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Multilevel Growth Modeling using R: A Case of GDP influencing Factors

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Abstract

Analysis of longitudinal data always remained a point of consideration among the researchers due to its complex nature. The analysis of longitudinal data becomes complex when data collected from the same individuals over the different time intervals. One of the techniques to overcome these types of issues is called Multilevel Growth Curve modeling. This study used a panel data of twenty four different countries comprising the data on two independent variables (Gross National Income and Gross National Expenditure) with one dependent variable (Gross Domestic Product) over the period of twenty four years ranging from 1991 to 2014 which was collected from world development indicators.

Three different Unit Root tests i.e. Augmented Dickey-Fuller (ADF) choi Z statistic, Im et al. (1997) and ADF Fisher Chi-square, provided the result that the variables used in this study are all stationary at level-1. Study utilized the approach provided by Johansen panel integration and proved the long run relationships for all the ten cases. Error Correction Model (ECM) does not show any short run relationships of GDP with Gross National Income and Gross National Expenditure.

The results of Multilevel Growth models showed that all the fitted models were good as independent variables explained around 98% variation in GDP. The results also exposed that both GLM and LMER provide the same results of the parameter estimated in the models.

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Based on the results we concluded that Gross National Income and Gross National Expenditure are the key factors to the growth of Gross Domestic Product of any country over long term.

Keywords

Multilevel Growth models, Panel data, Gross domestic product, Goodness of fit, GLM, LMER, Co-integration, ECM

1. Introduction

Longitudinal data, of all types of analytical data, is the one which creates problems in analysis for researchers. The analysis of longitudinal data is complex both in theoretical and conceptual framework. On the conceptual view, questions are about specific goals of analysis and type of process change (Chan, 1998). For example, in some case, purpose of analysis might be to find out mean differences in treatments over time. In another case, the purpose might be both to understand change patterns and also mean differences. So, the two cases are totally different i.e. first case is much simpler than the later one, and researcher will also have other issues in methodologies relevant to longitudinal data.

Further, these methodological challenges might be overwhelming. For instance, as the longitudinal data are collected over multiple time periods from same individuals, so there is some possibility that the multiple responses will not be independent. Which means the different responses will tend to be correlated, and correlation is that measure which violates the statistical assumption of independence which is required in many of the statistical data analysis techniques which are common for analysis (Kenny and Judd, 1986). Further complications are involved by thinking that there are some possibilities that nearby responses i.e. response 1 and 2 will be more strongly correlated than responses that are far apart i.e. response 1 and 5. Furthermore, it is also possible that responses will become more or less variable with time. For example an individual will have more variability in job performance, but with time, variation may reduce. So for analytical purpose of longitudinal data, models should account for all above multiple factors.

A final hurdle curtails that even if a researches is aware of all these methodological issues and is clear in vision for goals, he may lack knowledge of procedures for translating and relating theory with analysis. Whether this problem seems to be trifling with respect to the methodological and conceptual issue, but it is expected that somehow hurdles with the procedures prevents the compliance of longitudinal analysis.

In a nutshell, compared to cross sectional designs, longitudinal investigation studies need different intellectual methods and designs for analysis. This design incorporate very different methodological and theoretical concerns that do not exist in the cross sectional designs and not described in the statistical textbooks i.e. time is treated as an independent variable and error terms are correlated (Bliese and Ployhart, 2002).

1.1 Significance of Study: Gross Domestic Product (GDP) is one from most broadly used measures of a country's output or production. It is well-defined as the total worth of goods and services created within a state's boundaries in a definite period of time i.e. monthly, quarterly or annually. GDP is a perfect evidence of any economy's size, and GDP growth rate is perhaps the single top pointer of financial growth. As Samuelson and Nordhaus (2009) state it, "While GDP and the rest of the national income accounts may seem to be arcane concepts they are truly among the great inventions of the twentieth century." Here it comes to know why GDP empowers policymakers and central banks to identify whether the economy is shrinking or growing, whether it requires an enhancement or limitation and if any risk like deflation or inflation emerges on horizon. So, contribution this study in the literature is that we will find out which of the factors among different types of consumptions, investments, saving and expenditures effect the GDP growth over long term as we have a long term data of 24 years for different countries. So in this study, we will also examine for any country which of the variables does effect the GDP growth in long term, so that policymakers will be facilitated with this research for making different policies for long term planning and it will be easier for them to choose from the current policies for boosting their economies.

2. Literature review

Martinovic et al. (2015) inspected interethnic contacts of Germany with the help of multilevel latent growth approach. A longitudinal data comprising of German Socio Economic Board consisting of 15 years statistics was used for this research. For interethnic study, in literature, cross sectional studies were practiced but the researchers applied this on the longitudinal data. The Multilevel model confirmed some of the previous recognized factors, and also questioned the causal influence of others.

Maulana et al. (2014) examined student's academic motivation as an influence from teacher's personal involvement with the application of Multilevel Growth models. Data used for this study involved the 12 videotapes for thirteen teachers of mathematics among a schools year. The results exposed the same results in the Western background; they determined that teacher's contribution is a significant factor of self-directed motivation. Though, they found the teacher's participation as a significant indicator of controlled inspiration as well.

Ahmed et al. (2013) examined the achievement behavior in mathematics at schools level with the help of Growth Curve analysis. The study purposed to identify the trends in academic emotions and examination of relationship between emotions and self-regulatory approaches of students. Data of study comprised of 495 student's achievement in mathematics for 3 terms in school year. Growth model revealed the student's amusement and arrogance in mathematics weakened, as compared to monotony which improved over time. Anxiety persisted relatively steady through the study period. Generally, the results proposed that with the "will" and the "skill," students require the "thrill" to thrive in the school.

Nocentini et al. (2013) investigated the change patterns of high school intimidation behavior through Multilevel GCM approach. Data for this study was collected from 515 students of 41 classes over a period of three years. At beginning of the survey students were enrolled in the grade 9 and 10. A two level growth model was used to identify the behavior and results revealed that there was significant variation in both the levels. Level one was taken as to be individuals and level 2 was taken to be classes. At level 1, aggression, gender and struggle for success were the indicators of bullying behavior. In conclusion, a cross level interface emphasized the connection between anger and intimidation was diluted with pro-bullying performances inside each class.

Drozd et al. (2013) used Multilevel Growth model to a randomized control trial of a web based intrusion for reduction of stress. The study aimed at investigating that mindfulness are possible mediators of some treatment effects, and examine the effect of intervention for females and males, all ages, at all stages of education. The data of study comprised of 259 individuals from which 126 were cases and 133 were controlled. Multilevel approach indicated that cases recovered more efficiently more quickly from stress than the controls. Results suggested about the usefulness of web based intervention in relieving stress among all the normal population.

Fernandes (2012) evaluated the mathematical structures for estimating growth of young bulls. Data used in this study consisted of 20 Nellore bulls having initial weight 129 kg and ultimate weight 405 kg. They selected animals randomly and allocated them in four plans. Authors evaluated five different models for describing growth i.e. Logarithmic, Gompertz, Multiphase, Linear, and Logistic models. Tests for the goodness of fit, correlations, and standard errors of estimates and simultaneous F tests were used and their comparison was made. All the test predicted the variability between animals but the study concluded that logistic and gompertz model predictions were not satisfactory and Multiphase model was extra competent than the others for prediction of cattle growth.

Rudolph et al. (2011) applied latent growth models on mental health in elementary schools. They observed about prediction of aggressiveness and depressive indications with the help of early and later oppression. The results were drawn on the basis of latent growth curve modeling which indicated depressiveness and aggressive behavior is a result of both early and later victimization and both of these victimizations contribute individually. They also discovered, over time, victimization lead to mental health.

3. Data and methodology

This study used three variables; Gross Domestic Product, Gross National Income and Gross National Expenditure. These variables contain twenty four years (1991-2014) panel data of twenty four countries; Argentina, Azerbaijan, Bangladesh, Canada, Colombia, Denmark, Egypt, Georgia, Hungary, Japan, Malaysia, Mexico, Pakistan, Russian Federation, New Zealand, Iceland, South Africa, Mauritius, Thailand, Norway, United Kingdom, India, Indonesia and United States. This data is obtained from International Financial Statistics (IFS). All variables are transformed into Billion U.S. Dollars.

3.1 Multilevel Growth models: A wide range of approaches are there for the analysis of longitudinal data. All those methods have some features and specific methods to incorporate on the basis of requirements and type of research. Multilevel Growth modeling is a method that can be used to analyze panel (grouped) data with missing observations at random. The parameters do not get biased with missing data at random (DeShon, et al. 1998, Little and Schlenker, 1995). Multilevel Growth models are also used for repeated measures data analysis. Another advantage of Multilevel Growth models is that it can be used to the data which have different number of time measurements for different individuals (Buxton, 2008).

The Multilevel Growth model might be thought of some system of equations which utilized the independent variables used in a level of analysis as dependent variables in the other level of analysis. This approach is discussed to be as a "growth" model (Webb et al, 2002).

3.2 Benefits of Multilevel Growth models: In a Multilevel Growth model, we can use random variables for modeling variation between the groups. An alternative way to model variation between groups is to use Simple Regression model, but we have to include dummy variables in the model to represent the group differences. The Multilevel Growth modeling approach provides several advantages.

- Fewer parameters need to be estimated
- Between groups Information can be shared
- Generalization can be done to a wider population
- Missing observations do not affect the model estimation

3.3 Difference between Multilevel Growth models and Regression models: Multilevel Growth models differ from the Simple Regression approach in the sense that these incorporate two random variables, the overall level random variable and the subject level random variable. Multilevel Growth models include two terms, Fixed Effect and Random Effects. Fixed Effects are the estimates of the overall model and Random Effects are the estimates which are obtained for individual level. Regression model includes only one term called Fixed Effects of the model. In Regression, there are assumptions of independence of variables, uncorrelated error terms, which are not incorporated into the Multilevel Growth models. The most serious problem in using Regression approach to the longitudinal data is that analysis fails to account for the non-independence of the observation i.e. it does not consider that each individual contains multiple responses thus leading to large standard errors and we cannot get the significant results if there exist any (Bliese and Ployhart, 2002, Kenny and Judd, 1986).

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- Fewer parameters need to be estimated
- Between groups Information can be shared
- Generalization can be done to a wider population
- Can tackle the missing observations

3.5 Extension of Multilevel Growth modeling: Multilevel Growth models can be extended, in case of complex data or requirements.

- Group level inclusion of predictors
- Multiple grouping levels
- Multivariate responses
- Cross-classified data (e.g. children in a school could come from some different areas and the same area children could go to different schools
- Usual extensions like in Regression categorical predictors, multiple predictors and responses etc (Buxton, 2008).

3.6 Specification of model: This study used the Multilevel Growth Curve model function purposed by Hox and Stoel (2005) to check the effect of different factors on GDP.

3.7 Structural Form: For a variable y_{ti} having repeated measures of individual *i* at time *t*, the general Multilevel Growth Curve model may be written as:

$$y_{ti} = \lambda_{0t}\eta_{0i} + \lambda_{1t}\eta_{1i} + \gamma_{2i}x_{ti} + \varepsilon_{ti} \quad (3.1)$$

$$\eta_{0i} = v_0 + \gamma_0 z_i + \zeta_{0i} \quad (3.2)$$

$$\eta_{1i} = v_1 + \gamma_1 z_i + \zeta_{1i} \quad (3.3)$$

where,

λ_{1t} denotes the measurement time and λ_{0t}a constant having value equal to 1.

Also note that in a fixed events model λ_{1t}will be classically a successive sequence of integers (i.e. 0, 1, 2, 3,…, T) equal to all entities, in our case equal to no. of years i.e. 24. The individual intercept and slope of Growth Curve model are represented by η_{0i} and η_{0i}. Their respective expected values are v_0 and v_1 while random errors are equal to ζ_{0i} and ζ_{0i}. Here, γ_{2i} shows the effect of time varying variable (covariate) x_{ti}. Also $\gamma_0$and $\gamma_1$are the effects of time varying variable (covariate) on the first level and varying linear slope. Variations which are time specific are shown by independent and identical standard normal variable ε_{ti} whose variance is σ_{ε}^2. It is assumed that

$cov(\varepsilon_{it}, \varepsilon_{it'}) = 0, (\varepsilon_{it}, \eta_{i0}) = 0 , cov(\varepsilon_{it}, \eta_{i1}) = 0.$

3.8 Model 1: GDP and GNI:

$\boldsymbol{i} = \ cross\ sectionals = 1,2, \ldots ,24$

$\boldsymbol{t} = \ time\ periods = 1,2, \ldots ,24$

In this model, y_{ti} denotes GDP (dependent variable) and x_{ti} denotes GNI (independent variable) and λ_{1t} denotes time in years.

3.9: Model 2: GDP and GNE:

$\boldsymbol{i} = \ cross\ sectionals = 1,2, \ldots ,24$

$\boldsymbol{t} = \ time\ periods = 1,2, \ldots ,24$

In this model, y_{ti} denotes GDP (dependent variable) and x_{ti} denotes GNI (independent variable) and λ_{1t} denotes time in years.

3.10 Empirical Methodology: Panel data are the data containing both times series and cross sectional data. And when time is included in the data, before moving to estimation we have to test whether there is any long run or short run relationship between variables. There are both univariate (Engle and Granger, 1987) and multivariate techniques to test co-integration but we also have another problem of non-stationarity in time series data. So, first of all we have to test the stationarity of the series. We have different Unit Root test in literature to test for stationarity.

3.10.1 Unit Root Test: For an AR(1), model

$$Z_t = \ \varphi Z_{t-1} + e_t \qquad (3.4)$$

where,

$e_t \sim white\ noise$

If we consider $\boldsymbol{\varphi}$, three possibilities are there i.e. $|\boldsymbol{\varphi}|>1$, $|\boldsymbol{\varphi}|<1$ and $|\boldsymbol{\varphi}|=1$. For a series to be stationary $|\boldsymbol{\varphi}|$ should be less than one. For $|\boldsymbol{\varphi}|>1$, stationarity does not hold and for $|\boldsymbol{\varphi}|=1$, series will be stationary and also there will be Unit Root of this series. So, if we have |**φ**|=1, by subtracting Z_{t-1} from both sides of the eq. (3.1), we get

$$Z_t - Z_{t-1} = Z_{t-1} - Z_{t-1} + e_t$$

$$\Delta Z_t = e_t \quad (3.5)$$

Now, ΔZ_t has been converted to a stationary series and e_t is a white noise process. Hence. by differencing Z_t stationarity can be achieved which means Z_t has a Unit Root.

3.10.2 Co-integration: A non-stationary time series shows trend. In the presence of non stationarity our OLS estimates might be erroneous, that is why one way to deal with non stationarity and make the series stationary is differencing but it has some limitations. One problem due to differencing is that error process is also differenced which produces a non-invertible moving average process. Another problem after differencing, model does not have a unique long run solution. So, for dealing with these types of issues, here comes the concept of co-integration, which deals with the problem of non stationarity without including above mentioned disputes.

For two variables being non stationary, the cumulated error process for these variables will also be non-stationary. If in s special case this error process becomes stationary, the variables will be called as co-integrated. Suppose, we have two stationary variables let's say X_t and Y_t. Their linear relationship can be obtained by

$$Y_t = a_1 + a_2 X_t + u_t \quad (3.8)$$

And the residuals are obtained by

$$u_t = Y_t - a_1 - a_2 X_t \quad (3.9)$$

The variables X_t and Y_t are said to have long run relationship or co-integrated if $u_t \sim I(0)$. The Error Correction Model (ECM) provides us both the long run and short run relationships between these two time series variables.

3.11 R functions: R is freeware software and is used for advanced statistical computing. In this study, we have used two R functions to estimate the Growth Curve model of GDP i.e. GLM and LMER. GLM stands for Generalized Linear model and LMER function exists in the library of LME4. LMER function is used to estimate the Maximum Likelihood or Restricted Maximum Likelihood estimates of the parameters of the Mixed Effects Models. Mixed Effects Models are said to be mixed as these include both fixed term and random term in the linear models.

For example, if we have two variables i.e. three test results and IQ for a school in four class sections. Let, TR denotes the test result, IQ for IQ, class for the different four sections and T for three test times i.e 1 for *1st* test, 2 for *2nd* and 3 for *3rd* test, thwen we can show the GLM and LMER models as follows:

Model 1: GLM(TR ~ T + IQ + Class + IQ:Class, data = abc)
Model 2: LMER(TR ~ T + IQ + Class + IQ:Class + (1 | Class), data = abc, REML = FALSE)
Here, in LMER, REML = FALSE meaning that we don't need Restricted Maximum Likelihood estimates and are interested in the Maximum Likelihood estimates of the model.

Then a function "summary()" is used to show all the estimates and degrees of freedom concerned with residuals and overall model. Another function "coef()" can be used to find the results of the above models. It gives the residual degrees of freedom, estimates of parameters, their respective standard errors, t-statistics and also the p-values concerned with each estimate to find that the any parameter estimate is significant or insignificant.

4. Results and discussion

Here, we show the findings of Unit Root tests which were used to check the level of integration of variables. We also show the Lag Length Criteria to test the Johansson base panel co-integration approach. Then we show the estimates of our models which were estimated in R through two different approaches i.e. GLM and LMER.

4.1 Unit Root results: This study used three tests of Unit Root for checking stationarity of variables; Im, et al. (1997), Fisher, ADF Choi-Z stat and Augmented Dickey Fuller Fisher Chi-square. Normally every time series has intercept and trend. That is why we tested the data with trend. Table 1 shows that the variables; GDP, GNI and GNE are not stationary at their level in all three tests. But, when we apply first difference on these series, they meet the stationarity. These results reveal that all the variables are I(1). All these results are significant at 5% level of significance.

4.2 Lag criteria: Vector Auto Regressive (VAR) technique is used to select for the lag order before moving to second step of cointegration. Five tests used in eviews for this technique are Likelihood Ratio, Akaike Information Criteria, Final Prediction Error, Schwarz Criteria, and Hannan-Quinn Criteria. Table 2 describes the results of VAR of our case. The results recommend us to select lag order 2 as a maximum lag order showing that all the tests LR, FPE, AIC, SC and HQ are significant at lag 2.

4.3 Co-integration results: We moved to co-integration test after selecting the appropriate lag order. Now we run the Johansen base panel co-integration technique to evaluate the relationship of long run and short run among the variables. Table 3 gives the long run results of our case. It shows two long run co-integrations running among the variables as it has two maximum eigen value which are greater than their corresponding critical values.

Now, VECM (Vector Error Correction Model) can be run because of the co-integration of all variable. Table 4 provides the results of VECM which are resultant from co-integrated equation where GDP is a dependent variable. ECM is a speediness of change to long run stability. The value of ECM is (1.2965) and is significant at 5% level of significance, which does not show long run causation running from two independent variables (GNI and GNE) to dependent variable (GDP). Also the value of R-square (0.0947) is showing that how much OLS explained our model.

4.4 Model estimation: We have a dependent variable GDP, and two independent variables GNI and GNE. So we have two different models one model for each independent variable. Furthermore, we run two tests in R named GLM and MLER to find the estimates of our models. In the following tables the estimates, standard errors of estimates, t-test statistics value and their corresponding p-values are given.

4.4.1 Model 1: Table 5 contains the results of Model 1 from both methods i.e. GLM and LMER. Column 4 shows the probability of rejecting true null hypothesis that the estimated parameter has any significant effect on the model or not? If p-value is less than alpha value which is taken to be 5 % in this study, we will reject the null hypothesis and conclude that estimated parameter has significant effect on the model.

While looking to the Table 5, we find that Fixed Effects coefficient of slope (GNI) is significant in both GLM and LMER estimates. In Random Effects, intercept for Japan, Russian Federation, United Kingdom and United states are significant in both GLM and LMER estimates. The slope for Japan and United States are significant for the Model 1 in both GLM and LMER estimates. It is also clear from the Table 6 that both GLM and LMER give the same results with a minor variation in the estimates. Finally r square is a statistical measure of how close the data are to the fitted model. Here, in this model, percentage of explained variation of GDP with independent variable GNI is 99.9% with GLM and 99.9% with LMER.

4.4.1.1 Model comparison: Model comparison for best fitted model can be done with the help of Information criteria given by Akaike and Bayes values. We choose that model for which value of AIC and BIC is minimum. We can see from Table 6 that, both AIC and BIC are least in GLM, than LMER. Also GLM gives more degrees of freedom for residuals than LMER.

4.4.2 Model 2: Table 7 describes the results of Model 2 from both methods i.e. GLM and LMER. While looking to the Table 7, we find that Fixed Effects coefficients for intercept, time and slope (GNE) are significant in both GLM and LMER estimates. In Random Effects, intercept for Canada, Japan, United Kingdom and United States are significant in both GLM and LMER estimates. The slope for Russian Federation is significant for the Model_2 in both GLM and LMER estimates. It is also clear from the Table 7 that both GLM and LMER give the same results with a minor variation in the estimates. Finally r square shows that percentage of explained variation of GDP with independent variable GNE is 99.9% with GLM and 99.9% with LMER.

4.4.2.1 Model comparison: We can see in Table 8 that both AIC and BIC are minimum in GLM than LMER. Also GLM gives more degrees of freedom for residuals but not much difference from LMER is there.

5. Conclusion

In a nutshell, from the results of this study, we conclude that Gross national income and Gross national expenditure are the vital factors contributing to the growth of Gross Domestic Product of any country over the long period of time. Hence policy makers should consider these factors for country’s GDP growth in long run policies. And the efficacy of these factors differs from developed to developing countries. Also both GLM and LMER provide almost same results but if we have to decide from any one of them then we can select GLM for better results in the case of liner multilevel growth modeling on the basis of AIC and BIC.

Table 1: Results of Unit Root test

Variable Name	Fisher Chi-square		Choi-Z Stat		Im, Pesaran and Shin W-stat	
	Level	1st Diff	Level	1st Diff	Level	1st Diff
GDP	6.12	159.002 *	10.45	-8.050 *	11.58	-8.075 *
GNI	6.25	157.418 *	10.75	-7.937 *	12.09	-7.96 *
GNE	5.67	146.376 *	10.82	-7.266 *	12.26	-7.25 *

Table 2: Lag Order selection criteria (VAR)

Lag	**LogL**	**LR**	**FPE**	**AIC**	**SC**	**HQ**
0	-133838.8	NA	5.4e+217	535.4032	535.5043	535.4429
1	-123866.3	19426.44	4.6e+200	496.0892	497.4041	496.6051
2	-121989.2	3566.522*	5.4e+197*	489.1567*	489.1567*	490.1490*
* Indicates lag order selected by the criterion (ρ < 0.05)						

Table 3: Maximum eigenvalue rank test

Hypothesized No. of Co-integrations	**Eigenvalue**	**Max. Eigen Statistic**	**0.05 Critical Value**	**Prob.****
None *	0.182466	104.5596	21.131	0.0001
At most 1 *	0.050599	26.9489	14.2646	0.0003
At most 2	0.0029422	1.5292	3.8414	0.2162
* denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				

Table 4: Estimates of Vector Error Correction

Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**Prob.**
ECM	1.2965	0.2673	4.8499	0.0000 *
C	3.57E+10	1.16E+10	3.0706	0.0022 *
D(GNI)	-0.0704	0.6313	-0.1115	0.9112
D(GNE)	-0.3217	0.4830	-0.6661	0.5056
R Squared		0.0947	S.E Regression	2.67E+11
Adjusted R Square		0.0880	SS Residuals	3.88E+25

Table 5: Model 1 results

	GLM				LMER			
	Estimate	**Std. Error**	**t value**	**Pr(>|t|)**	**Estimate**	**Std. Error**	**t value**	**Pr(>|t|)**
Fixed Effects								
(Intercept)	7.23E+11	6.58E+11	1.099554	0.272	7.23E+11	6.28E+11	1.150967	0.206
T	-3.6E+08	3.3E+08	-1.09835	0.273	-3.6E+08	3.15E+08	-1.14971	0.206
GNI	1.041724	0.035586	29.27303	0.000 *	1.041724	0.033997	30.64179	0.000 *
Random Effects								
Intercept								
Azerbaijan	1.35E+09	1.34E+10	0.100393	0.920	1.35E+09	1.28E+10	0.105087	0.397
Bangladesh	2.51E+08	1.49E+10	0.016829	0.987	2.51E+08	1.42E+10	0.017616	0.399
Canada	1.42E+10	1.61E+10	0.884266	0.377	1.42E+10	1.54E+10	0.925612	0.260
Colombia	-2.93E+09	1.42E+10	-0.20598	0.837	-2.93E+09	1.36E+10	-0.21561	0.390
Denmark	7.52E+09	1.86E+10	0.403312	0.687	7.52E+09	1.78E+10	0.422171	0.365
Egypt	-2.46E+09	1.45E+10	-0.16974	0.865	-2.46E+09	1.39E+10	-0.17768	0.392
Georgia	7.38E+08	1.53E+10	0.048182	0.962	7.38E+08	1.46E+10	0.050435	0.398
Hungary	-2.1E+09	1.65E+10	-0.12706	0.899	-2.1E+09	1.57E+10	-0.133	0.395
Iceland	-2.6E+09	1.82E+10	-0.14355	0.886	-2.6E+09	1.74E+10	-0.15026	0.394
India	-4.7E+09	1.41E+10	-0.33508	0.738	-4.7E+09	1.35E+10	-0.35075	0.375
Indonesia	8.23E+09	1.38E+10	0.597005	0.551	8.23E+09	1.32E+10	0.62492	0.328
Japan	1.69E+11	3.77E+10	4.484813	0.000 *	1.69E+11	3.60E+10	4.694516	0.000 *
Malaysia	2.41E+09	1.45E+10	0.165949	0.868	2.41E+09	1.39E+10	0.173709	0.393
Mauritius	-2E+09	1.58E+10	-0.12477	0.901	-2E+09	1.51E+10	-0.1306	0.395
Mexico	1.70E+08	1.71E+10	0.009944	0.992	1.70E+08	1.64E+10	0.010409	0.399
New Zealand	-1.3E+09	1.68E+10	-0.07608	0.939	-1.3E+09	1.61E+10	-0.07964	0.397
Norway	3.19E+09	1.52E+10	0.210606	0.833	3.19E+09	1.45E+10	0.220453	0.389
Pakistan	1.99E+09	1.46E+10	0.136062	0.892	1.99E+09	1.40E+10	0.142424	0.395
Russia	-4.84E+10	1.42E+10	-3.41544	0.000 *	-4.84E+10	1.35E+10	-3.57514	0.000 *
South Africa	-2.4E+09	1.61E+10	-0.14909	0.882	-2.4E+09	1.54E+10	-0.15606	0.394
Thailand	-5.3E+09	1.57E+10	-0.33906	0.735	-5.3E+09	1.50E+10	-0.35492	0.374
United Kingdom	4.03E+10	1.87E+10	2.153357	0.000 *	4.03E+10	1.79E+10	2.254044	0.000 *
United States	2.96E+11	1.98E+10	14.93016	0.000 *	2.96E+11	1.89E+10	15.62827	0.000 *

Slope								
Azerbaijan	0.119886	0.209741	0.571592	0.568	0.119886	0.200372	0.598318	0.333
Bangladesh	-0.07275	0.118462	-0.61414	0.539	-0.07275	0.11317	-0.64285	0.324
Canada	-0.03054	0.035736	-0.85461	0.393	-0.03054	0.03414	-0.89457	0.267
Colombia	0.029287	0.056411	0.519175	0.604	0.029287	0.053891	0.543451	0.344
Denmark	-0.06438	0.068477	-0.94015	0.348	-0.06438	0.065418	-0.9841	0.246
Egypt	-0.0061	0.072803	-0.08381	0.933	-0.0061	0.069551	-0.08772	0.397
Georgia	0.369877	1.119243	0.330471	0.741	0.369877	1.069247	0.345923	0.376
Hungary	0.072251	0.134989	0.535236	0.593	0.072251	0.128959	0.560263	0.341
Iceland	0.554401	1.328226	0.4174	0.677	0.554401	1.268894	0.436917	0.362
India	-0.01178	0.035443	-0.3325	0.740	-0.01178	0.033859	-0.34805	0.375
Indonesia	-0.01269	0.037845	-0.33541	0.737	-0.01269	0.036154	-0.35109	0.375
Japan	-0.09708	0.035678	-2.72104	0.000 *	-0.09708	0.034084	-2.84827	0.000 *
Malaysia	0.004422	0.063021	0.070173	0.944	0.004422	0.060205	0.073454	0.398
Mauritius	0.745159	1.678563	0.443926	0.657	0.745159	1.603582	0.464684	0.358
Mexico	-0.01665	0.037265	-0.44686	0.655	-0.01665	0.0356	-0.46775	0.357
New Zealand	0.065285	0.149482	0.436742	0.662	0.065285	0.142805	0.457163	0.359
Norway	-0.04416	0.046195	-0.95597	0.340	-0.04416	0.044131	-1.00067	0.242
Pakistan	-0.08518	0.077446	-1.09989	0.272	-0.08518	0.073987	-1.15132	0.205
Russia	0.023958	0.035558	0.673785	0.501	0.023958	0.033969	0.70529	0.311
South Africa	0.008242	0.058775	0.140223	0.889	0.008242	0.05615	0.146779	0.394
Thailand	0.036479	0.058713	0.621318	0.535	0.036479	0.05609	0.65037	0.323
United Kingdom	-0.06151	0.035347	-1.74007	0.082	-0.06151	0.033768	-1.82144	0.076
United States	-0.07548	0.035404	-2.13185	0.000 *	-0.07548	0.033823	-2.23154	0.000 *
R Square	0.999				0.999			
***Indicates significance at 5%**								

Table 6: Model 1 comparison

GLM	**LMER**	**GLM**	**LMER**	**GLM**	**LMER**
d.f.		**AIC**		**BIC**	
512	510	28,395	28,397	28,611	28,617

Table 7: Model 2 results

	GLM				LMER			
	Estimate	**Std. Error**	**t value**	**Pr(>|t|)**	**Estimate**	**Std. Error**	**t value**	**Pr(>|t|)**
Fixed Effects								
(Intercept)	-2.74E+12	8.28E+11	-3.30898	0.001 *	-2.74E+12	7.91E+11	-3.46311	0.001 *
T	1.38E+09	4.16E+08	3.312587	0.001 *	1.38E+09	3.97E+08	3.466889	0.001 *
GNE	0.966979	0.046201	20.92978	0.000 *	0.966979	0.044145	21.9047	0.000 *
Random Effects								
Intercept								
Azerbaijan	-1.19E+10	1.77E+10	-0.67427	0.500	-1.19E+10	1.69E+10	-0.70568	0.311
Bangladesh	-1.06E+09	1.99E+10	-0.05319	0.958	-1.06E+09	1.90E+10	-0.05567	0.398
Canada	4.40E+10	2.11E+10	2.083273	0.038 *	4.40E+10	2.02E+10	2.180313	0.037 *
Colombia	-5.09E+09	1.88E+10	-0.2703	0.787	-5.09E+09	1.80E+10	-0.2829	0.383
Denmark	1.42E+10	2.52E+10	0.565619	0.572	1.42E+10	2.40E+10	0.591965	0.335
Egypt	-2.96E+08	1.90E+10	-0.01557	0.988	-2.96E+08	1.82E+10	-0.0163	0.399
Georgia	-9.16E+09	2.02E+10	-0.45404	0.650	-9.16E+09	1.93E+10	-0.47519	0.356
Hungary	-2.3E+09	2.11E+10	-0.10802	0.914	-2.3E+09	2.01E+10	-0.11306	0.396
Iceland	1.07E+09	2.24E+10	0.048018	0.962	1.07E+09	2.14E+10	0.050255	0.398
India	1.07E+10	1.86E+10	0.573165	0.567	1.07E+10	1.78E+10	0.599863	0.333
Indonesia	3.49E+09	1.82E+10	0.191692	0.848	3.49E+09	1.74E+10	0.200621	0.391
Japan	3.28E+11	4.88E+10	6.731059	0.000 *	3.28E+11	4.66E+10	7.044595	0.000 *
Malaysia	1.87E+07	1.93E+10	0.000971	0.999	1.87E+07	1.84E+10	0.001016	0.399
Mauritius	8.14E+08	2.03E+10	0.040136	0.968	8.14E+08	1.94E+10	0.042006	0.398
Mexico	1.21E+10	2.27E+10	0.533938	0.594	1.21E+10	2.17E+10	0.55881	0.341
New Zealand	3.58E+09	2.22E+10	0.161116	0.872	3.58E+09	2.12E+10	0.168621	0.393
Norway	-4.5E+09	2.04E+10	-0.22122	0.825	-4.5E+09	1.95E+10	-0.23152	0.388
Pakistan	1.07E+09	1.94E+10	0.055345	0.956	1.07E+09	1.85E+10	0.057923	0.398
Russia	-3.53E+10	1.87E+10	-1.88948	0.059	-3.53E+10	1.78E+10	-1.9775	0.057
South Africa	8.57E+09	2.10E+10	0.407362	0.684	8.57E+09	2.01E+10	0.426338	0.364
Thailand	-1.5E+09	2.08E+10	-0.07047	0.944	-1.5E+09	1.98E+10	-0.07375	0.398
United	5.32E+10	2.48E+10	2.142929	0.033 *	5.32E+10	2.37E+10	2.242748	0.032 *

Kingdom								
United States	3.43E+11	2.69E+10	12.75263	0.000 *	3.43E+11	2.57E+10	13.34665	0.000 *
Slope								
Azerbaijan	0.000858	0.36105	0.002377	0.998	0.000858	0.34498	0.002487	0.399
Bangladesh	-0.24389	0.159237	-1.5316	0.126	-0.24389	0.15215	-1.60295	0.110
Canada	-0.01513	0.046589	-0.32484	0.745	-0.01513	0.044516	-0.33997	0.376
Colombia	-0.06767	0.071192	-0.95047	0.342	-0.06767	0.068023	-0.99475	0.243
Denmark	-0.05176	0.097281	-0.53202	0.595	-0.05176	0.092951	-0.55681	0.341
Egypt	-0.16834	0.088335	-1.90574	0.057	-0.16834	0.084404	-1.99451	0.055
Georgia	-1.31459	1.207464	-1.08872	0.277	-1.31459	1.153723	-1.13943	0.208
Hungary	-0.12865	0.169645	-0.75833	0.449	-0.12865	0.162095	-0.79365	0.291
Iceland	-1.54279	1.379829	-1.1181	0.264	-1.54279	1.318416	-1.17019	0.201
India	-0.01989	0.046142	-0.431	0.667	-0.01989	0.044088	-0.45107	0.360
Indonesia	-0.00191	0.049393	-0.03868	0.969	-0.00191	0.047194	-0.04048	0.398
Japan	-0.03333	0.046489	-0.7169	0.474	-0.03333	0.04442	-0.75029	0.301
Malaysia	0.063344	0.090958	0.696401	0.486	0.063344	0.08691	0.72884	0.306
Mauritius	-2.61245	1.868832	-1.3979	0.163	-2.61245	1.785655	-1.46302	0.137
Mexico	-0.01689	0.048573	-0.34768	0.728	-0.01689	0.046412	-0.36387	0.373
New Zealand	-0.17937	0.186949	-0.95945	0.338	-0.17937	0.178628	-1.00414	0.241
Norway	0.115096	0.066671	1.726326	0.085	0.115096	0.063704	1.806739	0.078
Pakistan	-0.19491	0.100378	-1.94174	0.053	-0.19491	0.095911	-2.03219	0.051
Russia	0.119203	0.046402	2.568934	0.010 *	0.119203	0.044337	2.688596	0.011 *
South Africa	-0.07863	0.074847	-1.05058	0.294	-0.07863	0.071516	-1.09952	0.218
Thailand	-0.01147	0.076821	-0.14928	0.881	-0.01147	0.073402	-0.15623	0.394
United Kingdom	-0.02098	0.046055	-0.45547	0.649	-0.02098	0.044005	-0.47668	0.356
United States	-0.02957	0.046004	-0.64271	0.521	-0.02957	0.043957	-0.67264	0.318
R Square	0.999				0.999			
***Indicates significance at 5%**								

Table 8: Model 2 comparison

GLM	LMER	GLM	LMER	GLM	LMER
d.f.		**AIC**		**BIC**	
514	512	28,815	28,817	29,031	29,038

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