Journal of Statistics ***ISSN-P: 1684-8403***
Volume 28, 2024, pp. 66-80 ***ISSN-E: 2957-8981***

Efficient Circular Partially Balanced RMDs-II for *v* odd

*Jahangir Saeed [1], Farrukh Jamal[*1], Zawar Hussain[1], Abdul Salam[1],*
H. M. Kashif Rasheed[1] and Humera Hayyat[2]

Abstract

Being economical, RMDs are used in the experiments of almost every field of life. When RMDs are applied, there are chances to rise the residual effects. Residual effects can be removed through washout periods, but washout periods cannot be applied in many situations. To balance the residual effects, minimal circular balanced RMDs are preferred which can only be constructed for odd $v = ip+1$ through method of cyclic shifts (Rule I). Partially balanced RMDs-II can be used for odd $v = ip+3$ which are not available in literature. In this article, therefore, these designs are constructed using Rule I for $3 \leq p \leq 10$. Efficiency measures of residual effects and of Separability for each design are also calculated.

Keywords

Design Construction, Experimental Balancing, Cyclic Shift Methodology, Design Efficiency.

1. Introduction

Repeated measurements designs (RMDs) are always economical but with the use of RMDs residual effects may arise, therefore, choice of RMDs must be made in a way that the treatments can be compared fairly after allowing for the residual effects. Minimal circular balanced RMDs (MCBRMDs) are appropriate to control the residual effects economically but these designs can only be constructed for odd $v = ip+1$. Minimal circular partially balanced RMDs-II (MCPBRMDs-II) can be used for odd $v = ip+3$.For better understanding, following are some important definitions.

- RMD is balanced for first order residual effects if each treatment is immediately preceded once by all others (excluding itself).
- RMD is strongly balanced if each treatment is immediately preceded once by all others (including itself).
- RMD is MCPBRMD-I if all order pairs of distinct treatments appear once except $v/2$pairs which do not appear as preceded values.
- RMD is MCPBRMD-II if all order pairs of distinct treatments appear once except v pairs which do not appear as preceded values.

[*] Corresponding author
Email: farukkh.jamal@iub.edu.pk
[1] Department of Statistics, The Islamia University of Bahawalpur, Bahawalpur - 63100, Pakistan.
[2] Department of Social and Allied Sciences, Cholistan University of Veterinary and Animal Sciences, DHA, Bahawalpur, Pakistan.

- If periods are divided in circular form where the treatments applied in the last period is considered the preceding value of first period, then periods are considered as circular.
- The effect which a treatment has during its period of application (its direct effect) persists into the following period(s)then it is called residual effect or carry over effect.

Williams (1949) introduced the RMDs in linear formation. They constructed some classes of balanced RMDs (BRMDs) for $p = v$. Hedayat and Afsarinejad (1990) constructed non-circular BRMDs for $p \leq v$. Afsarinejad (1994) constructed BRMDs for unequal period sizes in linear formation. Sharma et al. (2003) constructed non- circular minimal BRMDs for v odd. Bate and Jones (2006) discussed some non-circular BRMDs. Magda (1980) introduced the idea of a circular BRMDs. Mendal et al. (2016) constructed BRMDs in circular periods through integer programming.

Following are the constructions in circular periods through method of cyclic shifts which are already available in literature.

- Iqbal and Jones (1994) presented RMDs for some limited cases.
- Iqbal and Tahir (2009); Iqbal et al. (2010) constructed some CBRMD.
- Bashir et al. (2018); Riaz et al. (2023); Shabbir et al. (2023) constructed CBRMDs for some cases.
- Some generators are developed by Rajab et al. (2018, 2022) to obtain CBRMDs for some cases.
- Ahmed et al. (2019) developed some generators for CBRMDs in periods of two different sizes.
- Daniyal et al. (2020) constructed MCSBRMDs in equal and unequal period sizes.
- Bashir et al. (2022) presented MCBRMDs which can be converted directly into MCSBRMD and MCNSBRMD.
- Khan et al. (2023); Rasheed et al. (2021) constructed MCNSBRMDs which can be converted directly into MCBRMD and MCSBRMD.
- Shabbir et al. (2023) discussed the universal optimality of MCNRMDs.

Bailey et al. (2017) constructed optimal PBRMDs for $p = v$. Jabeen et al. (2019), Khan et al. (2019) and Hassan et al. (2022) developed some generators to obtain CPBRMDs for $p \leq v$. Rasheed et al. (2021), Riaz et al. (2023) and Rasheed et al. (2022) developed generators for minimal circular weakly BRMDs in periods of two different sizes for some cases. In this article, MCPBRMDs-II are constructed in equal period sizes through Rule I for v (odd) = ip+3 with $3 \leq p \leq 10$.

2. Method of cyclic shifts

Method of cyclic shifts (Rule I) introduced by Iqbal (1991) is explained here for the construction of MCBRMDs and MCPBRMDs-II.

2.1 Procedure to obtain the design from given sets of shifts, using Rule I

Procedure is explained through S_1 = [4,8,7], S_2 = [2,10,9] for v = 11, p = 4 in Table 1 as follows. Take v experimental subjects for S_1= [4,7,8]. To get the elements of second period for each subject, add 4 (mod 11) to each element of first period for all subjects which are 0, 1, …, v-1. Then add 8 (mod 11) to each element of second period for all subjects of third period. Similarly add 7.

Table 1: Minimal circular balanced RMDs using Rule I through S_1.

Periods	Subjects										
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
1	0	1	2	3	4	5	6	7	8	9	10
2	4	5	6	7	8	9	10	0	1	2	3
3	1	2	3	4	5	6	7	8	9	10	3
4	8	9	10	0	1	2	3	4	5	6	7

In Table 2, take v more subjects for S_2 = [2,10,9]. Get the design in the similar way as taken through S_1.

Table 2: Minimal circular balanced RMDs using Rule I through S_2.

Periods	Subjects										
	12	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
1	0	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	0	1
3	1	2	3	4	5	6	7	8	9	10	3
4	10	0	1	2	3	4	5	6	7	8	9

2.2 Procedure to obtain MCPBRMDs-II, using Rule I

Let S_j = [q_{j1}, q_{j2}, …,$q_{j(p-1)}$] be the sets of shifts with 1 ≤ q_{ji} ≤ v-1. If each element 1, 2, …, v-1 appears once in S*, except one element which does not appear along with its complement then it will be MCPBRMD-II, where S* contains (i) each element of all S_j, and (ii) [v-sum of set (mod v)] for each S_j.

Example: S_1 = [4,8,7], S_2 = [2,10,9] for v = 11 & p= 4.

Here S* = [4,8,7,**3**,2,10,9,**1**] contains each of 1, 2, …, 10 exactly once except 5 & 6 which do not appear. Hence S & S produce MCPBRMD-II.

3. Efficiency of separability

Divecha and Gondaliya (2014) developed following efficiency of Separability (Es) for BRMDs.

$$Es = \left[1 - 1/(v\sqrt{v-1})\right] \times 100\%$$

4. MCPBRMDs-II for *v* (odd) = *ip*+3

Here in Table 3, we present sets of shifts generated by (i) dividing [2, 3, …, *v*-2] in *i* groups, each with *p* elements such that sum of each group is divisible by *v*, (ii) then deleting the smallest element from each group. Following are sets of shifts to produce MCPBRMDs-II for $v < 100$ with $3 \leq p \leq 10$.

Table 3: Sets of shifts to produce MCPBRMDs.

V	*p*	Sets of shifts	Es	Er
9	3	[7, 8] + [2, 6]	0.77	0.66
15	3	[10, 11] + [5, 6] + [2, 12] + [13, 14]	0.87	0.67
21	3	[15, 19] + [5, 14] + [4, 16] + [6, 12] + [13, 20] + [17, 18]	0.91	0.68
27	3	[18, 19] + [20, 24] + [4, 21] + [22, 23] + [11, 15] + [25, 26] + [6, 16] + [8, 12]	0.93	0.69
33	3	[27, 29] + [8, 21] + [22, 25] + [11, 15] + [9, 18] + [12, 20] + [24, 28] + [31, 32] + [23, 30] + [5, 26]	0.95	0.69
39	3	[25, 29] + [14, 18] + [32, 37] + [15, 16] + [33, 35] + [6, 28] + [3, 34] + [23, 38] + [11, 27] + [30, 36] + [26, 31] + [13, 22]	0.95	0.70
45	3	[14, 25] + [15, 21] + [29, 43] + [33, 41] + [28, 42] + [10, 27] + [39, 44] + [5, 36] + [12, 30] + [34, 37] + [13, 31] + [11, 32] + [26, 40] + [35, 38]	0.96	0.71
51	3	[16, 31] + [9, 34] + [33, 40] + [32, 42] + [38, 44] + [11, 30] + [47, 49] + [13, 36] + [45, 50] + [5, 43] + [37, 48] + [18, 19] + [39, 41] + [23, 27] + [35, 46] + [15, 24]	0.97	0.71
57	3	[12, 34] + [16, 33] + [17, 26] + [40, 53] + [39, 48] + [9, 42] + [37, 52] + [45, 47] + [10, 44] + [7, 49] + [50, 51] + [19, 20] + [23, 30] + [43, 56] + [24, 31] + [38, 41] + [36, 46] + [54, 55]	0.97	0.72
63	3	[12, 50] + [34, 59] + [52, 54] + [22, 30] + [13, 48] + [39, 49] + [17, 36] + [40, 58] + [6, 53] + [27, 29] + [44, 45] + [51, 61] + [43, 57] + [9, 46] + [55, 56] + [41, 62] + [47, 60] + [16, 42] + [25, 35] + [21, 24]	0.97	0.73
69	3	[27, 31] + [15, 41] + [10, 50] + [8, 59] + [16, 46] + [28, 38] + [58, 63] + [19, 32] + [51, 57] + [64, 68] + [14, 54] + [12, 53] + [45, 67] + [55, 61] + [44, 65] + [24, 40] + [39, 66] + [56, 62] + [47, 48] + [23, 25] + [42, 60] + [49, 52]	0.97	0.75
75	3	[57, 69] + [66, 70] + [18, 44] + [31, 33] + [25, 43] + [28, 39] + [56, 67] + [63, 71] + [62, 73] + [53, 74] + [52, 64] + [17, 55] + [49, 60] + [22, 48] + [26, 29] + [50, 65] + [6, 68] + [47, 58] + [51, 59] + [21, 42] + [30, 36]+ [10, 61] + [46, 72] + [19, 54]	0.97	0.78

V	*p*	Sets of shifts	Es	Er
81	3	[21, 43] + [48, 78] + [34, 45] + [24, 38] + [56, 77] + [14, 55] + [64, 67] + [16, 62] + [53, 74] + [53, 74] + [5, 75] + [22, 52] + [50, 80] + [65, 70] + [18, 54] + [20, 51] + [63, 73] + [28, 30] + [8, 69] + [49, 71] + [47, 76] + [61, 68] + [59, 66] + [58, 79] + [46, 72] + [13, 57] + [15, 60]	0.97	0.80
87	3	[35, 47] + [13, 62] + [60, 65] + [63, 85] + [33, 37] + [24, 57] + [25, 40] + [28, 50] + [56, 76] + [15, 68] + [67, 77] + [58, 80] + [52, 83] + [23, 46] + [66, 81] + [19, 54] + [38, 48] + [20, 51] + [75, 78] + [32, 53] + [55, 74] + [10, 70] + [61, 72] + [82, 84] + [69, 71] + [11, 73] + [59, 86] + [64, 79]	0.97	0.81
93	3	[88, 90] + [77, 92] + [83, 87] + [34, 37] + [14, 68] + [23, 69] + [71, 80] + [31, 44] + [56, 81] + [19, 62] + [57, 91] + [29, 58] + [32, 52] + [36, 54] + [51, 85] + [61, 70] + [24, 65] + [66, 67] + [13, 75] + [28, 45] + [74, 86] + [30, 42] + [79, 82] + [40, 43] + [15, 76] + [60, 78] + [27, 59] + [72, 73] + [63, 84] + [64, 89]	0.97	0.82
99	3	[94, 95] + [14, 83] + [65, 89] + [27, 59] + [21, 62] + [6, 88] + [64, 77] + [74, 78] + [85, 98] + [34, 61] + [39, 48] + [73, 82] + [70, 96] + [67, 75] + [71, 86] + [33, 55] + [42, 47] + [79, 81] + [26, 53] + [84, 90] + [19, 63] + [23, 69] + [37,40] + [35, 36] + [25, 66] + [60, 80] + [30, 68] + [76, 91] + [72, 97] + [54, 92] + [87, 93] + [45, 51]	0.97	0.84
11	4	[4, 8, 7] + [2, 10, 9]	0.82	0.65
15	4	[5, 9, 14] + [4, 10, 13] + [6, 11, 12]	0.87	0.77
19	4	[16, 17, 18] + [7, 14, 15] + [3, 4, 11] + [8, 12, 13]	0.90	0.78
23	4	[7, 17, 18] + [5, 6, 9] + [10, 14, 20] + [8, 15, 22] + [16, 19, 21]	0.91	0.78
27	4	[6, 8, 11] + [20, 21, 23] + [7, 18, 25] + [10, 19, 22] + [5, 9, 12] + [16, 24, 26]	0.93	0.79
31	4	[17, 19, 22] + [6, 10, 14] + [11, 20, 28] + [5, 25, 30] + [24, 27, 29] + [8, 21, 26] + [12, 18, 23]	0.94	0.81
35	4	[2, 8, 24] + [11, 23, 33] + [13, 16, 29] + [15, 21, 27] + [10, 26, 28] + [14, 22, 30] + [20, 32, 34] + [9, 25, 31]	0.94	0.81
39	4	[7, 8, 23] + [14, 18, 34] + [21, 25, 26] + [9, 29, 37] + [11, 27, 30] + [13, 28, 32] + [15, 24, 35] + [31, 33, 36] + [16, 22, 38]	0.95	0.82
43	4	[31, 34, 40] + [11, 35, 37] + [17, 23, 41] + [28, 36, 39] + [15, 29, 33] + [18, 25, 42] + [20, 30, 32] + [8, 12, 16] + [19, 27, 38] + [10, 13, 14]	0.95	0.82

V	***p***	**Sets of shifts**	**Es**	**Er**
47	**4**	[20, 31, 42] + [19, 22, 43] + [12, 34, 46] + [25, 28, 35] + [33, 37, 41] + [11, 36, 44] + [15, 17, 8] + [18, 21, 39] + [5, 9, 29] + [32, 38, 45] + [14, 27, 40]	0.96	0.82
51	4	[18, 33, 47] + [23, 32, 36] + [14, 38, 43] + [21, 37, 42] + [5, 10, 35] + [19, 24, 50] + [45, 46, 49] + [8, 15, 22] + [17, 30, 39] + [8, 31, 40] + [20, 29, 41] + [34, 44, 48]	0.94	0.83
55	4	[14, 33, 54] + [7, 20, 25] + [13, 32, 53] + [10, 43, 52] + [16, 45, 47] + [18, 35, 46] + [40, 42, 49] + [17, 37, 50] + [22, 29, 51] + [26, 36, 44] + [23, 38, 48] + [21, 31, 39] + [24, 30, 41]	0.96	0.83
59	4	[28, 31, 37] + [6, 23, 27] + [33, 55, 57] + [19, 35, 53] + [26, 40, 47] + [21, 46, 50] + [17, 43, 54] + [34, 36, 41] + [18, 39, 51] + [25, 42, 49] + [24, 38, 44] + [14, 16, 20] + [48, 56, 58] + [13, 45, 52]	0.97	0.84
63	4	[22, 33, 57] + [20, 42, 51] + [7, 8, 47] + [49, 61, 62] + [12, 19, 26] + [29, 45, 50] + [25, 27, 59] + [52, 56, 60] + [46, 48, 55] + [24, 39, 53] + [34, 43, 44] + [9, 23, 28] + [16, 41, 58] + [30, 38, 54] + [35, 36, 37]	0.97	0.84
67	4	[36, 44, 53] + [28, 30, 60] + [17, 47, 55] + [25, 27, 61] + [11, 48, 65] + [24, 37, 51] + [19, 52, 59] + [42, 63, 64] + [9, 57, 66] + [35, 39, 54] + [26, 49, 56] + [13, 20, 29] + [38, 43, 46] + [50, 58, 62] + [14, 18, 23] + [40, 41, 45]	0.97	0.85
71	4	[17, 48, 66] + [30, 34, 70] + [28, 33, 56] + [20, 53, 59] + [26, 42, 52] + [15, 55, 63] + [27, 44, 69] + [18, 58, 60] + [13, 39, 61] + [51, 54, 62] + [24, 45, 68] + [37, 38, 64] + [31, 32, 67] + [41, 43, 57] + [29, 47, 50] + [21, 49, 65] + [13, 14, 40]	0.96	0.85
75	4	[53, 62, 70] + [42, 50, 54] + [27, 43, 58] + [21, 51, 65] + [7, 19, 46] + [15, 20, 34] + [11, 26, 28] + [33, 55, 60] + [35, 36, 61] + [25, 30, 71] + [44, 67, 73] + [17, 56, 69] + [32, 45, 72] + [39, 48, 49] + [23, 47, 68] + [57, 63, 74] + [16, 59, 66] + [29, 52, 64]	0.95	0.85
79	4	[41, 46, 53] + [64, 67, 69] + [30, 44, 78] + [34, 48, 49] + [11, 25, 42] + [22, 63, 71] + [38, 45, 61] + [9, 15, 52] + [32, 47, 58] + [8, 29, 35] + [24, 62, 68] + [36, 43, 59] + [65, 73, 76] + [31, 33, 75] + [28, 50, 54] + [16, 57, 72] + [10, 66, 77] + [17, 55, 74] + [56, 60, 70]	0.96	0.86
83	4	[40, 54, 71] + [36, 48, 52] + [28, 45, 69] + [35, 55, 66] + [21, 51, 76] + [25, 47, 82] + [39, 43, 68] + [20, 62, 70] + [8, 75, 80] + [49, 50, 60] + [13, 29, 32] + [34, 44, 57] + [17, 72, 73] + [27, 58, 79] + [38, 56, 61] + [59, 63, 74] +	0.98	0.86

V	*p*	Sets of shifts	Es	Er
		[6, 26, 46] + [33, 37, 77] + [67, 78, 81] + [22, 64, 65]		
87	4	[5, 9, 70] + [38, 47, 58] + [18, 69, 72] + [36, 37, 81] + [33, 54, 63] + [23, 73, 74] + [12, 71, 80] + [26, 59, 67] + [27, 46, 76] + [42, 60, 65] + [35, 60, 65] + [35, 53, 57] + [66, 77, 86] + [19, 21, 39] + [48, 61, 64] + [34, 50, 62] + [49, 51, 68] + [30, 56, 75] + [40, 45, 79] + [78, 82, 84] + [52, 83, 85] + [14, 16, 55]	0.97	0.87
91	4	[34, 54, 66] + [37, 53, 70] + [17, 68, 83] + [26, 47, 84] + [40, 51, 75] + [9, 24, 57] + [29, 58, 80] + [39, 64, 72] + [71, 76, 78] + [8, 82, 90] + [27, 61, 73] + [43, 44, 77] + [32, 59, 88] + [38, 52, 86] + [23, 60, 89] + [31, 50, 81] + [33, 67, 69] + [11, 79, 87] + [49, 56, 65] + [41, 63, 74] + [35, 55, 62] + [36, 42, 85]	0.96	0.87
95	4	[24, 65, 89] + [14, 21, 52] + [38, 61, 64] + [35, 66, 69] + [77, 78, 86] + [53, 59, 62] + [4, 17, 71] + [40, 58, 60] + [51, 54, 75] + [41, 55, 68] + [22, 63, 94] + [82, 92, 93] + [73, 88, 90] + [70, 76, 83] + [37, 50, 74] + [30, 57, 84] + [36, 39, 87] + [15, 33, 46] + [72, 80, 91] + [43, 45, 79] + [5, 7, 81] + [9, 31, 49] + [25, 67, 85]	0.98	0.88
99	4	[10, 34, 47] + [41, 61, 64] + [18, 21, 45] + [46, 52, 74] + [25, 58, 95] + [31, 48, 91] + [30, 73, 79] + [66, 86, 90] + [4, 6, 87] + [44, 60, 81] + [24, 62, 93] + [27, 63, 97] + [77, 84, 85] + [43, 56, 76] + [70, 78, 82] + [69, 92, 96] + [53, 59, 83] + [9, 14, 71] + [35, 57, 94] + [65, 80, 98] + [38, 68, 75] + [39, 42, 88] + [22, 33, 37] + [36, 72, 89]	0.98	0.89
13	5	[5, 11, 8, 12] + [2, 4, 10, 9]	0.85	0.67
23	5	[7, 19, 16, 22] + [13, 15, 17, 20] + [3, 6, 14, 21] + [8, 9, 10, 18]	0.92	0.74
33	5	[19, 20, 27, 29] + [7, 10, 23, 25] + [13, 22, 30, 31] + [11, 12, 15, 26] + [6, 9, 14, 32] + [18, 21, 24, 28]	0.88	0.75
43	5	[9, 16, 25, 35] + [13, 15, 24, 29] + [12, 19, 20, 31] + [7, 8, 32, 36] + [23, 28, 34, 42] + [14, 17, 18, 26] + [30, 37, 38, 40] + [10, 33, 39, 41]	0.96	0.77
53	5	[17, 19, 20, 35] + [16, 18, 21, 40] + [7, 31, 33, 34] + [24, 36, 47, 50] + [14, 22, 29, 32] + [13, 43, 44, 49] + [23, 28, 51, 52] + [25, 38, 39, 45] + [6, 8, 42, 46] + [30, 37, 41, 48]	0.96	0.81
63	5	[9, 17, 37, 61] + [19, 48, 56, 58] + [27, 41, 46, 59] + [29, 38, 47, 57] + [5, 7, 13, 35] + [23, 42, 52, 60] + [25, 39, 53, 62] + [14, 28, 30, 43] + [24, 45, 49, 50] + [4, 22, 44, 55] + [20, 26, 34, 40] + [33, 36, 51, 54]	0.94	0.81

V	*p*	Sets of shifts	Es	Er
73	5	[30, 35, 54, 71] + [23, 24, 39, 43] + [14, 21, 48, 59] + [15, 22, 46, 60] + [7, 38, 40, 56] + [41, 49, 55, 63] + [28, 51, 58, 70] + [18, 32, 33, 47] + [6, 27, 44, 68] + [13, 20, 45, 66] + [52, 65, 69, 72] + [31, 42, 57, 64] + [19, 61, 62, 67] + [9, 26, 50, 53]	0.98	0.81
83	5	[35, 64, 66, 76] + [6, 13, 16, 45] + [30, 56, 57, 77] + [15, 37, 49, 51] + [20, 27, 38, 70] + [17, 25, 48, 74] + [33, 65, 67, 72] + [24, 60, 73, 82] + [19, 68, 75, 80] + [58, 61, 62, 63] + [34, 36, 43, 52] + [32, 40, 78, 81] + [21, 26, 31, 79] + [46, 50, 54, 55] + [28, 59, 69, 71] + [23, 39, 47, 53]	0.98	0.82
93	5	[69, 76, 85, 86] + [7, 14, 24, 45] + [33, 64, 73, 87] + [55, 59, 62, 91] + [25, 37, 40, 71] + [51, 58, 68, 79] + [39, 66, 72, 82] + [19, 31, 49, 78] + [32, 48, 84, 88] + [8, 36, 60, 81] + [18, 21, 65, 77] + [75, 80, 83, 92] + [4, 43, 67, 70] + [17, 34, 35, 90] + [29, 38, 52, 61] + [28, 30, 50, 63] + [16, 26, 44, 89] + [53, 54, 57, 74]	0.98	0.83
15	6	[2, 4, 13, 11, 14] + [5, 6, 10, 9, 12]	0.87	0.76
21	6	[9, 13, 17, 18, 20] + [3, 5, 6, 8, 19] + [4, 12, 14, 15, 16]	0.91	0.80
27	6	[8, 12, 16, 19, 22] + [5, 10, 18, 21, 25] + [9, 11, 15, 20, 23] + [6, 7, 17, 24, 26]	0.91	0.80
33	6	[9, 13, 17, 18, 20] + [3, 5, 6, 8, 19] + [4, 12, 14, 15, 16]	0.91	0.81
39	6	[9, 10, 30, 33, 34] + [8, 16, 28, 31, 32] + [12, 18, 22, 25, 37] + [11, 17, 21, 27, 35] + [15, 24, 29, 36, 38] + [5, 7, 13, 23,26]	0.89	0.81
45	6	[4, 19, 17, 28, 30] + [13, 18, 25, 26, 41] + [15, 19, 31, 33, 36] + [14, 16, 21, 39, 40] + [20, 32, 35, 38, 44] + [8, 10, 29, 42, 43] + [7, 24, 27, 34, 37]	0.88	0.81
51	6	[5, 17, 30, 49, 50] + [20, 22, 27, 33, 47] + [10, 23, 32, 38, 43] + [16, 36, 45, 46, 48] + [9, 11, 14, 19, 41] + [15, 29, 34, 35, 37] + [6, 12, 24, 28, 31] + [21, 39, 40, 42, 44]	0.89	0.81
57	6	[7, 30, 37, 42, 53] + [33, 35, 38, 44, 54] + [11, 17, 18, 26, 34] + [4, 12, 15, 39, 41] + [21, 23, 27, 36, 51] + [9, 19, 40, 47, 55] + [16, 45, 48, 49, 56] + [6, 10, 22, 25, 46] + [31, 32, 43, 50, 52]	0.96	0.82
63	6	[17, 19, 20, 21, 45] + [36, 41, 56, 57, 60] + [13, 14, 24, 28, 40] + [18, 29, 35, 44, 55] + [15, 16, 25, 27, 33] + [3, 38, 43, 46, 58] + [12, 26, 39, 47, 54] + [30, 37, 48, 53, 62] + [34, 50, 51, 52, 59] + [9, 23, 42, 49, 61]	0.96	0.82

V	*p*	Sets of shifts	Es	Er
69	6	[12, 24, 40, 59, 68] + [48, 49, 51, 55, 60] + [15, 32, 42, 52, 58] + [21, 23, 38, 43, 62] + [22, 25, 33, 54, 64] + [31, 45, 53, 63, 66] + [14, 27, 44, 47, 65] + [7, 26, 46, 56, 67] + [19, 36, 37, 41, 57] + [3, 6, 16, 50, 61] + [11, 28, 29, 30, 39]	0.97	0.82
75	6	[24, 49, 66, 72, 74] + [29, 33, 39, 46, 69] + [26, 30, 47, 59, 60] + [53, 55, 61, 62, 68] + [42, 51, 57, 67, 71] + [20, 27, 28, 63, 73] + [40, 41, 44, 48, 50] + [21, 23, 31, 34, 36] + [6, 35, 54, 56, 70] + [18, 22, 43, 64, 65] + [16, 17, 19, 32, 58] + [10, 11, 25, 45, 52]	0.97	0.83
81	6	[25, 30, 46, 53, 74] + [20, 37, 39, 56, 75] + [24, 35, 43, 59, 68] + [47, 50, 63, 71, 72] + [32, 38, 45, 51, 73] + [18, 31, 62, 64, 65] + [7, 11, 26, 33, 79] + [8, 29, 58, 67, 76] + [9, 19, 23, 49, 61] + [44, 54, 57, 77, 80] + [27, 34, 48, 55, 66] + [10, 17, 28, 36, 69] + [42, 52, 60, 70, 78]	0.98	0.83
87	6	[18, 34, 37, 74, 82] + [51, 58, 60, 69, 84] + [27, 45, 54, 57, 75] + [11, 15, 68, 77, 83] + [19, 21, 29, 38, 63] + [25, 42, 56, 59, 71] + [22, 46, 48, 66, 70] + [41, 65, 67, 80, 81] + [47, 61, 76, 78, 85] + [6, 40, 55, 72, 86] + [31, 33, 39, 62, 73] + [32, 35, 49, 53, 79] + [10, 17, 28, 50, 64] + [20, 24, 30, 36, 52]	0.98	0.84
93	6	[36, 48, 50, 53, 71] + [19, 27, 32, 34, 63] + [3, 15, 42, 60, 65] + [30, 70, 73, 81, 89] + [41, 58, 77, 78, 90] + [52, 62, 66, 83, 91] + [26, 38, 56, 67, 75] + [13, 51, 57, 69, 85] + [9, 25, 33, 35, 79] + [23, 39, 44, 74, 92] + [40, 43, 49, 59, 76] + [20, 22, 61, 82, 88] + [45, 55, 68, 80, 87] + [10, 14, 24, 64, 72] + [16, 31, 54, 84, 86]	0.95	0.85
99	6	[10, 13, 28, 60, 81] + [18, 40, 52, 86, 92] + [22, 26, 67, 70, 93] + [36, 41, 56, 77, 82] + [4, 61, 72, 74, 83] + [11, 27, 30, 53, 76] + [17, 39, 54, 75, 97] + [23, 34, 65, 69, 85] + [24, 31, 35, 38, 62] + [32, 45, 47, 78, 88] + [58, 73, 84, 89, 90] + [55, 63, 64, 87, 94] + [42, 44, 48, 68, 79] + [20, 51, 57, 59, 98] + [37, 43, 46, 66, 91] + [29, 71, 80, 95, 96]	0.96	0.86
17	7	[5, 7, 10, 13, 14, 16] + [2, 4, 6, 11, 12, 15]	0.88	0.67

V	***p***	**Sets of shifts**	**Es**	**Er**
31	7	[4, 9, 12, 19, 22, 26] + [7, 10, 11, 13, 23, 24] + [6, 14, 21, 25, 27, 29] + [8, 17, 18, 20, 28, 3]	0.87	0.69
45	7	[6, 13, 25, 26, 31, 33] + [12, 15, 19, 24, 27, 29] + [35, 37, 41, 42, 43, 44] + [7, 11, 20, 21, 34, 38] + [8, 10, 17, 18, 39, 40] + [5, 14, 16, 30, 32, 36]	0.91	0.83
59	7	[9, 10, 12, 16, 20, 47] + [21, 23, 34, 43, 44, 53] + [7, 14, 27, 36, 37, 51] + [15, 32, 38, 46, 49, 54] + [13, 50, 52, 55, 56, 58] + [17, 19, 24, 26, 28, 57] + [3, 22, 31, 39, 40, 41] + [25, 33, 35, 42, 45, 48]	0.97	0.83
73	7	[20, 39, 42, 59, 61, 64] + [28, 32, 48, 49, 50, 68] + [6, 9, 34, 53, 54, 60] + [22, 29, 35, 62, 65, 66] + [12, 16, 19, 43, 52, 69] + [14, 23, 25, 30, 58, 67] + [24, 26, 31, 40, 46, 47] + [11, 18, 21, 38, 56, 71] + [15, 27, 33, 41, 45, 57] + [44, 51, 55, 63, 70, 72]	0.97	0.84
87	7	[35, 48, 56, 58, 73, 74] + [23, 40, 53, 67, 78, 84] + [49, 50, 69, 76, 81, 86] + [11, 18, 21, 22, 32, 68] + [31, 45, 57, 60, 62, 80] + [9, 14, 51, 55, 61, 63] + [28, 34, 47, 65, 83, 85] + [29, 46, 52, 59, 72, 75] + [17, 26, 30, 33, 64, 79] + [7, 10, 16, 19, 39, 82] + [38, 41, 42, 54, 66, 70] + [20, 25, 27, 36, 71, 77]	0.98	0.84
19	8	[4, 5, 8, 13, 14, 15, 16] + [3, 6, 11, 12, 17, 18]	0.87	0.69
27	8	[3, 4, 9, 11, 12, 16, 25] + [5, 7, 10, 19, 20, 22, 23] + [8, 15, 17, 18, 21, 24, 26]	0.93	0.71
35	8	[4, 5, 12, 19, 20, 21, 23] + [6, 14, 15, 16, 27, 28, 32] + [11, 13, 25, 26, 29, 31, 33] + [8, 9, 10, 22, 24, 30, 34]	0.95	0.75
43	8	[5, 10, 16, 25, 36, 38, 41] + [7, 11, 14, 15, 20, 28, 32] + [13, 24, 30, 31, 35, 39, 40] + [8, 12, 18, 23, 26, 37, 42] + [9, 17, 19, 27, 29, 33, 34]	0.96	0.79
51	8	[5, 9, 22, 28, 45, 46, 47] + [8, 16, 24, 29, 37, 38, 48] + [19, 20, 34, 35, 36, 49, 50] + [13, 14, 21, 30, 33, 42, 44] + [3, 11, 18, 23, 27, 31, 39] + [10, 15, 17, 32, 40, 41, 43]	0.96	0.79
59	8	[23, 28, 32, 36, 48, 51, 56] + [11, 13, 15, 16, 19, 44, 54] + [7, 20, 31, 39, 40, 47, 50] + [4, 22, 24, 35, 45, 46, 57] + [12, 17, 26, 37, 41, 42, 55] + [9, 10, 18, 25, 33, 38, 43] + [14, 27, 34, 49, 52, 53, 58]	0.97	0.80
67	8	[6, 18, 19, 32, 36, 39, 46] + [26, 31, 40, 49, 53, 58, 63] + [11, 22, 28, 42, 47, 50, 61] + [16, 24, 29, 35, 41, 56, 66] + [21, 25, 44, 52, 54, 62, 65] + [4, 9, 20, 27, 37, 45, 57] + [8, 14, 30, 43, 51, 59, 60] + [13, 17, 23, 38, 48, 55, 64]	0.97	0.81

V	***p***	**Sets of shifts**	**Es**	**Er**
75	8	[30, 41, 44, 50, 62, 68, 74] + [18, 24, 28, 32, 56, 64, 65] + [20, 42, 52, 54, 58, 66, 72] + [10, 14, 17, 22, 36, 48, 71] + [12, 27, 31, 47, 51, 60, 70] + [5, 15, 21, 35, 43, 45, 57] + [23, 25, 26, 40, 49, 61, 73] + [9, 16, 29, 53, 55, 63, 67] + [19, 33, 34, 39, 46, 59, 69]	0.97	0.83
83	8	[4, 22, 39, 60, 63, 64, 77] + [24, 25, 30, 38, 62, 71, 74] + [17, 32, 33, 40, 53, 65, 82] + [5, 16, 23, 35, 36, 51, 81] + [9, 26, 27, 28, 47, 55, 56] + [13, 20, 49, 50, 52, 57, 79] + [14, 15, 43, 54, 59, 61, 75] + [19, 21, 66, 68, 72, 73, 78] + [34, 45, 46, 48, 67, 70, 76] + [7, 31, 37, 44, 58, 69, 80]	0.95	0.84
91	8	[4, 11, 18, 26, 55, 71, 87] + [23, 29, 53, 79, 81, 82, 86] + [13, 14, 25, 35, 39, 66, 73] + [38, 48, 50, 62, 68, 70, 83] + [15, 16, 52, 60, 64, 72, 75] + [27, 33, 42, 44, 51, 69, 78] + [12, 28, 37, 43, 61, 84, 90] + [17, 19, 47, 54, 67, 74, 80] + [21, 31, 40, 56, 57, 63, 89] + [24, 32, 34, 49, 58, 77, 85] + [3, 30, 41, 59, 65, 76, 88]	0.92	0.85
99	8	[41, 57, 61, 67, 77, 79, 80] + [20, 21, 28, 29, 54, 65, 71] + [17, 35, 46, 56, 63, 84, 89] + [26, 39, 40, 53, 68, 70, 82] + [10, 38, 48, 52, 73, 83, 85] + [22, 44, 47, 86, 92, 93, 97] + [12, 31, 42, 58, 74, 81, 87] + [5, 23, 32, 64, 76, 96, 98] + [15, 30, 36, 51, 72, 88, 91] + [4, 19, 34, 69, 78, 94, 95] + [8, 27, 37, 43, 55, 60, 66] + [24, 25, 45, 59, 62, 75, 90]	0.94	0.86
21	9	[2, 4, 9, 15, 17, 18, 19, 20] + [5, 6, 7, 8, 12, 13, 14, 16]	0.91	0.61
39	9	[7, 8, 11, 12, 23, 27, 28, 36] + [10, 14, 21, 24, 25, 30, 34, 35] + [9, 13, 17, 18, 31, 32, 33, 37] + [3, 6, 15, 16, 22, 26, 29, 38]	0.93	0.77
57	9	[5, 9, 10, 13, 18, 25, 34, 56] + [16, 17, 21, 36, 40, 46, 50, 53] + [27, 32, 35, 37, 45, 47, 51, 54] + [8, 11, 19, 23, 26, 38, 44, 52] + [3, 12, 15, 22, 41, 42, 43, 48] + [20, 24, 30, 31, 33, 39, 49, 55]	0.97	0.80
75	9	[21, 23, 27, 28, 34, 43, 56, 63] + [9, 12, 15, 17, 30, 36, 44, 61] + [11, 18, 29, 33, 40, 42, 48, 71] + [14, 26, 35, 46, 58, 59, 66, 68] + [41, 45, 50, 51, 52, 62, 64, 69] + [7, 31, 47, 49, 54, 55, 57, 73] + [22, 24, 25, 32, 53, 60, 72, 74] + [6, 10, 19, 20, 39, 65, 67, 70]	0.98	0.82
93	9	[10, 37, 38, 48, 54, 58, 59, 65] + [35, 39, 45, 79, 82, 83, 84, 85] + [7, 12, 18, 29, 32, 36, 63, 81] + [33, 41, 44, 51, 55, 56, 69, 88] + [15, 20, 24, 27, 60, 68, 72, 77] + [16, 30, 31, 34, 40, 57, 76, 86] + [19, 42, 49, 52, 53, 73, 75, 89] + [17, 61, 62, 71, 78, 80, 87, 91] + [8, 21, 23, 43, 50, 64, 67, 90] + [5, 14, 22, 25, 66, 70, 74, 92]	0.98	0.88

V	*p*	Sets of shifts	Es	Er
33	10	[3, 10, 12, 19, 20, 22, 24, 25, 29] + [6, 7, 8, 9, 11, 14, 18, 23, 32] + [5, 13, 15, 21, 26, 27, 28, 30, 31]	0.95	0.63
43	10	[4, 12, 16, 17, 29, 31, 33, 35, 36] + [8, 9, 15, 18, 23, 26, 28, 39, 42] + [6, 10, 11, 13, 14, 20, 27, 30, 40] + [5, 19, 24, 25, 32, 34, 37, 38, 41]		
53	10	[11, 17, 28, 35, 36, 43, 45, 50, 52] + [5, 14, 16, 20, 33, 34, 44, 46, 49] + [18, 19, 22, 23, 29, 30, 31, 37, 48] + [9, 15, 24, 25, 32, 38, 39, 40, 41] + [6, 7, 10, 12, 13, 21, 42, 47, 51]	0.97	0.75
63	10	[5, 9, 13, 14, 24, 37, 40, 45, 62] + [12, 16, 28, 33, 36, 42, 43, 44, 59] + [7, 15, 22, 29, 35, 41, 50, 55, 60] + [10, 11, 17, 27, 34, 46, 51, 52, 61] + [8, 19, 20, 25, 30, 48, 49, 54, 58] + [21, 23, 26, 38, 39, 47, 53, 56, 57]	0.91	0.86
73	10	[10, 21, 23, 30, 41, 49, 52, 64, 70] + [14, 15, 32, 35, 40, 42, 54, 60, 71] + [7, 9, 27, 34, 45, 46, 58, 66, 67] + [17, 22, 24, 26, 39, 50, 56, 57, 62] + [18, 20, 33, 44, 47, 63, 65, 68, 69] + [16, 19, 25, 29, 43, 48, 51, 61, 72] + [4, 8, 13, 28, 31, 38, 53, 55, 59]	0.92	0.88
83	10	[9, 21, 22, 29, 35, 61, 74, 75, 82] + [6, 25, 32, 34, 38, 55, 70, 72, 79] + [14, 20, 30, 44, 48, 59, 60, 64, 73] + [12, 17, 36, 37, 49, 56, 58, 68, 81] + [10, 15, 23, 28, 39, 43, 46, 51, 69] + [5, 11, 40, 45, 50, 52, 63, 71, 76] + [18, 26, 27, 31, 47, 53, 54, 65, 78] + [19, 24, 33, 57, 62, 66, 67, 77, 80]	0.93	0.89
93	10	[8, 20, 39, 42, 54, 62, 70, 84, 85] + [12, 17, 21, 22, 53, 80, 82, 83, 90] + [7, 18, 26, 35, 65, 71, 73, 75, 92] + [14, 28, 32, 48, 56, 57, 63, 78, 87] + [19, 23, 40, 51, 52, 55, 67, 68, 77] + [6, 15, 37, 44, 59, 64, 69, 79, 88] + [31, 33, 49, 50, 58, 66, 74, 81, 89] + [10, 11, 25, 29, 30, 34, 61, 72, 91] + [24, 36, 38, 41, 43, 45, 60, 76, 86]	0.94	0.89

5. Discussion and conclusion

MCBRMDs are used to control the residual effects which can only be constructed for odd $v = ip+1$ through Rule I. MCPBRMDs-II can be used for odd $v = ip+3$ which were not available in literature. Catalogues are always useful for researchers, practitioners and experimenters because they can choose the design of their choice. Therefore, a catalogue of these designs along with their efficiencies is compiled in this article for $3 \leq p \leq 10$. Efficiency of proposed designs shows that our proposed designs are efficient to control the residual effects.

Acknowledgment

Authors are thankful to the reviewers for their valuable suggestions/corrections.

References

1. Afsarinejad, K. (1994). Repeated measurement design with unequal period sizes. *Journal of the Italian Statistical society, 2,* 161-168. https://doi.org/10.1007/BF02589224
2. Ahmed, R., Shehzad, F., Rajab, M., Daniyal, M., & Tahir, M. H. (2019). Minimal circular balanced repeated measurements design in periods of unequal sizes. *Communications in Statistics-Theory and Methods*, *48*(21), 5223-5232. https://doi.org/10.1080/03610926.2018.1508719
3. Bailey, R. A., Cemeron, P. J., Fillipiak, K., Kunert, J., & Markiewicz, A. (2017). On optimality and construction of circular repeated measurements designs. *Statistica Sinica*, *27*, 1-22. https://doi.org/10.5705/ss.202015.0045
4. Bashir, Z, Ahmed, R., Tahir, M. H., Ghazali, S. S. A., & Shehzad, F. (2018). Some extensions of circular balanced and circular strongly balanced repeated measurements designs. *Communications in Statistics - Theory and Methods*, *47*(9), 2183-2194. https://doi.org/10.1080/24754269.2023.2184607
5. Bashir, Z., Ahmed, R., Gondaliya, J., & Rasheed, H. M. K. (2022). Some classes of circular balanced RMDs and their conversion into circular strongly and nearly strongly balanced RMDs. *Communications in Statistics-Theory and Methods*, *51*(19), 6573-6584. https://doi.org/10.1080/03610926.2022.2101664
6. Bate, S. T., & Jones, B. (2006). The construction of nearly balanced and nearly strongly balanced uniform cross-over designs. *Journal of Statistical Planning and Inference, 136*, 3248-3267. https://doi.org/10.1016/j.jspi.2004.11.012
7. Daniyal, M., Ahmed, R., Shehzad, F., Tahir, M. H., & Iqbal, Z. (2020). Construction of repeated measurements designs strongly balanced for residual effects. *Communications in Statistics-Theory and Methods*, *49*(17), 4288-4297. https://doi.org/10.1080/03610926.2019.1599019
8. Divecha, J., & Gondaliya, J. (2014). Construction of minimal balanced crossover designs having good efficiency of separability. *Electronic Journal of Statistics*, *8*, 2923-2936. https://doi.org/10.1214/14-EJS979
9. Hedayat, A., & Afsarinejad, K. (1975). Repeated measurements designs, I. *In a Survey of Statistical Design and Linear Models* (J. N. Srivastava, Ed.), pp. 229–242 (North-Holland, Amsterdam). https://doi.org/10.1007/978-1-4612-3662-7_6
10. Hussain, S., Ahmed, R., Aslam, M., Shah, A., & Rasheed, H. M. K. (2019). Some new construction of circular weakly balanced repeated measurements designs in

periods of two different sizes. *Communications in Statistics-Theory and Methods*, *49*(9), 2253-2263. https://doi.org/10.1080/03610926.2019.1570263
11. Jabeen, R., Rasheed, H. M. K., Ahmed, R., & Shehzad, F. (2019). Construction of circular strongly partially balanced repeated measurements designs. *Journal of King Saud University- Science*, *31*, 345-351. https://doi.org/10.1016/j.jksus.2019.02.002
12. James, A. T., & Wilkinson, G.N. (1971). Factorization of the residual operator and canonical decomposition of non-orthogonal factors in the analysis of variance. *Biometrika*, *58*, 258-294. https://doi.org/10.53560/PPASA(60-1)665
13. Iqbal, I. (1991). Construction of experimental designs using cyclic Shifts. Ph.D. thesis, University of Kent at Canterbury, UK.
14. Iqbal, I., & Jones, B. (1994). Efficient repeated measurements designs with equal and unequal period sizes. *Journal of Statistical Planning and Inference*, *42*, 79-88. https://doi.org/10.1016/0378-3758(94)90190-2
15. Iqbal, I., & Tahir, M. H. (2009). Circular strongly balanced repeated measurements designs. *Communications in Statistics—Theory and Methods*, *38*, 3686-3696. https://doi.org/10.1080/03610920802642566
16. Iqbal, I., Tahir, M. H., & Ghazali, S. S. A. (2010). Circular first-and second-order balanced repeated measurements designs. *Communications in Statistics-Theory and Methods*, *39*, 228-240. https://doi.org/10.1080/03610920902941728
17. Khan, A., Ahmed, R., Shehzad, F., Tahir, M. H., & Ghazali, S. S. A. (2019). Construction of circular partially balanced-repeated measurement designs using cyclic shifts. *Communications in Statistics - Simulation and Computation*. *48*(2), 506-515. https://doi.org/10.1080/03610918.2017.1387661
18. Khan, A., Bashir, Z., Rasheed, H. M. K., Ulhassan, M., & Ahmed, R. (2023). Construction of minimal circular nearly strongly balanced repeated measurements designs and their conversion. *Communications in Statistics-Simulation and Computation*, *52 (12)*, 5749-5758. https://doi.org/10.1080/03610918.2021.2016831
19. Magda, C. G. (1980). Circular balanced repeated measurements designs. *Communications in Statistics-Theory and Methods*, *9*, 1901-1918.
20. Mendal, B. N., Parsad, R., & Gupta, V. K. (2016). Cyclic circular balanced and strongly balanced crossover designs through integer programming. *Communications in Statistics: Theory and Methods*, *45*(4), 859-871. https://doi.org/10.1080/03610926.2013.853787
21. Pearce, S.C., Calinski, T., & Marshall, T. F. de C. (1974). The basic contrasts of an experimental design with special reference to the analysis of data. *Biometrika*, *61*, 449-460. https://doi.org/10.1093/biomet/61.3.449
22. Rajab, M., Ahmed, R., Shehzad, F., & Tahir, M. H. (2018). Some new constructions of circular balanced repeated measurements designs. *Communications in Statistics -Theory and Methods*, *47*(17), 4142-4152. https://doi.org/10.1080/03610926.2017.1367817
23. Rasheed, H. M. K., Jabeen, R., Hussain, S., Shah, A., & Ahmed, R. (2021). Catalogues of efficient circular weakly balanced repeated measurements designs in periods of two different sizes. *Aligarh Journal of Statistics*, *39*, 93-116.
24. Riaz, M., Arshad, H. M., Jabeen, R., Rashid, M. S., Afzal, A., & Ahmed, R. (2023). Catalogues of efficient minimal weakly balanced RMDs in circular periods of three different sizes. *Journal of Statistics Application and Probability*, 12(1), 75-82. https://doi.org/10.18576/jsap/120216
25. Riaz, M. Rasheed, H. M. K., Salam, A., Hussain, S., Hassan, J., & Ahmed, R. (2023). Catalogue of efficient repeated measurements designs for p_3 =5, 6, 7. *Wah University Journal of Science and Technology*, *7*(1), 19-25.

26. Rajab, R., Ahmed, R., Shehzad, F., & Daniyal, M. (2022). Universal Optimality of Circular Balanced Repeated Measurements Designs through Method of Cyclic Shifts. *Communications in Statistics-Theory and Methods*, *52*(19), 7057-7068. https://doi.org/10.1080/03610926.2022.2038628
27. Rasheed, H. M. K., Khan, H., Ahmed, R., & Jamal, F. (2021). Minimal Circular Nearly Strongly Balanced Repeated Measurements Designs in Unequal Period Sizes. *Proceedings of the Pakistan Academy of Sciences: A: Physical and Computational Sciences, 58*(4), 59-65. https://doi.org/10.53560/PPASA(58-4)633
28. Rasheed, K., Bashir, Z., Tahir, M. H., Shehzad, F., & Ahmed, R. (2022). Efficient minimal circular weakly balanced repeated measurements designs and their conversion into efficient minimal circular balanced and strongly balanced. *Communications in Statistics-Simulation and Computation*, *51*(10), 6197-6207. https://doi.org/10.1080/03610918.2022.2076871
29. Shabbir, J., Rasheed, H. M. K., Noreen, K., Khan, A., Ghani, M. A., & Ahmed, R. (2023). An efficient class of repeated measurements designs to control the residual effects using periods of three different sizes. *Proceedings of the Pakistan Academy of Sciences: A: Physical and Computational Sciences*, *60*(1), 25-32. https://doi.org/10.53560/PPASA(60-1)665
30. Sharma, V. K., Jaggi, S., & Varghese, C. (2003). Minimal balanced repeated measurements designs. *Journal of Applied Statistics*, *30*, 867-872. https://doi.org/10.1080/0266476032000075958
31. Hassan, M. U., Jabeen, R., Ahmed, R., & Sajjad, M. (2022). Efficient minimal circular strongly partially balanced RMDs in periods of two different sizes. *Thailand Statistician*, *20*(1), 80-97. https://doi.org/10.18576/jsap/120216
32. Williams, E. R. (1949). Experimental designs balanced for the estimation of residual effects of treatments. *Australian Journal of Science Research*, *A2*, 149-168. https://doi.org/10.1071/CH9490149