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Economical Group and Modified Group Chain Sampling Plans for Weibull Distribution

Kinza Tanveer[*1] *and Yaseen Arbab Khattak*[2]

Abstract

The acceptance sampling plan/strategy is one of the essential techniques in statistical quality control. When the testing is very destructive and expensive, a generally small sample is reserved from the lot with zero acceptance number. This situation is biased for the producer because, with the minimum increase in the proportion of defective, the lot acceptance probability drops immediately. In order to save time, money, and sample size, the Group chain sampling plan (GChSP) and Modified Group Chain Sampling plan (MGChSP) are highly beneficial in destructive testing for high-quality items. They also provide a more accurate lot acceptance probability $L(P)$ than the chain sampling plan. In this study, the economic model of the group chain and modified group chain are constructed for the minimum total cost of Weibull distribution, and these plans are compared. The study is generally divided into three phases; first, to develop minimized total cost procedures for Weibull distribution; the second stage is to obtain OC function using Weibull distributions; the last stage is to compare the total cost of GChSP and MGChSP. The result displays the minimum sample size, lot acceptance probability, and proportion defective for GChSP and MGChSP. Tables and the total cost needed for the life test are presented.

Keywords

Weibull distribution, Group chain sampling plan, Modified Group Chain Sampling plan.

1. Introduction

The technique of Acceptance Sampling is beneficial for determining the average lifetime of electronic devices, especially for the test time, t and the number of defectives c. However, there is a situation in which the testing is very destructive and costly, so a small sample is taken from the lot, usually with zero acceptance number. This situation is unfair for producers and consumers because the lot acceptance probability $L(p)$ drops rapidly with a small quantity of rise in the proportion of defectives. The chain sampling plan is beneficial in this situation because the lot decision is based on previous lot information.

Sometimes when one hundred percent inspection is impossible for the experimenter and the testing is very costly, acceptance sampling provides tools to handle it. For example, when the testing products are destructive products like electronic items (mobiles, bulbs,

* Corresponding author
[1] Faculty of Management Sciences, HITEC University Taxila, Pakistan.
Email: kinzatanveer1990@gmail.com
[2] Department of Statistics, Islamia College Peshawar, Pakistan.

energy savers, and tube lights) and the producer wants to test the average lifetime of the product, then it is not possible to test every item because it is very destructive to test each item. It is also very costly for the producer to test every item and observe them. Therefore, acceptance sampling helps to take a sample from the lot, which is accepted or rejected after observing these samples. The producer wants to improve its quality so that the rejection chances of the loT are minimized. Eventually, the history of the supplier is made as a good one, and due to this good history of the producer, confidence is made in its product/lot, and it is very costly and time-consuming to inspect all items. With the history of the supplier and the lot, a sample is drawn from the lot and then based on this information, the lot is accepted or rejected.

Teh *et al.* (2019a, 2019b) proposed the group chain and modified Group Chain Acceptance Sampling Plans built on the minimum angle method. Aziz *et al.* (2020) established the performance and comparison of the GChSP and MGChSP based on mean product lifetime for Rayleigh distribution. Recently, Teh *et al.* (2020, 2021) introduced a new method for GChSP and new group chain sampling plans (NGChSP-1) for the generalized exponential distribution.

2. Methodology

Mughal *et al.* (2015) introduced the GChSP for Pareto distribution of second kind in order to reduce the sample size. These acceptance plans are the extension of the work of Dodge (1955) and Govindaraju and Lai (1998), and lot acceptance probability for the GChSP and MGChSP can be calculated using acceptance sampling procedures, as shown in Figures 1 and 2, respectively.

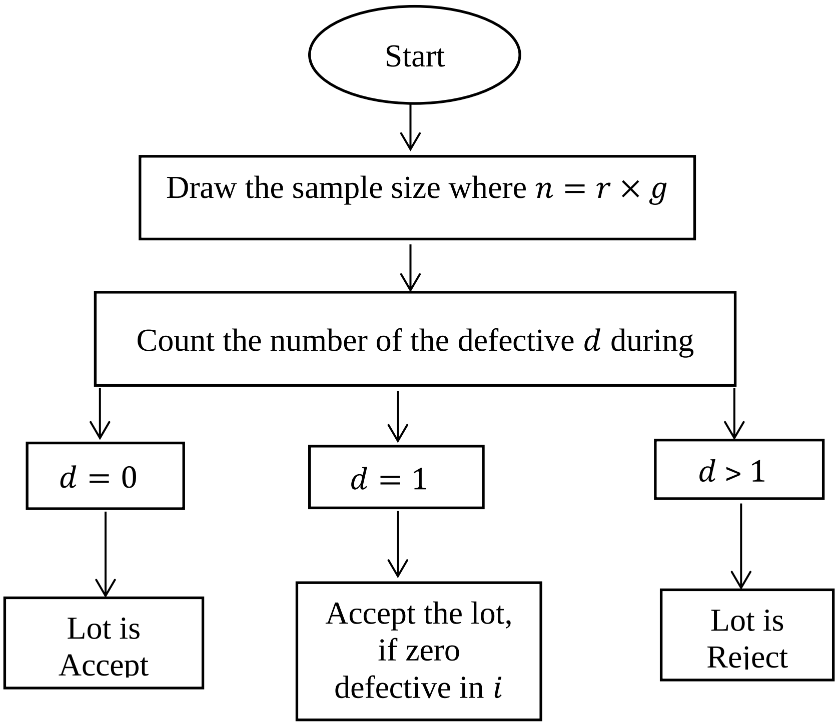

Figure 1: Flow chart for group chain sampling plan.

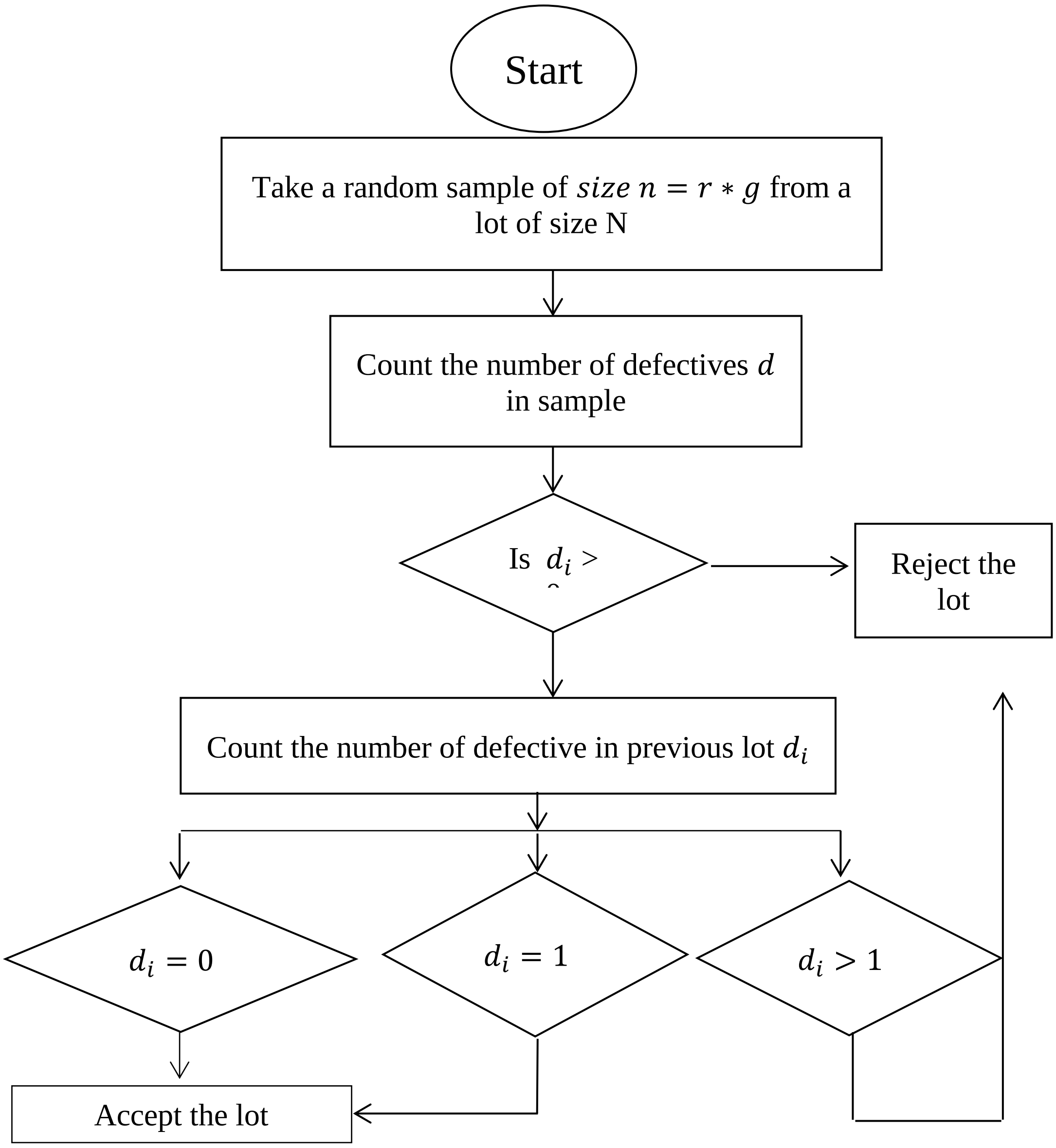

Figure 2: Flow chart for MGChSP.

The proportion of defective p is computed using the lifetime distribution. Mughal *et al.* (2015) suggested GChSP for truncated life tests under the Pareto distribution of the second kind. The CDF of Weibull distribution is:

$$F(t;\lambda,m) = \left[1 - exp\left\{-\left(\frac{t}{\lambda}\right)^m\right\}\right], \qquad t \geq 0 \tag{1}$$

Also, the mean μ of the Weibull distribution is

$$\mu = \frac{\lambda}{m}\Gamma\left(\frac{1}{m}\right) \tag{2}$$

where, the Weibull distribution mean, μ is established on *m* scale and λ shape parameters. The proportion of defective, p is

$$p = F(t_0;\ \lambda, m) = 1 - exp\left(-a^m\left(\frac{\mu}{\mu 0}\right)^{-m}\left(\frac{\Gamma\left(\frac{1}{m}\right)}{m}\right)^m\right). \quad (3)$$

To determine the value of p, mean ration=1 and some pre-specified values of testing time a are used. Now the values of p obtained are presented in Table 1 as follows.

Table 1: Lot proportion defectives, p.

	a								
m	**0.1**	**0.3**	**0.5**	**0.7**	**0.8**	**1.0**	**1.2**	**1.5**	**2.0**
1	0.0952	0.2592	0.3935	0.5034	0.5506	0.6321	0.6988	0.777	0.8647
2	0.0079	0.0682	0.1783	0.3194	0.3951	0.5441	0.6773	0.8292	0.9568
3	0.0014	0.0190	0.0852	0.2167	0.3055	0.5094	0.7079	0.9096	0.9966

Table 1 observes that the proportion of defective decreases as the value of the shape parameter of the Weibull distribution increases from 1 to 3. Also, as the testing time a increases, the proportion of detective p increases to all values of shape parameter m. e.g., for $m = 1$, as the testing time is increasing from 0.1 to 2.0, i.e., pre-specified values of a, the proportion of defective p also increases from 1 percent to 87 percent.

Hsu (2009) proposes an economical design for a single sampling plan. Similarly, Aslam *et al.* (2014) developed the economic layout of a group sampling plan using the Bayesian approach. Initially, the economic model for the group chain and modified group chain sampling plans is developed for the truncated life test with the same design parameter that fulfilled the consumer for Weibull distribution. The mathematical form of the model for GChSP is:

$$Minimize\ (TC) = C_i(ATI) + C_f D_d + C_0 D_n + gC_g \quad (4)$$

The Bayesian approach was used by Aslam *et al.* (2014) to estimate the value of p for the group acceptance sampling plan. Hsu (2009) fixed the value of p to find the economic model's parameter. In equation (4), C_i denoted the inspection cost per item, C_f Denotes the internal failure cost, C_0 denote the cost of outgoing defective and C_g Denote the cost per group. The upstairs costs (see Hsu, 2009 and Hsu and Hsu, 2012) are known as cost parameters. Let D_d Denote the number of defective items detected and D_n Denote the number of defective items being not detected. We have

$$D_d = [rgp + \{1 - L(p)\}\{N - (rg)\}p] \quad (5)$$

$$D_n = L(p)(N - rg)p \quad (6)$$

Now, the average total inspection (ATI) is

$$ATI = [rg + \{1 - L(p)\}\{N - (rg)\}]\ . \quad (7)$$

Figure 3 characterizes the economic model using Weibull distribution and comparison of the plans, such as GChSP and MGChSP.

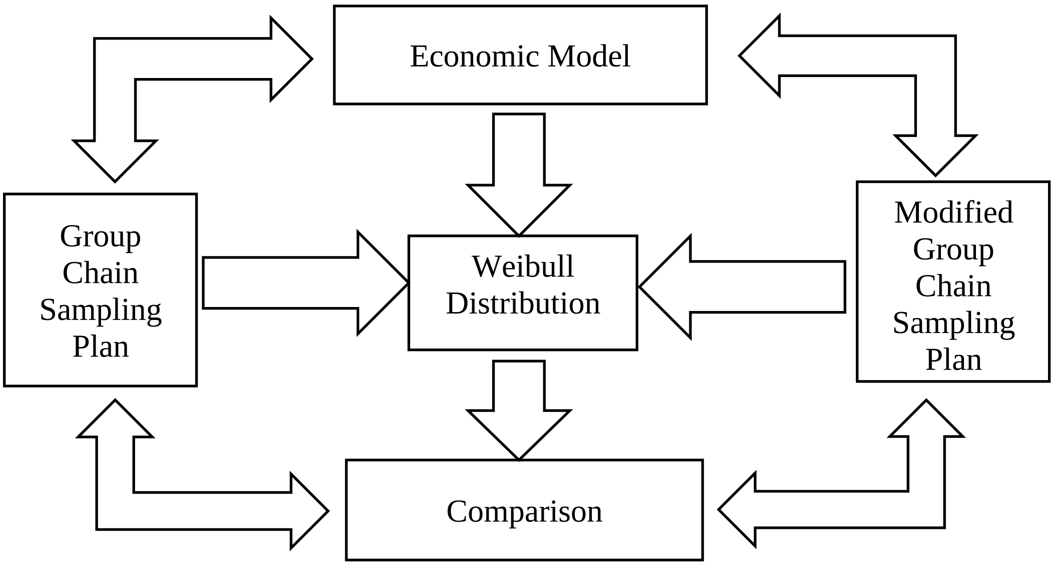

Figure 3: Flow chart for comparison of GChSP and MGChSP using Economic model.

3. Analysis of data

The whole procedure is already defined with the help of Figures. Now the GChSP is fully established to ensure that the acceptance probability increases by increasing the mean ratio of the product. In chain sampling plans, the lot acceptance is based on the results of previous lots, and the group concept is more concise than the results by increasing the acceptance probability. The lot that comes for inspection is accepted if the number of failures exceeds the acceptance number c. The $L(p)$, for GChSP is shown in equation (8) as:

$$L(p) = (1-p)^{r\times g} + (r\times g)\times p(1-p)^{(r\times g)-1}(1-p)^{r\times g\times i} \leq \beta \quad (8)$$

As here, the main concern is the consumer and its risk for different levels of consumer's risk β $(0.25, 0.10, 0.05, 0.01)$, the pre-specified testing time a $(0.5, 0.7, 0.8, 1.0, 1.2, 1.5, 2.0)$, number of testers in a group's, $r(2, 3, 4, 5)$ and allowable preceding lots i $(1, 2, 3, 4)$ are used to obtain the number of groups. Mughal *et al.* (2015a), Mughal and Aslam (2011), and Aslam *et al.* (2010) have also been used in their research. The optimal number of groups for GChSP for the shape parameter m=1 is shown in Table 2 and satisfies the preceding inequality.

In MGChSP, the lot that is about to be inspected is accepted if the current lot has no defective products, but the preceding i lot has only one defective product, and the remaining $i-1$ lots have none. The outcomes for MGChSP can be illustrated in equation (9).

$$L(p) = [\, \{(1-p)^{r\times g}\}^{i+1} \;+ i\{(1-p)^{(r\times g)}\}^{i} \,\{(r\times g)(p)(1-p)^{(r\times g)-1}\}\,] \tag{9}$$

The minimum number of group size for MGChSP is obtained and satisfy the following inequality:

$$L(p)_{MGChSP} \leq \beta$$

The optimal number of groups for MGChSP when the shape parameter of Weibull distribution $m = 1$ is shown in Table 3.

Table 2. Number of optimal groups for GChSP with shape parameter $m = 1$.

			a					
β	***r***	***i***	**0.7**	**0.8**	**1.0**	**1.2**	**1.5**	**2.0**
0.25	2	1	2	2	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	2	2	2	2	1	1
	3	2	2	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.05	2	1	3	2	2	2	2	1
	3	2	2	2	2	1	1	1
	4	3	2	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	4	3	3	2	2	2
	3	2	3	2	2	2	2	1
	4	3	2	2	2	1	1	1
	5	4	2	2	1	1	1	1

Table 3: Number of optimal groups for MGChSP with shape parameter $m = 1$.

			a					
β	***r***	***i***	**0.7**	**0.8**	**1.0**	**1.2**	**1.5**	**2.0**
0.25	2	1	1	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.10	2	1	2	1	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.05	2	1	2	2	1	1	1	1
	3	2	1	1	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1
0.01	2	1	2	2	2	2	1	1
	3	2	2	2	1	1	1	1
	4	3	1	1	1	1	1	1
	5	4	1	1	1	1	1	1

The value of shape parameters is specified in the number of groups for Weibull distribution in Table 2 and Table 3. Using the several values of the design parameter $r = 2$ and $i = 1$ and cost parameters according to Aslam *et al.* (2014) ($C_i = 1, C_f = 2,\ C_0 = 1, C_g = 3$) and the population size$N = 1{,}000$. The number of groups g in Tables 1 and 2 are used and find the lot acceptance probability $L(p)$, average total inspectionATI, defective items detected D_d and not detected D_n .Furthermore, minimized the total cost of Weibull distribution for GChSP and MGChSP, shown in Tables 3 and 4 with shape parameter $m = 1$.

Table 4 displays the minimized total cost of GChSP for Weibull distribution and minimized total cost rises when the consumer's risk decreases. It can be observed that for $r = 2\ and\ i = 1\ \ when\ \beta = 0.05$ the total cost for GhSP increases from 1991.736 to 2690.268 as "a"pre-specified time rises from 0.7 to 2.0 and proportion of defectives p increases from 0.5034 to 0.8646. It is evident that the minimized total cost also decreases when the shape parameter values increase.

Table 4: Minimized total cost for GChSP.

β	g	a	p	$L(p)$	ATI	D_d	D_n	TC	n
0.25	2	0.7	0.5034	0.0758	924.498	465.406	38.008	1899.318	4
	2	0.8	0.5506	0.0489	951.288	523.846	26.824	2031.806	4
	1	1.0	0.6321	0.1982	802.118	507.035	125.085	1944.274	2
	1	1.2	06988	0.1289	871.351	608.905	89.900	2182.063	2
	1	1.5	0.7768	0.0670	933.089	724.886	51.983	2437.843	2
	1	2.0	0.8646	0.0226	977.442	845.160	19.504	2690.268	2
0.10	2	0.7	0.5034	0.0758	924.498	465.406	38.008	1899.318	4
	2	0.8	0.5506	0.0489	951.288	523.846	26.824	2031.806	4
	2	1.0	0.6321	0.0206	976.461	619.137	12.983	2236.719	4
	2	1.2	0.6988	0.0088	991.177	692.640	06.165	2388.623	4
	1	1.5	0.7768	0.0670	933.089	724.886	51.983	2437.843	2
	1	2.0	0.8646	0.0226	977.442	845.160	19.504	2690.268	2
0.05	3	0.7	0.5034	0.0163	983.734	495.226	08.188	1991.376	6
	2	0.8	0.5506	0.0489	951.288	523.846	26.824	2031.806	4
	2	1.0	0.6321	0.0206	976.461	619.137	12.983	2236.719	4
	2	1.2	0.6988	0.0088	991.177	692.640	06.165	2388.623	4
	2	1.5	0.7768	0.0025	997.445	774.885	01.984	2555.201	4
	1	2.0	0.8646	0.0226	977.442	845.160	19.504	2690.268	2
0.01	4	0.7	0.5034	0.0038	996.221	501.512	01.902	2013.149	8
	3	0.8	0.5506	0.0087	991.324	545.893	04.777	2096.88	6
	3	1.0	0.6321	0.0025	997.473	630.523	01.597	2269.117	6
	2	1.2	0.6988	0.0088	991.177	692.640	06.165	2388.623	4
	2	1.5	0.7768	0.0025	997.445	774.885	01.984	2555.201	4
	2	2.0	0.8646	0.0003	999.663	864.337	00.291	2734.701	4

Table 5: Minimized Total Cost for MGChSP.

β	g	a	p	$L(p)$	ATI	D_d	D_n	TC	n
0.25	1	0.7	0.5034	0.1220	878.209	442.103	61.311	1826.730	2
	1	0.8	0.5506	0.0856	914.516	503.597	47.073	1971.781	2
	1	1.0	0.6321	0.0414	958.612	605.958	26.162	2199.690	2
	1	1.2	0.6988	0.0197	980.308	685.045	13.761	2367.163	2
	1	1.5	0.7768	0.0063	993.683	771.962	04.907	2545.510	2
	1	2.0	0.8646	0.0009	999.086	863.875	00.790	2730.632	2
0.10	2	0.7	0.5034	0.0055	994.488	500.640	02.774	2004.540	4
	1	0.8	0.5506	0.0856	914.516	503.597	47.073	1971.781	2
	1	1.0	0.6321	0.0414	958.612	605.958	26.162	2199.690	2
	1	1.2	0.6988	0.0197	980.308	685.045	13.761	2367.163	2
	1	1.5	0.7768	0.0063	993.683	771.962	04.907	2545.510	2
	1	2.0	0.8646	0.0009	999.086	863.875	00.790	2730.632	2
0.05	2	0.7	0.5034	0.0055	994.488	500.640	02.774	2004.540	4
	2	0.8	0.5506	0.0024	997.609	549.354	01.316	2103.631	4
	1	1.0	0.6321	0.0414	958.612	605.958	26.162	2199.690	2
	1	1.2	0.6988	0.0197	980.308	685.045	13.761	2367.163	2
	1	1.5	0.7768	0.0063	993.683	771.962	04.907	2545.510	2
	1	2.0	0.8646	0.0009	999.086	863.875	00.790	2730.632	2
0.01	2	0.7	0.5034	0.0055	994.488	500.640	02.774	2004.540	4
	2	0.8	0.5506	0.0024	997.609	549.354	01.316	2103.631	4
	2	1.0	0.6321	0.0004	999.552	631.837	00.283	2269.510	4
	2	1.2	0.6988	0.0000	999.915	698.747	00.059	2403.471	4
	1	1.5	0.7768	0.0063	993.683	771.962	04.907	2545.510	2
	1	2.0	0.8646	0.0009	999.086	863.875	00.790	2730.632	2

4. Data example with discussion

The data are based on the number of million spins before 23 ball bearings failed in the truncated life tests that Rao and Ramesh (2016) have already discussed. In this data example, we study the set of input parameters such as population size 1000, number of the tester in a group r, allowable acceptance numbers i , testing time a, and fixed value of cost parameters C_i , C_f , C_0 , C_g Aslam *et al.* (2014). We use Microsoft Excel computer software to obtain minimum sample size $n = rg$ for GChSP and MGChSP in Weibull distribution under$L(p) \leq \beta$. To specify the performance dimensions, Tables 1 and 2 show the number of groups, and Tables 3 and 4 show the proportion of defective p, average total inspection ATI, defective items detected D_d and not detected D_d And minimum total cost. Figures 4 and 5 show that the proportion of defective, average total inspection, and total cost increased, and the lot acceptance probability decreased. Also, the minimum total cost increases when the consumer's risk decreases and termination and pre-specified testing times increase. This data example specifies that without integrating the producer's and the consumer's risk requirements into the economic design of the GChSP and MGChSP, the plans obtained by the model, although reducing the producer's and the consumer's total quality cost.

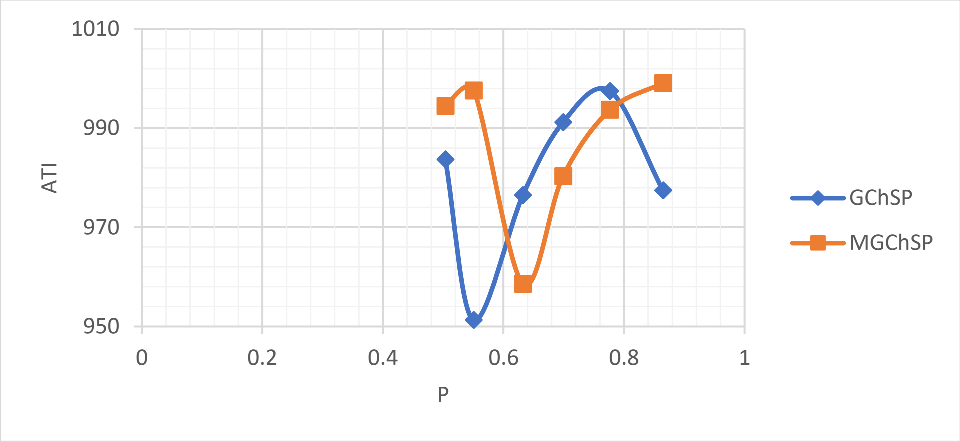

Figure 4: Average total inspection versus proportion defective of GChSP and MGChSP.

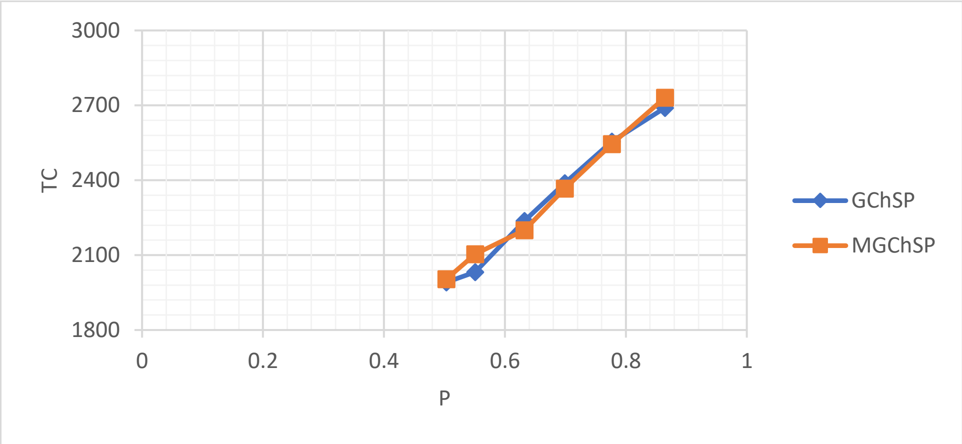

Figure 5: Minimize total cost versus proportion defective of GChSP and MGChSP.

Table 6: Comparison of the total cost of the GChSP and MGChSP, $r = 2, i = 1, \beta = 0.05$

a	p	T.C. through the economic model of GChSP	T.C. through the economic model of MGChSP
0.7	0.5034	1991.376	2004.540
0.8	0.5506	2031.806	2103.631
1.0	0.6321	2236.719	2199.690
1.2	0.6988	2388.623	2367.163
1.5	0.7768	2555.201	2545.510
2.0	0.8646	2690.268	2730.632

From Table 6, it seems that for the design parameter value $r = 2$ and $i = 1$ and $\beta = 0.05$, the total cost of GChSP increases from 1191.376 to 2690.268 and the total cost of MGChSP increases from 2004.540 to 2730.632 as the value of pre-specified testing time a and proportion of defectives p rises from 0.5034 to 0.8646. This section provides an analysis of the GChSP and MGChSP under optimal total cost, and the total cost of GChSP is the least as compared to the MGChSP due to the different factors.

5. Conclusions

The proposed plan provides a straightforward methodology for estimating the total cost. The method for finding the optimal plan parameters of the GChSP having the lowest overall total cost has been provided. As a result of this study, we can say that the GChSP is more cost-time effective than the MGChSP. This study can be extended through the economic model in a two-stage and modified two-stage group chain sampling plan for future work.

6. Future recommendation

According to the suggestion of anonymous reviewer, we will develop a new plan in which we consider the proportions of defectives that are less than 0.1.

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