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New Exponential Ratio Estimators using Auxiliary Information in Adaptive Cluster Sampling

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Abstract

In this paper, two Exponential Ratio estimators using auxiliary information in Adaptive Cluster Sampling are proposed to estimate the finite population mean of the study variable. The estimators are proposed by using population coefficient of correlation and standard deviation of the auxiliary information. The expressions of biases and Mean Square Errors of the two proposed estimators are derived. A simulation study is also performed to reveal the efficiency of the two proposed estimators and compare with other estimators in Conventional Sampling and Adaptive Cluster Sampling. The proposed Ratio estimators are proved more efficient than the existing Ratio estimators both in Conventional Sampling and Adaptive Cluster Sampling.

Keywords

Auxiliary information, Simulated values, Average values of the networks, Estimated relative bias, Comparative percentage relative efficiencies, Efficiency conditions

1. Introduction

The Adaptive Cluster Sampling (ACS) design is efficient for rare and clustered populations. The ecological populations like animals and plants of dying out species are the most suitable examples of ACS designs. In ACS, the selection of units to include in the sample depends on the observed values of the study variable during the survey.

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In ACS, the first sample is selected by usual sampling design like Simple Random Sampling then the neighborhood of each unit selected is included in the sample if and only if they meet the predefined condition *C* (usually, y > 0). The process continues until new unit does not satisfy the condition *C*. All the first sample units and the neighboring units compose the final sample.

Thompson (1990) initially gave the concept of ACS method for the rare and patched population. Smith et al. (1995) estimate the species of waterfowl in central Florida, America with ACS design. Chutiman and Kumphon (2008) proposed a Multiplicative Generalized Ratio estimator in ACS. Chutiman (2013) developed Ratio estimators by using population coefficient of variation and coefficient of kurtosis, a Regression and a difference estimator with single auxiliary variable by utilizing the averages of networks. Chaudhry and Hanif (2015) proposed two regression-cum-exponential ratio estimators to estimate the population mean in ACS using two auxiliary variables. The simulation results showed that these estimators are more efficient than the mean per unit estimator, modified ratio and exponential ratio estimators in ACS. Chaudhry and Hanif (2016) proposed modified regression-cum-exponential ratio estimators in ACS with one auxiliary variable.

2. Some available estimators in Simple Random Sampling

Consider a sample of size *n* is drawn with Simple Random Sampling without replacement (SRSWOR) from the *N* units of the population. The study variable *y* and auxiliary variable *x* has $\overline{Y}$ and $\overline{X}$ as the population means, S_y and S_x represents population standard deviations, C_y and C_x represents coefficients of variation, respectively. Also, let ρ_{xy} denotes population correlation coefficient between auxiliary and study variables.

Let, $\theta = \frac{1}{n} - \frac{1}{N}$ and $H_{jk} = \rho_{jk} \frac{C_j}{C_k}$, $j \neq k$.

The mean per unit estimator for a sample of size *n* drawn from a population of size *N* is defined as $t_0 = \overline{y}$ with the variance: $VAR(t_0) = \theta \overline{Y}^2 C_y^2$.

Cochran (1940) proposed the Ratio estimator with the bias and Mean Square Error as:

$$t_1 = \overline{y}\left[\frac{\overline{X}}{\overline{x}}\right] \tag{2.1}$$

$$Bias\left(t_1\right)=\theta\bar{Y}\left(C_x^2-\rho C_x C_y\right) \tag{2.2}$$

$$MSE\left(t_1\right)=\theta\bar{Y}^2\left(C_y^2+C_x^2-2\rho C_x C_y\right) \tag{2.3}$$

Bahl and Tuteja (1991) developed the Exponential Ratio estimator with the bias and Mean Square Error as:

$$t_2=\bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right] \tag{2.4}$$

$$Bias\left(t_2\right)=\theta\bar{Y}\left[\frac{3}{8}C_x^2-\frac{\rho_{xy}C_x C_y}{2}\right] \tag{2.5}$$

$$MSE\left(t_2\right)=\theta\bar{Y}^2\left[C_y^2+\frac{C_x^2}{4}-\rho_{xy}C_x C_y\right] \tag{2.6}$$

Subramani (2013) proposed a Modified Ratio estimator using the population median and standard deviation of the auxiliary variable with the bias and Mean Square Error as:

$$t_3=\bar{y}\left[\frac{\bar{X}M_d+S_x}{\bar{x}M_d+S_x}\right] \tag{2.7}$$

$$Bias\left(t_3\right)=\theta\bar{Y}\left(\theta_1^2 C_x^2-\rho\theta_1 C_x C_y\right), \quad \theta_1=\frac{\bar{X}M_d}{\bar{X}M_d+S_x} \tag{2.8}$$

$$MSE\left(t_3\right)=\theta\bar{Y}^2\left(C_y^2+\theta_1^2 C_x^2-2\rho\theta_1 C_x C_y\right) \tag{2.9}$$

Subramani (2013) proposed another Modified Ratio estimator using the population median and the correlation coefficient with the bias and Mean Square Error as:

$$t_4=\bar{y}\left[\frac{\bar{X}M_d+\rho}{\bar{x}M_d+\rho}\right] \tag{2.10}$$

$$Bias\left(t_4\right)=\theta\bar{Y}\left(\theta_2^2 C_x^2-\rho\theta_2 C_x C_y\right), \quad \theta_2=\frac{\bar{X}M_d}{\bar{X}M_d+\rho} \tag{2.11}$$

$$MSE\left(t_4\right)=\theta\bar{Y}^2\left(C_y^2+\theta_2^2 C_x^2-2\rho\theta_2 C_x C_y\right) \tag{2.12}$$

3. Some available estimators in Adaptive Cluster Sampling

Let an initial sample of *n* units is selected from a finite population of size *N* with SRSWOR. The average y-value and x-value in the network which includes unit are $w_{yi} = \frac{1}{m_i} \sum_{j \varepsilon A} y_j$ and $w_{xi} = \frac{1}{m_i} \sum_{j \varepsilon A} x_j$ respectively. According to Dryver and Chao (2007) when we consider averages of networks then the ACS can be regarded as SRS. The sample means of the study and auxiliary variable in the transformed population are $\bar{w}_y = \frac{1}{n} \sum_{i=1}^{n} w_{yi}$ and $\bar{w}_x = \frac{1}{n} \sum_{i=1}^{n} w_{xi}$.

The sampling error of the study and auxiliary variable are $\bar{e}_{wx}$ and $\bar{e}_{wy}$.

where,

$$\bar{e}_{wy} = \frac{\bar{w}_y - \bar{Y}}{\bar{Y}} \text{ and } \bar{e}_{wx} = \frac{\bar{w}_x - \bar{X}}{\bar{X}}. \tag{3.1}$$

$$E(\bar{e}_{wx}) = 0 \text{ , } E(\bar{e}_{wy}) = 0, \text{and } E(\bar{e}_{wx}\bar{e}_{wy}) = \theta\rho C_{wx} C_{wy}. \tag{3.2}$$

$$E(\bar{e}_{wx}^2) = \theta C_{wx}^2 \text{ and } E(\bar{e}_{wy}^2) = \theta C_{wy}^2. \tag{3.3}$$

Let, S_{wx} and ρ represents the standard deviation and co-efficient of correlation of the average values of network of study and auxiliary variables. Also M_{dw} denote median of the average values of network of auxiliary variable and let $\theta = \frac{1}{n} - \frac{1}{N}$.

Thompson (1990) proposed an unbiased estimator for population mean in ACS which is a modification of the Hansen-Hurwitz estimator (1943).

$$t_5 = \frac{1}{n} \sum_{i=1}^{n} w_{yi} = \bar{w}_y \tag{3.4}$$

$$\operatorname{var}(t_5) = \frac{\theta}{N-1} \sum_{i=1}^{N} \left(w_{yi} - \bar{Y} \right)^2 = \theta \bar{Y}^2 C_{wy}^2 \tag{3.5}$$

Dryver and Chao (2007) suggested a Modified Ratio estimator

$$t_6 = \left[\frac{\sum_{i \in s_o} w_{yi}}{\sum_{i \in s_o} w_{xi}} \right] \bar{X} = \hat{R}\,\bar{X} \tag{3.6}$$

$$\operatorname{var}(t_6) = \frac{\theta}{N-1} \sum_{i=1}^{N} \left(w_{yi} - R w_{xi} \right)^2 \tag{3.7}$$

Chaudhry and Hanif (2014) following the Bahl and Tuteja (1991) proposed a Generalized Exponential Ratio estimator for the population mean in ACS with the bias and Mean Square Error as:

$$t_{(GE)} = \bar{w}_y \exp\left[\alpha\left\{\frac{\bar{X} - \bar{w}_x}{(b-1)\bar{X} + \bar{w}_x}\right\}\right] \tag{3.8}$$

$$Bias\left(t_{(GE)}\right) = \theta\bar{Y}\left[\frac{\alpha C_{wx}^2}{b^2} + \frac{\alpha^2 C_{wx}^2}{2b^2} - \frac{\alpha\rho_{wxwy}C_{wx}C_{wy}}{b}\right] \tag{3.9}$$

$$MSE\left(t_{(GE)}\right) = \theta\bar{Y}^2\left[C_{wy}^2 + \frac{\alpha^2 C_{wx}^2}{b^2} - \frac{2\alpha\rho_{wxy}C_{wx}C_{wy}}{b}\right] \tag{3.10}$$

4. The Proposed Exponential Ratio Estimators

The Proposed Exponential Ratio estimators are the modification of Bahl and Tuteja (1991) and Subramani (2013) Ratio estimators

$$t_7 = \bar{w}_y \exp\left[\frac{\bar{X}M_{dw} - \rho}{\bar{w}_x M_{dw} + \rho}\right] \tag{4.1}$$

$$t_8 = \bar{w}_y \exp\left[\frac{\bar{X}M_{dw} - S_{wx}}{\bar{w}_x M_{dw} + S_{wx}}\right] \tag{4.2}$$

4.1 The bias and Mean Square Error of the Proposed estimator t_7:

$$t_7 = \left(\bar{Y}\bar{e}_{wy} + \bar{Y}\right)\exp\left[\frac{\bar{X}M_{dw} - \rho}{\bar{X}M_{dw}\bar{e}_{wx} + \bar{X}M_{dw} + \rho}\right] \tag{4.3}$$

$$t_7 = \left(\bar{Y}\bar{e}_{wy} + \bar{Y}\right)\exp\left[\left(1 - \frac{\rho}{\bar{X}M_{dw}}\right)\left(1 + \bar{e}_{wx} + \frac{\rho}{\bar{X}M_{dw}}\right)^{-1}\right] \tag{4.4}$$

$$t_7 = \left(\bar{Y}\bar{e}_{wy} + \bar{Y}\right)\exp\left[\left(1 - \frac{\rho}{\bar{X}M_{dw}}\right)\left(1 - \bar{e}_{wx} - \frac{\rho}{\bar{X}M_{dw}}\right)\right] \tag{4.5}$$

Let,

$$\theta_3 = \frac{\bar{X}M_{dw} - \rho}{\bar{X}M_{dw}}$$

$$t_7 = \left(\bar{Y}\bar{e}_{wy} + \bar{Y}\right)\exp\left[\theta_3\left(\theta_3 - \bar{e}_{wx}\right)\right] \tag{4.6}$$

$$t_7 = \left(\bar{Y}\bar{e}_{wy} + \bar{Y}\right)\left[1 + \theta_3\left(\theta_3 - \bar{e}_{wx}\right) + \frac{1}{2}\theta_3^2\left(\theta_3 - \bar{e}_{wx}\right)^2\right] \tag{4.7}$$

$$t_7 = \left(\bar{Y}\bar{e}_{wy} + \bar{Y}\right)\left[1 + \theta_3^2 - \theta_3\bar{e}_{wx} + \frac{\theta_3^4}{2} + \frac{\theta_3^2}{2}\bar{e}_{wx}{}^2 - \theta_3^3\bar{e}_{wx}\right] \tag{4.8}$$

$$t_7 = \bar{Y} + \bar{Y}\left[\begin{array}{l} \theta_3^2 - \theta_3\bar{e}_{wx} + \dfrac{\theta_3^4}{2} + \dfrac{\theta_3^2}{2}\bar{e}_{wx}{}^2 - \theta_3^3\bar{e}_{wx} + \bar{e}_{wy} \\ +\theta_3^2\bar{e}_{wy} - \theta_3\bar{e}_{wx}\bar{e}_{wy} + \dfrac{\theta_3^4}{2}\bar{e}_{wy} - \theta_3^3\bar{e}_{wx}\bar{e}_{wy} \end{array}\right] \tag{4.9}$$

Taking expectation of above equation we get the following equation

$$E\left(t_7 - \bar{Y}\right) = \bar{Y}\left[\begin{array}{l} \theta_3^2 - \theta_3 E\left(\bar{e}_{wx}\right) + \dfrac{\theta_3^4}{2} + \dfrac{\theta_3^2}{2}E\left(\bar{e}_{wx}{}^2\right) - \theta_3^3 E\left(\bar{e}_{wx}\right) + E\left(\bar{e}_{wy}\right) \\ +\theta_3^2 E\left(\bar{e}_{wy}\right) - \theta_3 E\left(\bar{e}_{wx}\bar{e}_{wy}\right) + \dfrac{\theta_3^4}{2}E\left(\bar{e}_{wy}\right) - \theta_3^3 E\left(\bar{e}_{wx}\bar{e}_{wy}\right) \end{array}\right] \tag{4.10}$$

$$Bias\left(t_7\right) = \bar{Y}\left[\theta_3^2 + \frac{\theta_3^4}{2} + \frac{\theta_3^2}{2}\theta C_{wx}^2 - \rho\theta C_{wx}C_{wy}\left(\theta_3 - \theta_3^3\right)\right] \tag{4.11}$$

For Mean Square Error we use eq. (4.6) and neglect the terms of order more than one

$$t_7 = \left(\bar{Y}\bar{e}_{wy} + \bar{Y}\right)\left[1 + \theta_3\left(\theta_3 - \bar{e}_{wx}\right)\right] \tag{4.12}$$

$$t_7 = \bar{Y} + \bar{Y}\left[\theta_3^2 - \theta_3\bar{e}_{wx} + \bar{e}_{wy} + \theta_3^2\bar{e}_{wy} - \theta_3\bar{e}_{wx}\bar{e}_{wy}\right] \tag{4.13}$$

$$t_7 - \bar{Y} = \bar{Y}\left[\theta_3^2 - \theta_3\bar{e}_{wx} + \bar{e}_{wy}\left(1 + \theta_3^2\right) - \theta_3\bar{e}_{wx}\bar{e}_{wy}\right] \tag{4.14}$$

Taking expectation and square on both side of eq. (4.14)

$$E\left(t_7 - \bar{Y}\right)^2 = \bar{Y}^2\left[\begin{array}{l} \theta_3^4 - \theta_3^2 E\left(\bar{e}_{wx}^2\right) + \left(1 + \theta_3^2\right)^2 E\left(\bar{e}_{wy}^2\right) - 2\theta_3^3\bar{e}_{wx} \\ +2\theta_3^2\left(1 + \theta_3^2\right)E\left(\bar{e}_{wx}\right) - 2\theta_3\left(1 + \theta_3^2\right)E\left(\bar{e}_{wx}\bar{e}_{wy}\right) \end{array}\right] \tag{4.15}$$

$$MSE\left(t_7\right) = \bar{Y}^2\left[\theta_3^4 + \theta_3^2\theta C_{wx}^2 + \left(1 + \theta_3^2\right)^2\theta C_{wy}^2 - 2\theta_3\left(1 + \theta_3^2\right)\theta\rho C_{wx}C_{wy}\right] \tag{4.16}$$

4.2 The bias and Mean Square Error of the Proposed estimator t_8:

$$t_8 = \left(\bar{Y}\bar{e}_{wy} + \bar{Y}\right)\exp\left[\frac{\bar{X}M_{dw} - S_{wx}}{\bar{X}M_{dw}\bar{e}_{wx} + \bar{X}M_{dw} + S_{wx}}\right] \quad (4.17)$$

$$t_8 = \left(\bar{Y}\bar{e}_{wy} + \bar{Y}\right)\exp\left[\left(1 - \frac{S_{wx}}{\bar{X}M_{dw}}\right)\left(1 + \bar{e}_{wx} + \frac{S_{wx}}{\bar{X}M_{dw}}\right)^{-1}\right] \quad (4.18)$$

Let,

$$\theta_4 = \frac{\bar{X}M_{dw} - S_{wx}}{\bar{X}M_{dw}}$$

$$t_8 = \left(\bar{Y}\bar{e}_{wy} + \bar{Y}\right)\exp\left[\theta_4\left(\theta_4 - \bar{e}_{wx}\right)\right] \quad (4.19)$$

$$t_8 = \left(\bar{Y}\bar{e}_{wy} + \bar{Y}\right)\left[1 + \theta_4\left(\theta_4 - \bar{e}_{wx}\right) + \frac{1}{2}\theta_4^2\left(\theta_4 - \bar{e}_{wx}\right)^2\right] \quad (4.20)$$

$$t_8 = \left(\bar{Y}\bar{e}_{wy} + \bar{Y}\right)\left[1 + \theta_4^2 - \theta_4\bar{e}_{wx} + \frac{\theta_4^4}{2} + \frac{\theta_4^2}{2}\bar{e}_{wx}^{\,2} - \theta_4^3\bar{e}_{wx}\right] \quad (4.21)$$

$$t_8 = \bar{Y} + \bar{Y}\left[\begin{array}{l}\theta_4^2 - \theta_4\bar{e}_{wx} + \dfrac{\theta_4^4}{2} + \dfrac{\theta_4^2}{2}\bar{e}_{wx}^{\,2} - \theta_4^3\bar{e}_{wx} + \bar{e}_{wy} \\ +\theta_4^2\bar{e}_{wy} - \theta_4\bar{e}_{wx}\bar{e}_{wy} + \dfrac{\theta_4^4}{2}\bar{e}_{wy} - \theta_4^3\bar{e}_{wx}\bar{e}_{wy}\end{array}\right] \quad (4.22)$$

Taking expectation of above equation we get the following equation

$$E\left(t_8 - \bar{Y}\right) = \bar{Y}\left[\begin{array}{l}\theta_4^2 - \theta_4E\left(\bar{e}_{wx}\right) + \dfrac{\theta_4^4}{2} + \dfrac{\theta_4^2}{2}E\left(\bar{e}_{wx}^{\,2}\right) - \theta_4^3E\left(\bar{e}_{wx}\right) + E\left(\bar{e}_{wy}\right) \\ +\theta_4^2E\left(\bar{e}_{wy}\right) - \theta_4E\left(\bar{e}_{wx}\bar{e}_{wy}\right) + \dfrac{\theta_4^4}{2}E\left(\bar{e}_{wy}\right) - \theta_4^3E\left(\bar{e}_{wx}\bar{e}_{wy}\right)\end{array}\right] \quad (4.23)$$

$$Bias\left(t_8\right) = \bar{Y}\left[\theta_4^2 + \frac{\theta_4^4}{2} + \frac{\theta_4^2}{2}\theta C_{wx}^2 - \rho\theta C_{wx}C_{wy}\left(\theta_4 - \theta_4^3\right)\right] \quad (4.24)$$

For Mean Square Error we use eq. (4.19) and neglect the terms of order more than one:

$$t_8 = \left(\bar{Y}\bar{e}_{wy} + \bar{Y}\right)\left[1 + \theta_4\left(\theta_4 - \bar{e}_{wx}\right)\right] \quad (4.25)$$

$$t_8 = \bar{Y} + \bar{Y}\left[\theta_4^2 - \theta_4\bar{e}_{wx} + \bar{e}_{wy} + \theta_4^2\bar{e}_{wy} - \theta_4\bar{e}_{wx}\bar{e}_{wy}\right] \quad (4.26)$$

$$t_8 - \bar{Y} = \bar{Y}\left[\theta_4^2 - \theta_4 \bar{e}_{wx} + \bar{e}_{wy}\left(1+\theta_4^2\right) - \theta_4 \bar{e}_{wx}\bar{e}_{wy} \right] \tag{4.27}$$

Taking expectation and square on both side of eq. (4.27)

$$E\left(t_8 - \bar{Y}\right)^2 = \bar{Y}^2 \begin{bmatrix} \theta_4^4 - \theta_4^2 E\left(\bar{e}_{wx}^2\right) + \left(1+\theta_4^2\right)^2 E\left(\bar{e}_{wy}^2\right) - 2\theta_4^3 \bar{e}_{wx} \\ +2\theta_4^2\left(1+\theta_4^2\right)E\left(\bar{e}_{wx}\right) - 2\theta_4\left(1+\theta_4^2\right)E\left(\bar{e}_{wx}\bar{e}_{wy}\right) \end{bmatrix} \tag{4.28}$$

$$MSE\left(t_8\right) = \bar{Y}^2\left[\theta_4^4 + \theta_4^2 \theta C_{wx}^2 + \left(1+\theta_4^2\right)^2 \theta C_{wy}^2 - 2\theta_4\left(1+\theta_4^2\right)\theta\rho C_{wx} C_{wy} \right] \tag{4.29}$$

5. Simulation Study

In ACS the expected final sample size changes from sample to sample and E(v) represents the estimated final sample size. The E(v) is the addition of probabilities of inclusion of all quadrats. The variance of sample mean from a SRSWOR using E(v) can be computed from the following formula

$$Var(\bar{y}) = \frac{\sigma^2 (N - E(v))}{NE(v)} \tag{5.1}$$

The estimated Mean Squared Error of the estimated mean is given by

$$\hat{MSE}\left(t_*\right) = \frac{1}{r}\sum_{i=1}^{r}(t_* - \bar{Y})^2 \tag{5.2}$$

where,

t_* and r represents the value for the relevant estimator for sample i and the number of iterations respectively.

The percentage relative efficiency is defined as:

$$PRE = \frac{Var(\bar{y})}{\hat{MSE}\left(t_*\right)} \times 100 \tag{5.3}$$

5.1 Population: The population is taken from Smith et al. (1995) the total area for original data was 5000 km, which was divided into 25 200-km^2 quadrants in central Florida. Dryver and Chao (2007) used the blue-winged teal data (which was divided into 50 100-km^2) to compare the performance of their Purposed Ratio estimators in ACS with the Ratio estimators in SRSWOR. They used the following two models to generate the valuess

$$y_i = 4x_i + \varepsilon_i \qquad \varepsilon_i \sim N\left(0, x_i\right) \tag{5.4}$$

$$y_i = 4w_{xi} + \varepsilon_i \qquad \varepsilon_i \sim N\left(0, w_{xi}\right) \tag{5.5}$$

The variability of the study variable *y* is proportional to the auxiliary variable *x* in model eq. (5.4) while it is proportional to the within-network mean level of the auxiliary variable in model eq. (5.5). Then, the within network variances of the study variable in the two networks consisting of more than one units are much larger in the population generated by model eq. (5.4).The population simulated by model eq. (5.5) has a high correlation with the transformed population but not a high correlation with the original and thus it assumed that the there is a high correlation on a region level between the study variable and the auxiliary variable, but not on the sampling unit level (Dryver and Chao 2007).

5.2 Thompson efficiency condition: Thompson (2002) studied that ACS is preferable than the comparable usual sampling methods when the within network variance is sufficiently large as compared to overall variance of the study variable. He offered the condition when the modified Hansen-Hurwitz estimator (1943) for ACS has lesser variance than variance of the mean per unit for a SRSWOR of size $E(v)$ if and only if:

$$S_y^2 \le \frac{E(v)\left[N-n\right]}{N\left[E(v)-n\right]} S_{wy^D}^2 \tag{5.6}$$

$$S_y^2 \le hS_{wy}^2 \; D \tag{5.7}$$

where,

the within network variance is defined by

$$S_{wy}^2 D = \frac{1}{(N-1)} \sum_{k=1}^{k} \sum\nolimits_{i\varepsilon A_i} \left(y_i - w_i\right)^2 \tag{5.8}$$

Thomson efficiency conditions eq. (5.7) are satisfied Table 3, so Adaptive estimators are expected to perform better than the usual estimators. ACS is preferable than the comparable Conventional Sampling methods if overall variance of the study variable is sufficiently large as compared to the transformed population variance. For the simulation study ten thousand iterations were performed for each estimator to get accurate estimates with the simple random sampling without replacement with the sample sizes of 10,20,30,40 and 50.

6. Conclusions

A simulation study was conducted to evaluate the performance of the new Proposed Exponential Ratio estimators in ACS. The estimated relative bias (Table 4) of all the estimators decreases as sample sizes increase. The percentage relative efficiency (Table 5) of the ACS estimators is much higher than their equivalent SRS estimators. Dryver and Chao (2007) assumed 0/0 as zero for the Ratio estimator but in this research we did not consider 0/0 as 0 in simulation study. Some Ratio and Exponential Ratio estimators did not perform and return no value (*). The Proposed estimators in ACS proved to be efficient by using the coefficient of correlation and the standard deviation. This may be due to the fact that the data was highly skewed.

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Table 1: Blue-Winged Teal (Smith et al., 1995)

0	0	0	5	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	20	2	0	0	0	103	0
0	0	0	0	0	0	0	0	150	1
0	0	0	0	2	0	0	0	6	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	122
0	0	0	0	0	0	0	0	0	114
0	0	0	0	0	0	0	0	0	3
0	0	0	0	0	0	0	0	0	0
0	0	3	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	4	12	0	0	10	0	0
0	0	0	0	0	3	0	0	7144	0
0	0	0	0	0	0	2	0	6339	0
0	0	0	0	0	0	0	0	14	60
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	2	0	0	0

Table 2: Simulated *y* values, based on model eq. (5.4)

0	0	0	20	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	80	8	0	0	0	412	0
0	0	0	0	0	0	0	0	600	4
0	0	0	0	8	0	0	0	24	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	488
0	0	0	0	0	0	0	0	0	456
0	0	0	0	0	0	0	0	0	12
0	0	0	0	0	0	0	0	0	0
0	0	12	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	16	48	0	0	40	0	0
0	0	0	0	0	12	0	0	28576	0

0	0	0	0	0	0	8	0	25356	0
0	0	0	0	0	0	0	0	56	240
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	8	0	0	0

Table 3: Efficiency condition for population

Sample Sizes		**Thompson Efficiency Condition**	
N	***E(v)***	S^2_{wy}	gS^2_y
10	13.29	3619028	5368359
20	25.93	3619028	5414015
30	38.01	3619028	5460402
40	49.57	3619028	5507270
50	60.69	3619028	5554371

Table 4: Estimated relative bias

N	$E(v)$	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8
10	13.29	*	-0.49	1.72	1.75	*	*	-0.63	-0.63
20	25.93	*	-0.42	1.75	1.74	*	*	-0.63	-0.63
30	38.01	*	-0.36	1.70	1.73	*	*	-0.63	-0.63
40	49.57	*	-0.32	1.72	1.72	*	*	-0.64	-0.63
50	60.69	*	-0.27	1.73	1.66	*	*	-0.64	-0.64

Table 5: Estimated percentage relative efficiency

E(v)	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8
13.29	100	*	462.71	12.94	12.93	197.71	*	657.82	649.30
25.93	100	*	349.07	12.29	12.34	198.83	*	450.84	450.69
38.01	100	*	293.50	11.74	11.70	196.64	*	342.00	339.21
49.57	100	*	253.50	11.10	11.13	200.40	*	261.54	264.33
60.69	100	*	231.37	10.47	10.60	199.03	*	210.92	211.01

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