ISSN 1684-8403
Journal of Statistics
Volume 24, 2017. pp. 120-132

A Class of Improved Estimators for Estimating the Population Mean in Double Sampling and its properties

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Abstract

In this paper a class of improved estimators has been proposed for estimating population mean in Two Phase (Double) Sampling when information of auxiliary variable is available. Under Simple Random Sampling (SRSWOR), expressions of Mean Square Error and bias have been derived to make comparison of suggested class with wide range of other estimators. Empirical study has also been given using five different natural populations. Empirical study confirmed that the suggested class of improved estimators is more efficient under relative efficiency criterion. We have also developed some ranges of constant where we get minimum mean square error. These constant values are useful for surveyor.

Keywords

Double sampling, Auxiliary variable, Bias, Mean square error

1. Introduction

Consider the problem of estimating population mean of variable of interest in the presence of auxiliary variable but it is expensive to collect information on study variable for a large sample and information on auxiliary variables is not known in advance.

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In such a situation, Two-Phase Sampling techniques can be used to decrease the cost for little sacrifice on efficiency. Samiuddin and Hanif (2006) have discussed two possibilities of availability of information of single auxiliary variable. The first situation is when information of auxiliary variables is available for the whole population and in the second it is not available for the complete population. Samiuddin and Hanif (2007) have argued that Single Phase Sampling; if cost is not matter of concern; is a suitable technique in first situation. In Two Phase Sampling a larger sample is selected from population and information of auxiliary variable is recorded and at second phase information on auxiliary variable and variable of interest is recorded for estimation. Two Phase Sampling can, therefore, be used when cost of drawing large sample is too high or when information of auxiliary variable is not available for population.

In Simple Random Sampling WithOut Replacement design in each phase, the Two Phase Sampling scheme is as follows:

- At first-phase a large sample of size $n_1 (n_1 \subset N)$ is selected by Simple Random Sampling With and WithOut Replacement SRSWR(WOR) from a finite population of *N* and only auxiliary variable information is observed for these units. This sample is called as first-phase sample.
- At second-phase another sample of size $n_2 (n_2 \subset n_1)$ is chosen by SRSWR(WOR) from the first-phase sample n_1, and information on the variable understudy may be obtained. This sample is called as second-phase sample. Second-phase sample can also be taken independent of the first sample, Bose (1943).

Let us consider the finite population of size N and let $\bar{Y}, \bar{X}$ and $\bar{Z}$ are the population means of the variables y, x and z respectively. The sample of size 'n_1'at the first phase is drawn from the population and we assume that $\bar{x}_1 = \sum_{i=1}^{n_1} \frac{x_i}{n_1}$ be the sample mean of variable 'x' for the first phase sample, whereas $\bar{x}_2 = \sum_{i=1}^{n_2} \frac{x_i}{n_2}$, $\bar{y}_2 = \sum_{i=1}^{n_2} \frac{y_i}{n_2}$ and $\bar{z}_2 = \sum_{i=1}^{n_2} \frac{z_i}{n_2}$ are the means of variable x, y and z respectively for the sample of size 'n_2 obtained at second phase'.

1.1 Notations: In Simple Random Sampling With and WithOut Replacement with single auxiliary variables we may obtained as,

$$e_{0(2)} = \bar{e}_{y2} = \frac{\bar{y}_2 - \bar{Y}}{\bar{Y}}$$
$$e_{1(2)} = \bar{e}_{x2} = \frac{\bar{x}_2 - \bar{X}}{\bar{X}} \ ,$$
$$e_{2(2)} = \bar{e}_{s^2_{x2}} = \frac{s^2_{x2} - S^2_{x2}}{S^2_{x2}}$$
$$\mu_{rsl} = \frac{1}{N}\sum_{i=1}^{N}(X_i - \bar{X})^r (Y_i - \bar{Y})^s (Z_i - \bar{Z})^l \qquad n_1 \subset N, \quad n_2 \subset n_1$$
$$e_{\bar{y}(2)} = e_{o(2)}$$
$$\bar{y}_{(2)} = \bar{Y}\left(1 + e_{o(2)}\right)$$
$$e_{o(2)} = \frac{e_{\bar{y}(2)}}{\bar{Y}}$$
$$e_{\bar{x}(1)} = e_{1(1)}$$
$$\bar{x}_{(1)} = \bar{X}\left(1 + e_{1(1)}\right)$$
$$e_{1(1)} = \frac{e_{\bar{x}(1)}}{\bar{X}}$$
$$e_{s^2_{x(1)}} = e_{2(1)}$$
$$s^2_{x(1)} = S^2_x\left(1 + e_{2(1)}\right)$$
$$e_{2(1)} = \frac{e_{s^2_{x(1)}}}{S^2_x}$$
$$E\left(e^2_{o(2)}\right) = \theta_2 \frac{\mu_{2o}}{\bar{Y}^2}$$
$$E\left(e^2_{1(1)}\right) = \theta_1 \frac{\mu_{o2}}{\bar{X}^2}$$
$$E\left(e^2_{1(2)}\right) = \theta_2 \frac{\mu_{o2}}{\bar{X}^2}$$
$$E\left(e_{o(2)}e_{1(1)}\right) = \theta_1 \frac{\mu_{11}}{\bar{X}\bar{Y}}$$
$$E\left(e_{o(2)}e_{1(2)}\right) = \theta_2 \frac{\mu_{11}}{\bar{X}\bar{Y}}$$
$$E\left(e_{o(2)}e_{2(1)}\right) = \theta_1^* \frac{\mu_{12}}{\bar{Y}\mu_{o2}}$$
$$E\left(e^2_{2(1)}\right) = \theta_1^*\left(\beta_{o2(1)} - 1\right)$$
$$E\left(e_{1(1)}e_{2(1)}\right) = \theta_1^* \frac{\mu_{o3}}{\bar{X}\mu_{o2}} \qquad \text{where } \theta_2 = \frac{1-f}{n_2} \quad \theta_1 = \frac{1-f}{n_1} \qquad \theta_1^* = \frac{1}{n_1}$$

Ratio method of estimation has been very popular in Single Phase Sampling. In Double Sampling this method has attracted number of survey statisticians and large number of estimators has been proposed from time to time. The Ratio estimator, in Double Sampling, can be constructed in two ways namely known and unknown information of mean of auxiliary variable. The Ratio estimator when mean of auxiliary variable is known is given as:

$$\bar{y}_{R(2)} = \frac{\bar{y}_2}{\bar{x}_2}\bar{X} \tag{1.2}$$

and

$$\bar{y}_{P(2)} = \frac{\bar{y}_2}{\bar{X}}\bar{x}_2 \tag{1.3}$$

and Mean Square Errors of eq. (1.1) and eq. (1.2) are,

$$MSE\left(\bar{y}_{R(2)}\right) = \theta_2\bar{Y}^2\left(C_y^2 + C_x^2 - 2\rho_{xy}C_xC_y\right) \tag{1.4}$$

and

$$MSE\left(\bar{y}_{P(2)}\right) = \theta_2\bar{Y}^2\left(C_y^2 + C_x^2 + 2\rho_{xy}C_xC_y\right) \tag{1.5}$$

The Mean Square Error given in eq. (1.4) is precisely the Mean Square Error of Classical Ratio estimator of Single Phase Sampling based upon a sample of size n_2. Sahoo and Sahoo (1993, 1994) proposed Ratio and Product type estimators for population mean when population information of auxiliary variable is known

$$\bar{y}_{SSr(2)} = \frac{\bar{y}_2}{\bar{z}_1}\bar{Z} \tag{1.6}$$

and

$$\bar{y}_{SSp(2)} = \frac{\bar{y}_2}{\bar{Z}}\bar{z}_1 \tag{1.7}$$

Mean Square Errors of eq. (1.6) and eq. (1.7) are,

$$MSE\left(\bar{y}_{SSr(2)}\right) = \bar{Y}^2\left[\theta_2C_y^2 + \theta_1\left(C_z^2 - 2\rho_{yz}C_yC_z\right)\right] \tag{1.8}$$

$$and$$

$$MSE\left(\bar{y}_{SSp(2)}\right) = \bar{Y}^2\left[\theta_2C_y^2 + \theta_1\left(C_z^2 + 2\rho_{yz}C_yC_z\right)\right] \tag{1.9}$$

Comparison of eq. (1.6) with eq. (1.2) shows that estimator $\bar{y}_{Sr(2)}$ will be more precise than $\bar{y}_{SSr(2)}$ if $\rho_{xy} > \frac{C_x}{2C_y}$ holds.

2. Proposed estimator - I

In this paper, the Generalized Exponential Ratio and Product type estimators have been suggested under Double Sampling design when the information of auxiliary variable is known.

$$t_{1(2)}^{g} = \bar{y}_2 exp\left\{\frac{\bar{X}^{\frac{1}{h}} - \bar{x}_1^{\frac{1}{h}}}{\bar{X}^{\frac{1}{h}} + (a-1)\bar{x}_1^{\frac{1}{h}}}\right\} \tag{2.1}$$

$$t_{1(2)}^{g} = \bar{y}_2 exp\left\{\frac{\bar{x}_1^{\frac{1}{h}} - \bar{X}^{\frac{1}{h}}}{\bar{X}^{\frac{1}{h}} + (a-1)\bar{x}_1^{\frac{1}{h}}}\right\} \tag{2.2}$$

where a is optimized constant and h is the generalized constant. Using the notations eq. (1.1), we may write the estimator eq. (2.1) as:

$$t_{1(2)}^{g} = \bar{Y}\left(1+e_{o(2)}\right) exp\left\{\frac{\bar{X}^{\frac{1}{h}} - \bar{X}^{\frac{1}{h}}\left(1+e_{1(1)}\right)^{\frac{1}{h}}}{\bar{X}^{\frac{1}{h}} + (a-1)\bar{X}^{\frac{1}{h}}\left(1+e_{1(1)}\right)^{\frac{1}{h}}}\right\}$$

$$t_{1(2)}^{g} = \bar{Y}\left(1+e_{o(2)}\right) exp\left[\frac{-e_{1(1)}}{ah\left\{1+\frac{e_{1(1)}}{h}-\frac{e_{1(1)}}{ah}\right\}}\right]$$

$$t_{1(2)}^{g} = \bar{Y}\left(1+e_{o(2)}\right) exp\left[\frac{-e_1}{ah}\left\{1+\frac{e_{1(1)}}{h}-\frac{e_{1(1)}}{ah}\right\}^{-1}\right]$$

Expanding the series $\left\{1+\frac{e_{1(1)}}{h}-\frac{e_{1(1)}}{ah}\right\}^{-1}$ upto first order in e_1, we get

$$t_{1(2)}^{g} = \bar{Y}\left(1+e_{o(2)}\right) exp\left[\frac{-e_{1(1)}}{ah}\left(1-\frac{e_{1(1)}}{h}+\frac{e_{1(1)}}{ah}\right)\right]$$

Ignoring the highest order terms of errors, we get

$$t_{1(2)}^{g} = \bar{Y}\left(1+e_{o(2)}\right) exp\left[\frac{-e_{1(1)}}{ah}\right] \tag{2.3}$$

Expand $exp\left[\frac{-e_{1(1)}}{ah}\right]$ function upto order one as:

$$t_{1(2)}^{g} = \bar{Y}\left(1+e_{o(2)}\right)\left[1-\frac{e_{1(1)}}{ah}+\frac{e_{1(1)}^2}{2a^2h^2}\right]$$

$$t_{1(2)}^{g} = \bar{Y}\left(1+e_{o(1)}-\frac{e_{1(1)}}{ah}-\frac{e_{0(2)}e_{1(1)}}{ah}+\frac{e_{1(1)}^2}{2a^2h^2}\right) \tag{2.4}$$

Applying expectation on eq. (2.4) and using eq. (1.1), we get

$$MSE\left(t_{1(2)}^{g}\right) = \bar{Y}^2\left(\frac{\theta_2\mu_{20}}{\bar{Y}^2}+\frac{\theta_1\mu_{02}}{a^2h^2\bar{X}^2}-\frac{2\theta_1\mu_{11}}{2a\bar{X}\bar{Y}}\right)$$

Differentiate eq. (2.5) w.r.t. “a”, we get

$$a = \frac{\bar{Y}\mu_{02}}{h\bar{X}\mu_{11}} \tag{2.5}$$

Put the value of ‘a’ in eq. (2.5), we get the minimum $MSE\left(t_{1(2)}^{g}\right)$.

$$MSE\left(t_{1(2)}^{g}\right) = \mu_{20}(\theta_2 - \theta_1\rho^2) \tag{2.6}$$

The bias of $t_{1(2)}^{g}$ to first order of approximation, is given by

$$Bias\left(t_{1(2)}^{g}\right) = \bar{Y}\left[\frac{\theta_1\mu_{11}^2}{2\bar{Y}^2\mu_{02}} - \frac{\theta_1\mu_{11}^2}{\bar{Y}^2\mu_{02}}\right]$$

$$Bias\left(t_{1(2)}^{g}\right) = -\frac{\theta_1\mu_{11}^2}{2\bar{Y}\mu_{02}} \tag{2.7}$$

2.1 Family of estimator for $t_{1(2)}^{g}$ ***:*** Some members of $t_{1(2)}^{g}$ are given in Table 1.

3. Generalized Exponential Ratio type estimator-II when sampling is done with replacement

The following Generalized Exponential estimator has been suggested under Double Sampling with replacement when the information of auxiliary variable is known.

$$t_{2(2)}^{g} = \bar{y}_2 exp\left\{\frac{S_x^{2^{\frac{1}{h}}} - s_{x1}^{2^{\frac{1}{h}}}}{S_x^{2^{\frac{1}{h}}} + (a-1)s_{x1}^{2^{\frac{1}{h}}}}\right\} \tag{3.1}$$

Using the notations eq. (1.1), we may write the estimator eq. (3.1) as:

$$t_{2(2)}^{g} = \bar{Y}\left(1 + e_{o(2)}\right)exp\left\{\frac{S_x^{2^{\frac{1}{h}}} - S_x^{2^{\frac{1}{h}}}\left(1+e_{2(1)}\right)^{\frac{1}{h}}}{S_x^{2^{\frac{1}{h}}} + (a-1)S_x^{2^{\frac{1}{h}}}\left(1+e_{2(1)}\right)^{\frac{1}{h}}}\right\} \tag{3.2}$$

After some simplification, we get eq. (3.2) as:

$$t_{2(2)}^{g} = \bar{Y}\left(1 + e_{o(2)}\right)exp\left[\frac{S_x^{2^{\frac{1}{h}}}}{S_x^{2^{\frac{1}{h}}}}\left\{\frac{1 - 1 - \frac{e_{2(1)}}{h}}{1 + a - 1 - \frac{ae_{2(1)}}{h} - \frac{e_{2(1)}}{h}}\right\}\right]$$

or

$$t_{2(2)}^{g} = \bar{Y}\left(1 + e_{o(2)}\right)exp\left[\frac{-e_{2(1)}}{ah\left[1 + \frac{e_{2(1)}}{h} - \frac{e_{2(1)}}{ah}\right]}\right]$$

or

$$t_{2(2)}^{g} = \bar{Y}\left(1 + e_{o(2)}\right)exp\left[\frac{-e_{2(1)}}{ah}\left[1 + \frac{e_{2(1)}}{h} - \frac{e_{2(1)}}{ah}\right]^{-1}\right] \tag{3.3}$$

Expanding the series up to first order and re-writing eq. (3.3) as:

$$t_{2(2)}^{g} = \bar{Y}\left(1+e_{o(2)}\right)exp\left[\frac{-e_{2(1)}}{ah}\left[1+\frac{e_{2(1)}}{h}-\frac{e_{2(1)}}{ah}\right]\right] \quad (3.4)$$

or

$$t_{2(2)}^{g} = \bar{Y}\left(1+e_{o(2)}\right)exp\left[\frac{-e_{2(1)}}{ah}\right] \quad (3.5)$$

Expanding the exponential series up to first order and re-writing eq. (3.5) as:

$$t_{2(2)}^{g} = \bar{Y}\left(1+e_{o(2)}\right)\left[1-\frac{e_{2(1)}}{ah}+\frac{1}{2!}\frac{e_{2(1)}^{2}}{a^{2}h^{2}}\right]$$

$$t_{2(2)}^{g} = \bar{Y}\left(1+e_{o(2)}\right)\left[1-\frac{e_{2(1)}}{ah}+\frac{e_{2(1)}^{2}}{2a^{2}h^{2}}\right]$$

or

$$t_{2(2)}^{g} = \bar{Y}\left(1+e_{o(2)}-\frac{e_{2(1)}}{h}+\frac{e_{2(1)}^{2}}{2a^{2}h^{2}}-\frac{e_{o(2)}e_{2(1)}}{ah}\right) \quad (3.6)$$

Taking square and applying expectation of the eq. (3.6) and ignore higher order teams, we get

$$E\left(t_{2(2)}^{g}-\bar{Y}\right)^{2} = \bar{Y}^{2}E\left[e_{o(2)}-\frac{e_{2(1)}}{ah}\right]^{2}$$

$$E\left(t_{2(2)}^{g}-\bar{Y}\right)^{2} = \bar{Y}^{2}\left(E\left(e_{o(2)}^{2}\right)+\frac{E\left(e_{2(1)}^{2}\right)}{a^{2}h^{2}}-\frac{2E\left(e_{o(2)}e_{2(1)}\right)}{ah}\right)$$

Using eq. (1.1), we get

$$MSE\left(t_{2(2)}^{g}\right) = \bar{Y}^{2}\left[\frac{\theta_{2}^{*}\mu_{2o}}{\bar{Y}^{2}}+\frac{\theta_{1}^{*}\left(\beta_{o2(1)}-1\right)}{a^{2}h^{2}}-\frac{2\theta_{1}^{*}\mu_{12}}{ah\bar{Y}\mu_{o2}}\right] \quad (3.7)$$

Differentiate eq. (3.7) w.r.t ‘a’ and h is generalized constant,

$$a = \frac{\bar{Y}\mu_{o2}\left(\beta_{o2(1)}-1\right)}{h\mu_{12}}$$

Put the value of ‘a’ in eq. (3.7), we have minimum $MSE\left(t_{2(2)}^{g}\right)$.

$$MSE\left(t_{2(2)}^{g}\right) = \mu_{2o}\left[\theta_{2}^{*}-\frac{\theta_{1}^{*}\mu_{12}^{2}}{\mu_{2o}\mu_{o2}^{2}\left(\beta_{o2(1)}-1\right)}\right] \quad (3.8)$$

The bias of $t_{2(2)}^{g}$ to first order of approximation, is given by

$$Bias\left(t_{2(2)}^{g}\right) = -\frac{\theta_{1}\mu_{12}^{2}}{\bar{Y}\mu_{o2}^{2}(\beta_{o2}-1)} \quad (3.9)$$

3.3.1 Family of Generalized Ratio type estimator for $t_{2(2)}^{g}$: Some members of $t_{2(2)}^{g}$ in Table 2.

4. Numerical comparison

In this paper, Mean Square Errors of the estimator proposed in eq. (2.6) and eq. (3.8) have been numerically compared with the Mean Square Errors of the existing estimators. For the comparison numbers of populations are considered from real life data. The description of the populations is given in Table 3. In this section, numerical comparison between the proposed estimators and existing estimators using Two Double Sampling with single auxiliary variable has been made for each of the population described in Table 3. Table 4 – Table 8 show the Mean Square Errors and the relative efficiencies for each population with information of single auxiliary variable, respectively. The sample of size n_1 at the first phase is taken equal to 60% of the total sample size n and sample of size n_2 at phase two is taken 67% of n_1.

5. Conclusion

The study of the Table 4 - Table 8 shows that in case of single known auxiliary variable information the proposed estimators i.e. eq. (2.1) and eq. (3.1) are highly efficient and consistent than the existing estimators i.e. eq. (1.2) and eq. (1.6) for each of the population under study. It is also observed that for few choices of h and a detail of which are given in respective Tables, the estimators i.e. eq. (2.1) and eq. (3.1) are highly efficient and consistent than the existing estimators. The choices of the constant are $1.0 \leq h \leq 2.0$ and $3.5 \leq a \leq 6.0$. These constant values are helpful for surveyors.

Table 1: Family of estimators for $t_{1(2)}^{g}$

Family of Exponential-type Ratio Estimator $t_{1(2)R(h,a,)}^{g}$	**Family of Exponential –type product Estimators $t_{1(2)R(h,a)}^{g}$**	**h**	**a**
$t_{1(2)R(1,1)}^{1} = \bar{y}exp\left[\frac{\bar{X}-\bar{x}_1}{\bar{X}}\right]$	$t_{1(2)P(1,1)}^{2} = \bar{y}exp\left[\frac{\bar{x}_1-\bar{X}}{\bar{X}}\right]$	1	1
$t_{1(2)R(2,1)}^{3} = \bar{y}exp\left[\frac{\sqrt{\bar{X}}-\sqrt{\bar{x}_1}}{\sqrt{\bar{X}}}\right]$	$t_{1(2)P(2,1)}^{4} = \bar{y}exp\left[\frac{\sqrt{\bar{X}}-\sqrt{\bar{x}_1}}{\sqrt{\bar{X}}}\right]$	2	1

$t^5_{1(2)R(1,2)} = \bar{y}exp\left[\frac{\bar{X}-\bar{x}_1}{\bar{X}+\bar{x}_1}\right]$ Noor et al. (2012)	$t^6_{1(2)P(1,2)} = \bar{y}exp\left[\frac{\bar{x}_1-\bar{X}}{\bar{X}+\bar{x}_1}\right]$ Noor et al. (2012)	1	2
$t^7_{1(2)R(2,2)} = \bar{y}exp\left[\frac{\sqrt{\bar{X}}-\sqrt{\bar{x}_1}}{\sqrt{\bar{X}}+\sqrt{\bar{x}_1}}\right]$	$t^8_{1(2)P(2,2)} = \bar{y}exp\left[\frac{\sqrt{\bar{x}_1}-\sqrt{\bar{X}}}{\sqrt{\bar{X}}+\sqrt{\bar{x}_1}}\right]$	2	2

The MSE’s of family of estimators are in Appendix A.2.1.

Table 2: Family of estimators for $t^g_{2(2)}$

Family of Exponential-type ratio Estimators $t^g_{2(2)R(h,a)}$	Family of Exponential –type product Estimators $t^g_{2(2)P(h,a)}$	h	a
$t^1_{2(2)R(1,1)} = \bar{y}_2exp\left[\frac{S^2_x-s^2_{x1}}{S^2_x}\right]$	$t^2_{2(2)P(1,1)} = \bar{y}_2exp\left[\frac{s^2_{x1}-S^2_x}{S^2_x}\right]$	1	1
$t^3_{2(2)R(2,1)} = \bar{y}_2exp\left[\frac{\sqrt{S^2_x}-\sqrt{s^2_{x1}}}{\sqrt{S^2_x}}\right]$	$t^4_{2(2)P(2,1)} = \bar{y}_2exp\left[\frac{\sqrt{S^2_x}-\sqrt{s^2_{x1}}}{\sqrt{S^2_x}}\right]$	2	1
$t^5_{2(2)R(1,2)} = \bar{y}_2exp\left[\frac{S^2_x-s^2_{x1}}{S^2_x+s^2_{x1}}\right]$	$t^6_{2(2)P(1,2)} = \bar{y}_2exp\left[\frac{s^2_{x1}-S^2_x}{s^2_{x1}+S^2_x}\right]$	1	2
$t^7_{2(2)R(2,2)} = \bar{y}_2exp\left[\frac{\sqrt{S^2_x}-\sqrt{s^2_{x1}}}{\sqrt{S^2_x}+\sqrt{s^2_{x1}}}\right]$	$t^8_{2(2)P(2,2)} = \bar{y}_2\ \mathrm{exp}\left[\frac{\sqrt{s^2_{x1}}-\sqrt{S^2_x}}{\sqrt{s^2_{x1}}+\sqrt{S^2_x}}\right]$	2	2

The MSE’s of family of estimators are in Appendix A.3.2.

Table 3: Description of the populations

Parameters	Population 1	Population 2	Population 3	Population 4	Population 5
	Source: Applied Linear Statistical Models 2004, Pg 1348, data set 1	Source: Applied Linear Statistical Models 2004, Pg 1348, data set 1	Source: Applied Linear Statistical Models 2004, Pg 1348, data set 3	Source: Applied Linear Statistical Models 2004, Pg 1350, data set 3	Source: Applied Linear Statistical Models 2004, Pg 1352, data set 9
	X:ANB Y:AFS Z: ANN	X: ANB Y:ALS Z:AFS	X : GNRP Y: AMP Z: AMMS	X: AMMS Y: AMP Z: GNRP	X: NOC Y: NI Z: TC
	N=113	N=113	N=36	N=36	N=788
ρ_{xy}	0.7835	0.4093	-0.3877	-0.1885	0.1961
μ_{20}	231.0662	3.6537	0.02656	0.02656	31.3002
μ_{02}	37188.3	37188.3	28389.71	0.06989	0.06154
μ_{110}	1658.645	150.8594	-10.64885	-0.008122	0.2722

Table 4: Mean Square Error and relative efficiencies for Population 1

n_1	n_2	MSE R and RE		MSE SSR 1994 and RE		MSE $t^g_{1(2)}$	MSE $t^g_{2(2)}$	Constant		MSE$t^g_{1(2)}$
		WOR	WR	WOR	WR	WOR	WR	h	a	WOR
12	8	70.656 (302)* (432)**	76.039 (275)*	36.074 (154)* (220)**	41.458 2 (150)*	23.3655 (100)	27.6568 (100)	1 1 1	3.5 3.5 4.0	16.5703 16.9252 16.3633**
18	12	45.3096 (330)* (426)**	50.692 (281)*	26.447 (193*) (248)**	31.830 (176)*	13.7377 (100)	18.0291 (100)	1 1 1	3.5 3.5 4.0	10.7719 10.9944 10.6421**
30	20	25.0324 (415)* (413)**	30.415 (294)*	18.744 (311)* (309)**	24.128 (234)*	6.0355 (100)	10.3269 (100)	1 1 1	3.5 3.5 4.0	6.1332 6.250 6.060**

Table 5: Mean Square Error and relative efficiencies for Population 2

n_1	n_2	MSE R and RE		MSE SSR 1994 and RE		MSE $t^g_{1(2)}$	MSE $t^g_{2(2)}$	Constant		MSE$t^g_{1(2)}$
		WOR	WR	WOR	WR	WOR	WR	h	a	WOR
12	8	5.4069 (1321)* (1427)**	5.8188 (1291)*	1.474 (360)* (389)**	1.8866 (419)*	0.4094 (100)	0.4507 (100)	1.5 1.5 2.0	5.0 6.0 6.0	0.3818 0.3789** 0.3802
18	12	3.4673 (1349)* (1423)**	3.8792 (1300)*	1.322 (514)* (543)**	1.7343 (581)*	0.2571 (100)	0.2985 (100)	1.5 1.5 2.0	5.0 6.0 6.0	0.2454 0.2436** 0.2445
30	20	1.9156 (1415)* (1415)**	2.3275 (1317)*	1.2006 (887)* (887)**	1.6126 (913)*	0.1354 (100)	0.1767 (100)	1.5 1.5 2.0	5.0 6.0 6.0	0.1364 0.1354** 0.1358

Table 6: Mean Square Error and relative efficiencies for Population 3

n_1	n_2	MSE P and RE		MSE SSP 1994 and RE		MSE $t^g_{1(2)}$	MSE $t^g_{2(2)}$	Constants		MSE $t^g_{1(2)}$
		WOR	WR	WOR	WR	WOR	WR	h	a	WOR
6	4	0.2039 (3515)* (2134)**	0.2294 (3476)*	0.0310 (534)* (323)**	0.0561 (850)*	0.0058 (100)	0.0066 (100)	1.5 1.5 2.0	5.0 6.0 6.0	0.0108 0.0096 0.0103
12	8	0.0892 (3568) (2176)*	0.1147 (3584)	0.0273 (1092) (666)*	0.0528 (1650)	0.0025 (100)	0.0032 (100)	1.5 1.5 2.0	5.0 6.0 6.0	0.0045 0.0041** 0.0042
18	12	0.0510 (3643)* (2429)**	0.0765 (3643)*	0.0262 (1871)* (1248)**	0.0517 (2462)*	0.0014 (100)	0.0021 (100)	1.5 1.5 2.0	5.0 6.0 6.0	0.0024 0.0021** 0.0023

Table 7: Mean Square Error and relative efficiencies for Population 4

n_1	n_2	MSE P and RE		MSE SSP 1994 and RE		MSE$t_{1(2)}^{g}$	MSE$t_{2(2)}^{g}$	Constants		MSE$t_{1(2)}^{g}$
		WOR	WR	WOR	WR	WOR	WR	h	a	WOR
6	4	0.0146 (252)* (235)**	0.0164 (248)*	0.0070 (121)* (113)**	0.0088 (133)*	0.0058 (100)	0.0066 (100)	1.5 1.5 2.0	5.0 6.0 6.0	0.0063 0.0062** 0.0063
12	8	0.0064 (256)* (234)**	0.0082 (256)*	0.0037 (116)* (135)**	0.0055 (148)*	0.0025 (100)	0.0032 (100)	1.5 1.5 2.0	5.0 6.0 6.0	0.00276 0.00274 0.00273**
18	12	0.0036 (257)* (200)**	0.0055 (262)*	0.0026 (186)* (168)**	0.0044 (209)*	0.0014 (100)	0.0021 (100)	1.5 1.5 2.0	5.0 6.0 6.0	0.00155** 0.0054 0.00155

Table 8: Mean Square Error and relative efficiencies for Population 5

n_1	n_2	MSE R and RE		MSE SSR 1994 and RE		MSE$t_{1(2)}^{g}$	MSE$t_{2(2)}^{g}$	Constants		MSE$t_{1(2)}^{g}$
		WOR	WR	WOR	WR	WOR	WR	h	a	WOR
60	40	9.5989 (1297)* (1315)**	10.1122 (1296)*	1.5131 (204)* (207)**	2.0265 (260)*	0.7403 (100)	0.7801 (100)	1.5 1.5 2.0 2.0	5.0 6.0 5.0 6.0	0.7651 0.7455 0.7381 0.7299**
180	120	2.8574 (1307)* (1314)**	3.3707 (1304)*	0.9915 (454)* (456**	1.5048 (582)*	0.2186 (100)	0.2584 (100)	1.5 1.5 2.0 2.0	5.0 6.0 5.0 6.0	0.2273 0.2219 0.2198 0.2175**
300	200	1.5091 (1320)* (1312)**	2.0224 (1312)*	0.8871 (776)* (771)**	1.4005 (909)*	0.1143 (100)	0.1541 (100)	1.5 1.5 2.0 2.0	5.0 6.0 5.0 6.0	0.1198 0.1171 0.1161 0.1150**

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Appendix A

Table (A.2.1): Bias and MSE of Family of estimator for t_1^g

h	a	$MSE's\ and\ Bias\ \lambda = 1\ and\ \alpha = 1$
1	1	$MSE\left(t_{R(1,1)}^1\right) = \theta[\mu_{2o} + R^2\mu_{o2} - 2R\mu_{11}]$ $MSE\left(t_{P(1,1)}^2\right) = \theta[\mu_{2o} + R^2\mu_{o2} + 2R\mu_{11}]$ $Bias\left(t_{R(1,1)}^1\right) = \theta\bar{Y}\left[\frac{\mu_{o2}}{2\bar{X}^2} - \frac{\mu_{11}}{\bar{X}\bar{Y}}\right]$ $Bias\left(t_{P(1,1)}^2\right) = \theta\bar{Y}\left[\frac{\mu_{o2}}{2\bar{X}^2} + \frac{\mu_{11}}{\bar{X}\bar{Y}}\right]$
1	2	$MSE\left(t_{R(1,2)}^3\right) = \theta\left[\mu_{2o} + \frac{R^2\mu_{o2}}{4} - R\mu_{11}\right]$ $Bias\left(t_{R(1,2)}^3\right) = \theta\bar{Y}\left[\frac{\mu_{o2}}{8\bar{X}^2} - \frac{\mu_{11}}{2\bar{X}\bar{Y}}\right]$ $MSE\left(t_{P(1,2)}^4\right) = \theta\left[\mu_{2o} + \frac{R^2\mu_{o2}}{4} + R\mu_{11}\right]$ $Bias\left(t_{P(1,2)}^4\right) = \theta\bar{Y}\left[\frac{\mu_{o2}}{8\bar{X}^2} + \frac{\mu_{11}}{2\bar{X}\bar{Y}}\right]$
2	1	$MSE\left(t_{R(1,2)}^5\right) = \theta\left[\mu_{2o} + \frac{R^2\mu_{o2}}{4} - R\mu_{11}\right]$ $Bias\left(t_{R(1,2)}^5\right) = \theta\bar{Y}\left[\frac{\mu_{o2}}{8\bar{X}^2} - \frac{\mu_{11}}{2\bar{X}\bar{Y}}\right]$ $MSE\left(t_{P(1,2)}^6\right) = \theta\left[\mu_{2o} + \frac{R^2\mu_{o2}}{4} + R\mu_{11}\right]$ $Bias\left(t_{P(1,2)}^6\right) = \theta\bar{Y}\left[\frac{\mu_{o2}}{8\bar{X}^2} + \frac{\mu_{11}}{2\bar{X}\bar{Y}}\right]$
2	2	$MSE\left(t_{R(2,2)}^7\right) = \theta\left[\mu_{2o} + \frac{R^2\mu_{o2}}{16} - \frac{R\mu_{11}}{2}\right]$ $Bias\left(t_{R(2,2)}^7\right) = \theta\bar{Y}\left[\frac{\mu_{o2}}{32\bar{X}^2} - \frac{\mu_{11}}{4\bar{X}\bar{Y}}\right]$ $MSE\left(t_{P(2,2)}^8\right) = \theta\left[\mu_{2o} + \frac{R^2\mu_{o2}}{16} + \frac{R\mu_{11}}{2}\right]$

		$Bias\left(t^{8}_{P(2,2)}\right) = \theta\bar{Y}\left[\frac{\mu_{o2}}{32\bar{X}^2} + \frac{\mu_{11}}{4\bar{X}\bar{Y}}\right]$

Table (A.3.2): Bias and MSE of Family of estimator for t_2^g SRS WR and SRS WOR

h	**a**	$\boldsymbol{MSE's\ and\ Bias}\ \lambda = 1\ \ and\ \alpha = 1$
1	1	$MSE\left(t^{1}_{1,1}\right) = \theta^*\left[\mu_{2o} + \bar{Y}^2(\beta_{o2} - 1) - \frac{2\bar{Y}\mu_{12}}{\mu_{o2}}\right]$ $Bias\left(t^{1}_{1,1,1}\right) = \theta^*\left[\bar{Y}^2(\beta_{o2} - 1) - \frac{\mu_{12}}{\mu_{o2}}\right]$
1	2	$MSE\left(t^{3}_{1,2}\right) = \theta^*\left[\mu_{2o} + \frac{\bar{Y}^2(\beta_{o2} - 1)}{4} - \frac{2\bar{Y}\mu_{12}}{2\mu_{o2}}\right]$ $Bias\left(t^{3}_{1,2}\right) = \theta^*\left[\frac{\bar{Y}(\beta_{o2} - 1)}{4} - \frac{\mu_{12}}{2\mu_{o2}}\right]$
2	1	$MSE\left(t^{5}_{2,1}\right) = \theta^*\left[\mu_{2o} + \frac{\bar{Y}^2(\beta_{o2} - 1)}{4} - \frac{\bar{Y}\mu_{12}}{\mu_{o2}}\right]$ $Bias\left(t^{5}_{2,1}\right) = \theta^*\left[\frac{\bar{Y}(\beta_{o2} - 1)}{4} - \frac{\mu_{12}}{2\mu_{o2}}\right]$
2	2	$MSE\left(t^{7}_{2,2}\right) = \theta^*\left[\mu_{2o} + \frac{\bar{Y}^2(\beta_{o2} - 1)}{16} - \frac{1}{2}\frac{\bar{Y}\mu_{12}}{\mu_{o2}}\right]$ $Bias\left(t^{7}_{2,2}\right) = \theta^*\left[\frac{\bar{Y}(\beta_{o2} - 1)}{16} - \frac{\mu_{12}}{4\mu_{o2}}\right]$