ISSN 1684-8403
Journal of Statistics
Volume 24, 2017. pp. 133-149

The Optimization of Design of EWMA Control Chart using Box-Behnken Method under Type I Censoring

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Abstract

This study is conducted to evaluate the performance of the Exponentially Weighted Moving Average (EWMA) control chart in the presence of Type I censored data. For the purpose of performance evaluation the Average Run Length (ARL) index is used. The Box-Behnken design which is a response surface method in Design Of Experiment (DOE) is being used for obtaining the optimal design for the EWMA structure. The empirical result shows that the censored data following Rayleigh lifetimes has a significant effect on the value of the ARL. Another important finding is that, under the presence of censored data the control limits of the EWMA chart should be narrowed down to L = 2 for the best performance. On the other hand, the value of λ does not have a significant effect on performance measure. The EWMA control chart under the censored environment is designed in such a way that the practitioners can achieve the highest possible performance when deploying the EWMA chart.

Keywords

Censored, Average run length (ARL), Box-behnken design, Exponentially Weighted Moving Average (EWMA) chart

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1. Introduction

Grant and Leavenworth (1979) describe the Statistical Process Control (SPC) as a “useful and important tool used commonly in engineering field to monitor the overall process.” SPC can be applied to all kinds of engineering operations. The significant application of the SPC analysis of the process will make the process more consistent and reliable.

Statistical Process Control is usually referred as “SPC”. It is a method which includes:

- Monitoring
- Controlling
- Refining

The real life application clearly states that all processes inherent some variation. But sometimes the process shows a great level of discrepancy and results in the occurrence of offensive/unpredictable results. One of the uses of the SPC is to reduce variation to achieve the best objective value.

The control charts are the most important tool of SPC tool kit. It is commonly used to differentiate between the “assignable and un-assignable causes”. The purpose of the effective process monitoring system is to detect the presence of an “assignable cause” as rapidly as possible without stopping checking too often or too late. The control charts are of different types. Some are “memory control charts” and other is “memory-less control charts”. Shewhart (1931) are memory-less control charts and are being used to detect a large size shift whereas the memory type charts are used for dealing with small size shifts.

The industrial tools are emerging rapidly therefore, it is essential to design the products with high consistencies. By using the highly censored data collected from life time distribution the time and expenses can be minimized. An important issue in life-testing applications for industrial engineering is how we can develop the control chart for monitoring the mean life time of products when the data is censored under Type-I.

Lu and Tsai (2008) has done work on Type I censored data using Gamma distribution and proposed EWMA conditional expected values (CEV) control chart for monitoring mean level of the Gamma life times under Type I censored test.

Steiner and Mackay (2000, 2001a, 2001b) developed a one-side charting procedure based on the CEV which allows for rapid detection of deterioration in the process quality with highly censored data under normality.

Tsai and Lin (2009) proposed a EWMA control chart to identify mean shifts for the Gompertz distributed lifetimes with the decrease and increase in Type I censoring.

Zhang and Chen (2004) shows the practical implementation of censored data analysis in which we monitor a painting process regarding the rust resistant capabilities, scratch panels from a type of metal electrical box painted using this painting process are put in a salt spray chamber with the temperature maintained at 30 Celsius. It was concluded that when the data is censored the simple/traditional control charting such as $\bar{x}$ and R charts shows as large false alarm rates or low power.

Simple/traditional control charting methodology couldn't provide an effective analysis in the presence of censored data therefore to solve this issue statistician /researchers have developed different methods for different life time distributions. In this paper, a methods is discussed which provides an effective results in the presence of censored data for Rayleigh distribution. This paper is organized as following sections: The introduction is given in Section 1, in section 2 proposed methodology is presented and in section 3 Numerical computations for conditional expected values (CEVs) and charting tools are presented. Section 4 covers some concluding remarks about proposed methods.

2. Research objectives

- To evaluate the performance of the Exponentially Weighted Moving Average (EWMA) control chart under Type I censoring enviournment.
- To use the Box-Behnken design to carry out the optimal design of the EWMA parameters.

3. Alternative Hypothesis

H_1: The CEV Exponentially Weighted Moving Average (EWMA) control chart shows better performance in the presence of Type I censored data.
H_2: The optimal design of the EWMA parameters are achieved using the Box-Behnken design.

4. Research questions

Q_1: Does the CEV Exponentially Weighted Moving Average (EWMA) control chart show better performance in the presence of Type I censored data.
Q_2: Does the optimal design of the EWMA parameters achieved using the Box-Behnken design.

5. The CEV based EWMA control charts

The CEV Weighted control charts are used for detecting mean level shifts in the process. Using CEVs Weighted control charts each censored observation is replaced with its condition expected values. Now the test statistic of control chart is calculated. For our case, the EWMA test statistic is calculated and plotted in a manner similar to the traditional EWMA chart. With right censored data the goal of CEV Weighted control charts is to detect decrease in process mean and/or increase in process standard deviation. The proposed control charts is both one sided. We have considered decrease in mean level shifts and increase in standard deviation shifts using concept given by Steiner and Mackay (2000). The CEV EWMA chart has only upper sided control limits. In this paper, we have calculated CEV based EWMA control charts and apply Box-Behnken design for the improvement and monitoring of design structure.

5.1 Procedural details: Let the lifetime of items T_{i1}, T_{i2} ..... T_{in} (where "*i*" show subgroup number and "*n*" show the sample size) follows Raleigh distribution; all items are put on Type I censoring test. The lifetimes of items are exactly known only if they are less than or equal to the censoring time C. The practitioners predetermined the censoring time.
The mean life time of C is presented as $\mu_o = 1.25\sigma$.

The censoring rate is defined as $Pc=1-F(t;\sigma)$, where $F(t;\sigma)$ is the cumulative density function of Raleigh distribution and is given as: $P(T \leq t) = 1 - \exp\left(-t^2 / (2\sigma^2)\right)$.

If the process is under a statistical control state, the mean lifetime is assumed to be $\mu_o = 1.25\sigma$. If the process parameter σ is unknown it can be replaced by its MLE based on m pre-samples each of size n. The CEV are derived as follows:

$$E(T|T>c) = \frac{1}{F(c,\alpha)} \int_{c_o}^{\infty} t\left(\frac{2t}{\alpha} e^{\frac{-t^2}{\alpha}}\right) dt = \frac{\alpha \Gamma(z_c, 3/2)}{\exp(-z_c)}$$

So, CEV for Raleigh distribution is,

$$E(T|T>c) = \frac{\alpha_0 \Gamma(z_c, 3/2)}{\exp(-z_c)}$$

where, $\alpha > 0$.

And $z_c = (c/\alpha)^2$, c is the censoring time and $\alpha = (\sqrt{2})(\sigma)$.

Now, we transfer the Type I censored data set to

$$W_{ij} = \begin{cases} T_{ij}, \ if \ T_{ij} \leq C \ (uncensored) \ . \\ CEV(T_{ij}), if \ T_{ij} > C \ (censored) \ . \end{cases}$$

$$j=1,2,3,\ldots,n\ ,\ i=1,2,3,\ldots,m.$$

Now, using the transformed distribution (i.e. the distribution of W_{ij}) is used to make EWMA control chart. Control limits of EWMA control chart is given as:

$$\left.\begin{aligned} UCL &= \mu_0 + L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]} \\ CL &= \mu_0 \\ LCL &= \mu_0 - L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]} \end{aligned}\right\}$$

where, μ_0 is the average of preliminary data, L is the width of control limits and λ is the weight assigned to the observation. The typical CEV EWMA control chart for the simulated data is given in Figure 1.

To simulate a special cause, a shift of size δ_0 which is in the form of a step function is applied into a process at time t =50 as:

$$\delta(t) = \begin{cases} 0; t < t_{50} \\ \delta_0; t \geq t_{50} \end{cases}$$

Surface experiments are performed to fit either a first order model (linear function) or a second order model to the observations. The advantage of the Box-Behnken technique is that it does not include any points at the vertices of the cubic region and the resulting design is still rotatable.

We have applied the Box-Behnken design to obtain the optimal design for EWMA chart when the observations are censored (Figure 2). Due to the study, all the potential factors, namely censoring rater Pc, shift size and the EWMA chart's parameters, λ and L, are investigated to quantify the effect and optimize their values. Afterwards, the values of λ and L are characterized in order to minimize the Average Run Length (ARL) when an assignable cause does occur.

6. Result and discussion

As shown in Table 3, the statistical analysis was conducted in the form of the ANalysis Of VAriance (ANOVA). From the ANOVA table, three factors, A (Pc), C (shift) and D (L), along with the curvature (C2) has a significant effect on the value of ARL. The main effect plot is generated to quantify the influence of each significant factor.

The Figure 3 illustrates the relationship between censoring rate Pc and ARL when the values of the other factors were set at the mid-points (average) as: λ = 0.15, shift = 2 and L = 3.

Figure 4, the only EWMA design factor to be concerned (the effects of other factors are averaged at: Pc= 0.4, λ = 0.15 and shift = 2) is the multiple of sigma in the control limit (L) which should be assigned the value at 2 in order to minimize the out of control ARL. A-Pc has levels -1 showing low censoring rate <20%, 0 showing medium 30%-40% and 1 showing high >40% censoring rates. According to the cube plot (Figure 4), the censoring rate Pc has a significant effect on the out of control ARL (ARL_1) when λ is averaged at 0.15. The ARL_1 range is between 1.19 (A = 1) and 39.92 (A = -1). The response surface methodology is applied to have the optimal response point for ARL_1.

The Figure 5 and 6 shows the minimum ARL point in the response surface. The minimum ARL_1 is found as 1.19 which is the optimal value for this control chart. The table 4 given below shows that *CEV* EWMA control charts performs well as compared to traditional EWMA control chart. It is also observed that for with the increase in censoring rates the ARL performance gets for CEV EWMA control chart.

7. Conclusion

This study is used to evaluate the performance of the Exponentially Weighted Moving Average (EWMA) control chart under the situation that the observations are Type I censored. The performance is evaluated using the Average Run Length (ARL). The response surface method, Box-Behnken design is used to develop an optimal design of the EWMA parameters when data is Type I censored. The empirical results show that the censored data following Rayleigh lifetimes has a significant effect on the value of the ARL, i.e., the ability to detect a special cause and the occurrence frequency of a false alarm. Another important finding is that, under the censored data case the control limits of the EWMA chart should be narrowed down to L = 2 for the best performance. The EWMA chart under the censored environment is appropriately designed; the practitioners will have state of the art guidelines for achieving the highest possible performance when deploying the EWMA chart.

Table 1: Input factors and levels

Factor	-1	0	1
A (Pc)	-1	0	1
B (λ)	0.05	0.15	0.25
C (Shift size)	0	2	4
D (L)	2	3	4

Table 2: Design matrix

Order	**Pc**	λ	**Shift**	**L**	**ARL**
1	-1	0.05	2	3	8.5
2	1	0.05	2	3	1.60
3	-1	0.25	2	3	4.1
4	1	0.25	2	3	1.37
5	0	0.15	0	2	39.92
6	0	0.15	4	2	1.476
7	0	0.15	0	4	351.86

8	0	0.15	4	4	2.14
9	-1	0.15	2	2	2.41
10	1	0.15	2	2	1.19
11	-1	0.15	2	4	9.2
12	1	0.15	2	4	1.57
13	0	0.05	0	3	324.7
14	0	0.25	0	3	259.10
15	0	0.05	4	3	2.97
16	0	0.25	4	3	1.61
17	-1	0.15	0	3	50.24
18	1	0.15	0	3	2.64
19	-1	0.15	4	3	2.42
20	1	0.15	4	3	1.21
21	0	0.05	2	2	3.9
22	0	0.25	2	2	2.4
23	0	0.05	2	4	8.14
24	0	0.25	2	4	5.11
25	0	0.15	2	3	3.9
26	0	0.15	2	3	4.17
27	0	0.15	2	3	4.14

Table 3: ANOVA

Source	**SS**	**DF**	**MS**	**F**	**P-Value**
A-Pc	0.3951	2	0.3951	30.213	< 0.0001
C-Shift	0.8794	2	0.8794	67.244	< 0.0001
D-L	0.1065	1	0.1065	8.1436	0.0088
Residual	0.3138	22	0.013	--	--
Total	1.7686	27	--	--	--

Table 4: ARL's (out of control) for Censored and Traditional shewhart charts

Pc	*CEV EWMA chart (Mean(CEV))*	**Traditional EWMA chart (ignoring censoring)**
0.2	25	35
0.3	18	38
0.5	12	42
0.6	10	53

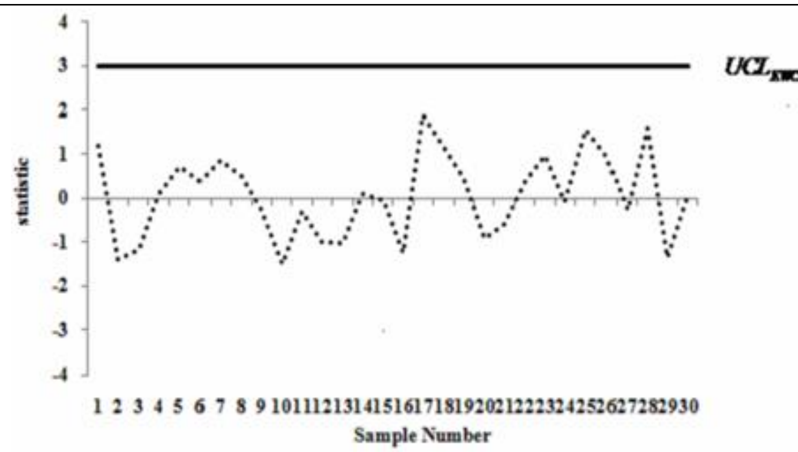

Figure 1: Typical CEV EWMA control chart for the simulated data

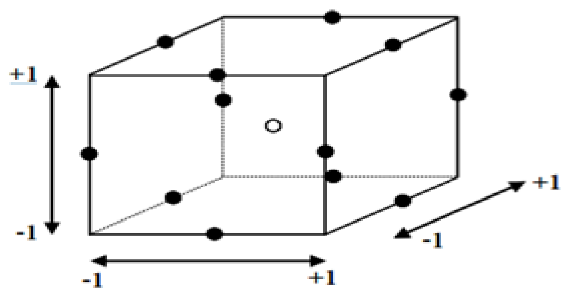

Figure 2: Box-Behnken design for three factors

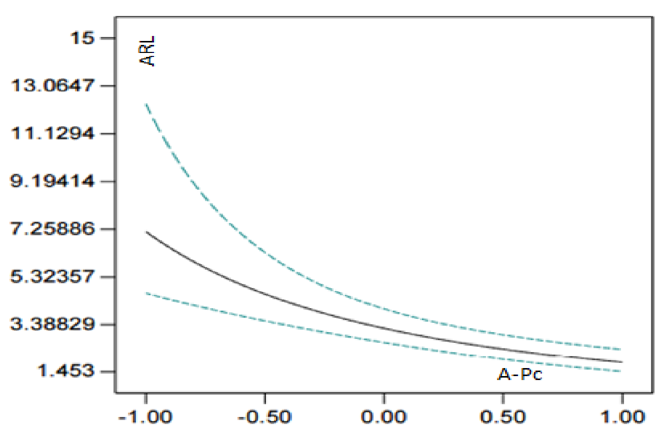

Figure 3: Main effect plot of Pc

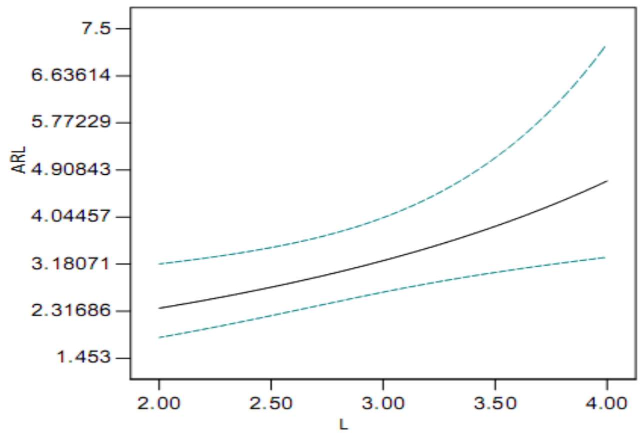

Figure 4: Main effect plot of L

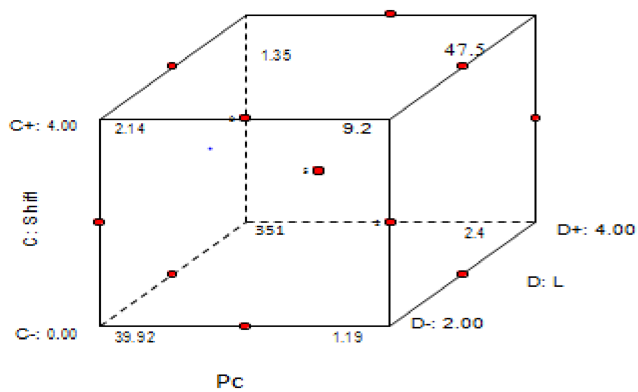

Figure 5: Cube plot of A, C and D

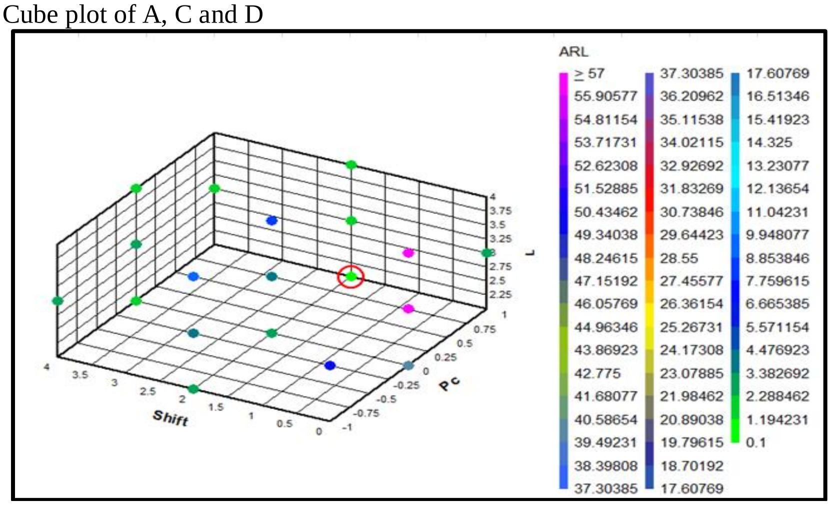

Figure 6: Response surface for ARL_1 using CEV EWMA

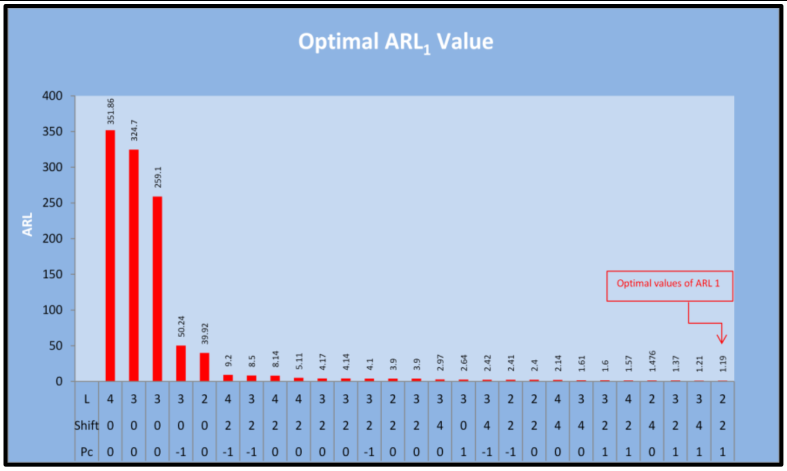

Figure 7: ARL_1 responses

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Appendix

Table: Samples of Size 3 and averages for CEV EWMA CHART

Sample #	Samples			Average
1	0.320215	0.243033	0.243033	0.268761
2	0.41314	0.320215	0.320215	0.35119
3	0.777092	0.41314	0.41314	0.534457
4	0.903412	0.777092	0.777092	0.819199
5	0.697438	0.903412	0.903412	0.834754
6	0.570595	0.697438	0.697438	0.655157
7	0.532079	0.570595	0.570595	0.557756
8	0.773757	0.532079	0.532079	0.612639
9	0.872513	0.773757	0.773757	0.806676
10	1.104709	0.872513	0.872513	0.949912
11	0.876027	1.104709	1.104709	1.028482
12	0.516186	0.876027	0.876027	0.75608
13	0.284508	0.516186	0.516186	0.43896
14	0.60261	0.284508	0.284508	0.390542
15	1.584483	0.60261	0.60261	0.929901
16	1.012579	1.584483	1.584483	1.393848
17	0.60306	1.012579	1.012579	0.876073
18	0.530398	0.60306	0.60306	0.578839
19	0.411835	0.530398	0.530398	0.490877
20	0.136045	0.411835	0.411835	0.319905
21	0.796219	0.136045	0.136045	0.356103
22	1.023111	0.796219	0.796219	0.87185
23	0.104093	1.023111	1.023111	0.716772
24	0.847569	0.104093	0.104093	0.351918
25	0.604725	0.847569	0.847569	0.766621
26	0.162138	0.604725	0.604725	0.457196
27	1.108384	0.162138	0.162138	0.477554
28	0.512842	1.108384	1.108384	0.90987
29	0.212224	0.512842	0.512842	0.412636
30	0.217176	0.212224	0.212224	0.213874
31	0.359186	0.217176	0.217176	0.264513
32	0.127676	0.359186	0.359186	0.282016
33	0.646662	0.127676	0.127676	0.300671
34	0.61784	0.646662	0.646662	0.637054
35	0.788793	0.61784	0.61784	0.674824
36	0.315561	0.788793	0.788793	0.631049

37	0.697715	0.315561	0.315561	0.442945
38	0.87716	0.697715	0.697715	0.75753
39	0.696208	0.87716	0.87716	0.816842
40	0.513732	0.696208	0.696208	0.635383
41	0.258275	0.513732	0.513732	0.42858
42	0.612514	0.258275	0.258275	0.376354
43	1.013663	0.612514	0.612514	0.74623
44	0.696686	1.013663	1.013663	0.908004
45	0.8113	0.696686	0.696686	0.734891
46	0.629346	0.8113	0.8113	0.750648
47	1.635398	0.629346	0.629346	0.964697
48	0.18984	1.635398	1.635398	1.153545
49	0.775267	0.18984	0.18984	0.384982
50	1.444906	0.775267	0.775267	0.99848
51	0.238693	1.444906	1.444906	1.042835
52	0.186231	0.238693	0.238693	0.221205
53	0.484144	0.186231	0.186231	0.285535
54	0.335071	0.484144	0.484144	0.434453
55	0.483728	0.335071	0.335071	0.384623
56	0.869526	0.483728	0.483728	0.612327
57	0.19629	0.869526	0.869526	0.645114
58	0.2415	0.19629	0.19629	0.21136
59	1.122872	0.2415	0.2415	0.535291
60	0.383713	1.122872	1.122872	0.876486
61	1.287971	0.383713	0.383713	0.685132
62	0.920267	1.287971	1.287971	1.165403
63	0.184166	0.920267	0.920267	0.6749
64	0.926254	0.184166	0.184166	0.431528
65	1.141714	0.926254	0.926254	0.998074
66	0.523867	1.141714	1.141714	0.935765
67	0.897705	0.523867	0.523867	0.64848
68	0.312986	0.897705	0.897705	0.702799
69	0.216683	0.312986	0.312986	0.280885
70	1.348513	0.216683	0.216683	0.59396
71	0.274503	1.348513	1.348513	0.99051
72	0.76489	0.274503	0.274503	0.437965
73	1.004752	0.76489	0.76489	0.844844
74	1.25993	1.004752	1.004752	1.089811
75	0.547003	1.25993	1.25993	1.022288
76	0.213267	0.547003	0.547003	0.435758
77	0.881632	0.213267	0.213267	0.436055

78	0.436647	0.881632	0.881632	0.733304
79	0.896505	0.436647	0.436647	0.589933
80	0.164257	0.896505	0.896505	0.652422
81	0.35702	0.164257	0.164257	0.228511
82	0.475521	0.35702	0.35702	0.396521
83	0.596323	0.475521	0.475521	0.515789
84	0.178322	0.596323	0.596323	0.456989
85	0.260094	0.178322	0.178322	0.205579
86	1.055258	0.260094	0.260094	0.525148
87	0.740624	1.055258	1.055258	0.95038
88	0.532067	0.740624	0.740624	0.671105
89	0.452685	0.532067	0.532067	0.505607
90	0.523802	0.452685	0.452685	0.476391
91	1.326547	0.523802	0.523802	0.791384
92	0.539118	1.326547	1.326547	1.064071
93	1.513007	0.539118	0.539118	0.863747
94	0.541971	1.513007	1.513007	1.189328
95	0.419079	0.541971	0.541971	0.501007
96	0.548695	0.419079	0.419079	0.462284
97	0.651748	0.548695	0.548695	0.583046
98	0.127368	0.651748	0.651748	0.476954
99	1.0307	0.127368	0.127368	0.428479
100	0.39341	1.0307	1.0307	0.81827
101	0.717014	0.39341	0.39341	0.501278
102	0.939604	0.717014	0.717014	0.791211
103	0.915727	0.939604	0.939604	0.931645
104	0.867727	0.915727	0.915727	0.899727
105	0.523942	0.867727	0.867727	0.753132
106	0.796686	0.523942	0.523942	0.614857
107	0.587664	0.796686	0.796686	0.727012
108	0.253512	0.587664	0.587664	0.47628
109	0.827581	0.253512	0.253512	0.444868
110	1.166588	0.827581	0.827581	0.940583
111	0.915314	1.166588	1.166588	1.08283
112	1.047115	0.915314	0.915314	0.959248
113	0.174685	1.047115	1.047115	0.756305
114	0.503516	0.174685	0.174685	0.284295
115	0.315956	0.503516	0.503516	0.440996
116	1.17521	0.315956	0.315956	0.602374
117	0.442353	1.17521	1.17521	0.930924

118	0.223236	0.442353	0.442353	0.369314
119	0.27735	0.223236	0.223236	0.241274
120	0.431367	0.27735	0.27735	0.328689
121	1.256289	0.431367	0.431367	0.706341
122	0.867353	1.256289	1.256289	1.126644
123	0.083868	0.867353	0.867353	0.606192
124	0.388353	0.083868	0.083868	0.185363
125	0.778712	0.388353	0.388353	0.518473
126	0.373184	0.778712	0.778712	0.643536
127	0.711574	0.373184	0.373184	0.485981
128	1.011067	0.711574	0.711574	0.811405
129	0.644614	1.011067	1.011067	0.888916
130	1.0133	0.644614	0.644614	0.767509
131	0.626637	1.0133	1.0133	0.884412
132	0.437745	0.626637	0.626637	0.563673
133	0.617205	0.437745	0.437745	0.497565
134	0.249515	0.617205	0.617205	0.494642
135	1.096525	0.249515	0.249515	0.531852
136	1.226053	1.096525	1.096525	1.139701
137	1.390645	1.226053	1.226053	1.280917
138	1.331108	1.390645	1.390645	1.370799
139	0.608729	1.331108	1.331108	1.090315
140	0.820405	0.608729	0.608729	0.679287
141	0.333356	0.820405	0.820405	0.658055
142	0.401261	0.333356	0.333356	0.355991
143	0.596657	0.401261	0.401261	0.466393
144	0.412408	0.596657	0.596657	0.535241
145	0.77939	0.412408	0.412408	0.534735
146	0.640406	0.77939	0.77939	0.733062
147	0.388734	0.640406	0.640406	0.556516
148	0.877276	0.388734	0.388734	0.551581
149	0.995271	0.877276	0.877276	0.916607
150	0.362266	0.995271	0.995271	0.784269
151	0.440291	0.362266	0.362266	0.388275
152	0.802693	0.440291	0.440291	0.561092
153	1.088377	0.802693	0.802693	0.897921
154	0.247911	1.088377	1.088377	0.808221
155	0.22499	0.247911	0.247911	0.24027
156	0.827335	0.22499	0.22499	0.425772
157	0.299275	0.827335	0.827335	0.651315
158	0.317785	0.299275	0.299275	0.305445

159	0.674625	0.317785	0.317785	0.436732
160	0.389653	0.674625	0.674625	0.579635
161	0.216653	0.389653	0.389653	0.331986
162	0.860312	0.216653	0.216653	0.431206
163	0.398941	0.860312	0.860312	0.706522
164	0.59089	0.398941	0.398941	0.462924
165	0.278437	0.59089	0.59089	0.486739
166	0.601693	0.278437	0.278437	0.386189
167	0.542214	0.601693	0.601693	0.581867
168	1.339555	0.542214	0.542214	0.807995
169	1.426644	1.339555	1.339555	1.368584
170	0.869625	1.426644	1.426644	1.240971
171	0.919966	0.869625	0.869625	0.886405
172	0.302833	0.919966	0.919966	0.714255
173	0.889593	0.302833	0.302833	0.498419
174	0.286015	0.889593	0.889593	0.6884
175	1.060207	0.286015	0.286015	0.544079
176	0.812484	1.060207	1.060207	0.977632
177	0.791272	0.812484	0.812484	0.805413
178	0.719098	0.791272	0.791272	0.767214
179	0.558621	0.719098	0.719098	0.665606
180	0.944903	0.558621	0.558621	0.687382
181	0.29611	0.944903	0.944903	0.728639
182	1.202004	0.29611	0.29611	0.598075
183	0.802417	1.202004	1.202004	1.068808
184	1.209838	0.802417	0.802417	0.938224
185	1.192424	1.209838	1.209838	1.204033
186	1.192177	1.192424	1.192424	1.192342
187	0.585412	1.192177	1.192177	0.989922
188	0.489945	0.585412	0.585412	0.55359
189	0.88812	0.489945	0.489945	0.62267
190	1.162216	0.88812	0.88812	0.979486
191	1.084667	1.162216	1.162216	1.136367
192	0.101194	1.084667	1.084667	0.756843
193	0.572875	0.101194	0.101194	0.258421
194	1.001845	0.572875	0.572875	0.715865
195	0.781385	1.001845	1.001845	0.928358
196	0.990525	0.781385	0.781385	0.851098
197	0.970429	0.990525	0.990525	0.983827
198	0.263234	0.970429	0.970429	0.734697

199	0.39559	0.263234	0.263234	0.307352

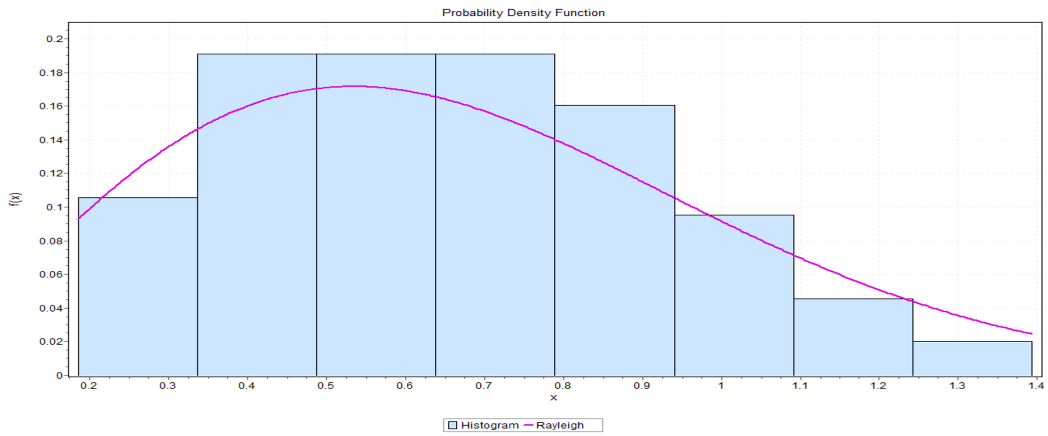

Figure: Distribution Fit of Data Averages