36, 10, 38, 39, 49, 44] + [0, 43, 41, 45, 46, 47, 48, 31]t	0.938
73	11	10	[64, 1, 2, 3, 4, 5, 6, 7, 8, 9] + [12, 13, 14, 15, 16, 17, 18, 19, 60, 21] + [23, 24, 25, 26, 27, 28, 20, 30, 31, 32] + [34, 55, 36, 37, 37, 38, 39, 40, 41, 42] + [44, 45, 46, 47, 48, 49, 70, 51, 52, 53] + [10, 50, 56, 58, 59, 29, 61, 62, 65] + [0, 63, 66, 67, 68, 69, 57, 43]t	0.950
95	11	10	[86, 2, 3, 4, 5, 6, 7, 8, 9, 10] + [12, 13, 14, 15, 16, 17, 18, 19, 32, 21] + [23, 24, 25, 26, 27, 64, 29, 30, 31, 75] + [34, 35, 36, 37, 38, 39, 40, 41, 42, 43] + [45, 46, 47, 1, 48, 44, 50, 51, 52, 53] + [55, 56, 57, 58, 59, 60, 61, 62, 63, 73] + [66, 76, 68, 69, 70, 71, 72, 28, 74, 93] + [87, 91, 89, 90, 81, 82, 83, 84, 92] + [0, 77, 78, 79, 80, 88, 49, 20]t	0.945

5. Conclusion

Catalogue of efficient MCGSBRMDs for $v = ip_1+2p_2-2$, i odd, p_1 odd and p_2 integer is not available in the literature. Considering the importance of these efficient proposed designs, a catalogue for $v = ip_1+2p_2-2$, $5 \leq p_1$ (odd) ≤ 11, $3 \leq p_2 \leq 10$ with i odd, $v \leq 99$ and $p_1 > p_2$ is presented in two different period sizes which is useful for experimenters and practitioners.

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List of abbreviations:

RMDs	Repeated measurements designs.
BRMDs	Balanced repeated measurements designs.
CBRMDs	Circular balanced repeated measurements designs.
SBRMDs	Strongly balanced repeated measurements designs.
CSBRMDs	Circular strongly balanced repeated measurements designs.
MCSBRMD	Minimal circular strongly balanced repeated measurements designs.
MCNSBRMD	Minimal circular nearly strongly balanced repeated measurements designs.
MCSPBRMD	Minimal circular strongly partially balanced repeated measurements designs.
MCGSBRMDs	Minimal circular generalized strongly balanced repeated measurements designs.
ES	Efficiency of Separability

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