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Kurtosis Statistics with Reference to Power Function Distribution

Azaz Ahmad[1] *and Ahmed Saeed Akhter*[2]

Abstract

Pearson statistics of skewness and kurtosis gave false impression to assess the peakedness and tailedness for skewed (moderately, J-shape or reverse J-shape) Distributions. A number of alternate measures were suggested in literature by Hosking (1992), Blest (2003), Elamir and Seheult (2003), and Fiori and Zenga (2005) that provided better interpretation than the Karl Pearson statistics. Power Function Distribution has the characteristics of symmetric, J-shape or reverse J-shape with varying magnitude of its shape parameter. In this paper, we derived the Blest's statistics of skewness and kurtosis, L-skewness and L-kurtosis and Trimmed L-skewness and Trimmed L-kurtosis for Power Function Distribution. Comparison is made with Karl Pearson statistics.

Keywords

Power function distribution, Blest's measure, L-moments, Trimmed L-moments

1. Introduction

The oldest and the most common measure of skewness and kurtosis is the standard fourth moment by Pearson (1905). These measures often concentrate only on symmetric Distribution. It does not provide true information about peakedness and tailedness for skewed Distribution.

[1] Government Postgraduate College, Okara, Pakistan
Email: azazpu@yahoo.com
[2] College of Statistical and Actuarial Sciences, University of the Punjab, Quaid e Azam Campus, Lahore, Pakistan
Email: asakther@yahoo.com

Poor performance of standardized central moment of skewness and kurtosis may lead to seek alternate statistics and definition to study the Distribution shape characteristics. Included in these Blest's measures (Blest, 2003), L-skewness and L-kurtosis (Hosking, 1992) and TL-skewness and TL-kurtosis (Elamir and Seheult, 2003).

Rahila and Memon (2012) also conducted such kind of comparison for Weibull Distribution but they just compared the Pearson with Blest's measure. Both Blest's and Karl Pearson measures are based on higher moments of a Distribution. So these measures can't evaluate for those Distributions like Cauchy and Inver Rayleigh Distribution whose higher moments do not exists. So how can we study the description of such Distributions? Answer is obtained by evaluating another alternate measures, the L-moments and Trimmed L-moments. So in our study, we computed also L-skewness and L-kurtosis, Trimmed L-skewness and Trimmed L-kurtosis for Power Function Distribution with the Blest's measures and Pearson.

Statistical Distributions have long been employed in the assessment of semi-conductor device and product reliability. The use of the Exponential Distribution which is frequently preferred over mathematically more complex Distributions, such as the Weibull and the Lognormal among others, suggest that most engineers favor the application of simpler models to obtain failure rates and reliability figures quickly. It is, therefore, proposed that the Power Function Distribution be considered as a simple alternative which, in some circumstances, may exhibit a better fit for failure data and provide more appropriate information about reliability and Hazard Rates (Meniconi ,1996).

The Distribution Function of Power Function Distribution as

$$F(x)=\left(\frac{x}{b}\right)^{c} \qquad 0\le x\le c$$

$$\text{shape parameter}=c \qquad (1.1)$$

$$\text{scale parameter}=b>0$$

2. Skewness and Kurtosis Statistics for Power Function Distribution

2.1 Karl Pearson Statistics of Skewness and Kurtosis: Karl Pearson measure of skewness is the standard third moment coefficient as

$$\alpha_3 = E\left(\frac{X-\mu}{\sigma}\right)^3 \tag{2.1.1}$$

standard fourth moment coefficient for kurtosis as

$$\alpha_4 = E\left(\frac{X-\mu}{\sigma}\right)^4 \tag{2.1.2}$$

We evaluated for Power Function Distribution as

$$E(x^r) = \int_0^b x^r . \frac{cx^{c-1}}{b^c} dx$$

$$E(x^r) = \frac{cb^r}{c+r} \tag{2.1.3}$$

$$\mu_2 = \mu_2' - (\mu_1')^2$$

$$= \frac{cb^2}{c+2} - (\frac{cb}{c+1})^2$$

$$\mu_2 = \frac{cb^2}{(c+1)^2(c+2)} \tag{2.1.4}$$

$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2(\mu_1')^3$$

$$= \frac{cb^3}{c+3} - 3\left(\frac{cb}{c+1}\right)\left(\frac{cb^2}{c+2}\right) + 2\left(\frac{cb}{c+1}\right)^3$$

$$= \frac{cb^3}{c+3} - \left(\frac{3c^2b^3}{(c+1)(c+2)}\right) + \left(\frac{2c^3b^3}{(c+1)^3}\right)$$

$$= \frac{cb^3\left[(c+1)^3(c+2) - 3c\left\{(c+1)^2(c+3)\right\} + 2c^2\left\{(c+2)(c+3)\right\}\right]}{(c+1)^3(c+2)(c+3)}$$

$$\mu_3 = \frac{2cb^3(1-c)}{(c+1)^3(c+2)(c+3)} \tag{2.1.5}$$

$$\mu_4 = \mu_4' - 4\mu_1'\mu_3' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$$

$$= \frac{cb^4}{c+4} - 4\left(\frac{c^2b^4}{(c+1)(c+3)}\right) + 6\left(\frac{c^3b^4}{(c+1)^2(c+2)}\right) - 3\left(\frac{c^4b^4}{(c+1)^4}\right)$$

$$= \frac{cb^4\begin{bmatrix}(c+1)^4(c+2)(c+3) - 4c(c+1)^3(c+2)(c+4)\\ +6c^2(c+1)^2(c+3)(c+4) - 3c^3(c+2)(c+3)(c+4)\end{bmatrix}}{(c+1)^4(c+2)(c+3)(c+4)}$$

$$\mu_4 = \frac{3cb^4\left(3c^2 - c + 2\right)}{(c+1)^4(c+2)(c+3)(c+4)} \qquad (2.1.6)$$

Now using eq. (2.1.1) to eq. (2.1.6), we obtain Karl Pearson measures of skewness and kurtosis respectively as

$$\alpha_3 = \frac{2(1-c)(c+2)^{\frac{1}{2}}}{(c+3)(c^{\frac{1}{2}})} \qquad (2.1.7)$$

$$\alpha_4 = \frac{3(c+2)(3c^2 - c + 2)}{c(c+3)(c+4)} \qquad (2.1.8)$$

It is interesting to note that the third and fourth moment about mean are the function of scale and shape parameters but the measures of skewness and kurtosis are function of shape parameter only. Power Function Distribution is symmetric for c = 1 and negatively skewed for c > 1 and positively skewed for c < 1.

The Figure 1 shows the skewness and kurtosis for Power Function Distribution with changing the values of shape parameter. The Figure 1 indicates the coefficient of kurtosis has a value 3 at two different points c= 0.32 and c = 2.87. For c = 0.32 the Distribution is positively skewed ($\alpha_3 = 1.10299$) for c = 2.87 the Distribution is negatively skewed ($\alpha_3 = -0.82996$) for c = 1 the Distribution is symmetric. As we noted that skewness decreased for increasing value of shape parameter, kurtosis also decreased rapidly for c<1 but for c>1 it increased gradually.

We are focusing the Distribution in three cases

- c=1
- c>1
- c<1

2.2 Blest's Statistics for Kurtosis: Blest suggested a new measures of kurtosis by which the effect of any skewness is deleted, allowing comparison of Distribution on the basis of kurtosis alone (Blest, 2003). Blest proposed a new measure of central tendency called Meson (from Greek mesos, meaning 'middle') is denoted by ζ and the standardized value of meson is $f=\dfrac{\zeta-\mu}{\sigma}$, third moment about meson is zero i.e $E\left(X-\zeta\right)^3=0$ and the r^{th} moment about meson are defined as $\mu_r^*=E(x-\zeta)^r$

So first four moment about meson are

$$\mu_1^*=E(x-\zeta)$$

$$=E(x-\mu-\sigma f)\ =\mathrm{E}(\mathrm{x}-\mu)-\sigma f\ \ [\zeta=\mu+\sigma f]$$

$$\mu_1^*=-\sigma f \qquad (2.2.1)$$

$$\mu_2^*=E(x-\zeta)^2$$

$$=E(x-\mu-\sigma f)^2$$

$$=E\left\{(x-\mu)-(\sigma f)\right\}^2$$

$$=E\left\{(x-\mu)^2-2(x-\mu)(\sigma f)+\sigma^2 f^2\right\} \quad =E(x-\mu)^2-2\sigma fE(x-\mu)+\sigma^2 f^2$$

$$=\sigma^2-0+\sigma^2 f^2$$

$$=\sigma^2+\sigma^2 f^2$$

$$\mu_2^*=\sigma^2(1+f^2) \qquad (2.2.2)$$

$$\mu_3^*=E(x-\zeta)^3$$

$$=E(x-\mu-\sigma f)^3 \quad [\zeta=\mu+\sigma f]$$

$$=\mathrm{E}\left\{(\mathrm{x}-\mu)-\sigma f\right\}^3$$

$$=E\left\{(\mathrm{x}-\mu)^3-3(\mathrm{x}-\mu)^2(\sigma f)+3(\mathrm{x}-\mu)(\sigma^2 f^2)-\sigma^3 f^3\right\}$$

$$=E(\mathrm{x}-\mu)^3-3(\sigma f)\,\mathrm{E}((\mathrm{x}-\mu)^2+3\sigma^2 f^2 E(\mathrm{x}-\mu)-\sigma^3 f^3$$

$$\mu_3^*=\mu_3-3\sigma^3 f-\sigma^3 f^3 \qquad (2.2.3)$$

$$\mu_4^* = E(x-\zeta)^4$$
$$= E(x-\mu-\sigma f)^4$$
$$= E\{(x-\mu)-(\sigma f)\}^4$$
$$= E\{(x-\mu)^4 - 4(x-\mu)^3(\sigma f) + 6(x-\mu)^2(\sigma f)^2 - 4(x-\mu)(\sigma f)^3 + (\sigma f)^4\}$$
$$= E(x-\mu)^4 - 4\sigma f E(x-\mu)^3 + 6\sigma^2 f^2 \mathrm{E}(x-\mu)^2 - 4\sigma^3 f^3 \mathrm{E}(x-\mu) + \sigma^4 f^4$$
$$= \mu_4 - 4\sigma f(\mu_3) + 6\sigma^4 f^2 - 0 + \sigma^4 f^4$$
$$= \mu_4 - 4\sigma f(3\sigma^3 f + \sigma^3 f^3) + 6\sigma^4 f^2 + \sigma^4 f^4$$
$$= \mu_4 - 12\sigma^4 f^2 - 4\sigma^4 f^4 + 6\sigma^4 f^2 + \sigma^4 f^4$$
$$= \mu_4 + (-12\sigma^4 f^2 + 6\sigma^4 f^2) + (-4\sigma^4 f^4 + \sigma^4 f^4)$$
$$\mu_4^* = \mu_4 - 6\sigma^4 f^2 - 3\sigma^4 f^4 \qquad (2.2.4)$$

Blest’s proposed the following Moment Ratio

$$\alpha_3^* = \frac{\mu_3^*}{\sigma^3} \qquad (2.2.5)$$

$$\alpha_4^* = \frac{\mu_4^*}{\sigma^4} \qquad (2.2.6)$$

So from the eq. (2.2.3), we evaluate

$$\mu_3^* = \mu_3 - 3\sigma^3 f - \sigma^3 f^3$$
$$\frac{\mu_3^*}{\sigma^3} = \frac{\mu_3}{\sigma^3} - \frac{3\sigma^3 f}{\sigma^3} - \frac{\sigma^3 f^3}{\sigma^3}$$
$$\alpha_3^* = \alpha_3 - 3f - f^3$$

third moment as $E(X-\zeta)^3 = 0$

so, $f^3 + 3f - \alpha_3 = 0$ (2.2.7)

so, the real roots of this equation is the measure of skewness in term of standardized value of meson.

put $f = y - \frac{1}{y}$ *in* (2.2.7)

$$\left(y - \frac{1}{y}\right)^3 + 3\left(y - \frac{1}{y}\right) - \alpha_3 = 0$$

$$y^3 - \frac{1}{y^3} + \frac{3}{y} - 3y + 3y - \frac{3}{y} - \alpha_3 = 0$$

Let,

$$y^3 - \frac{1}{y^3} - \alpha_3 = 0$$

$$(y^3)^2 - 1 - \alpha_3 y^3 = 0$$

$$(y^3)^2 - \alpha_3 y^3 - 1 = 0 \qquad (2.2.8)$$

Put $y^3 = w$ *it reduced in quadratic equation*

$$w^2 - \alpha_3 w - 1 = 0$$

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1, b = -\alpha_3,\ c = -1$$

$$w = \frac{\alpha_3 \pm \sqrt{\alpha_3^{\ 2} - 4(1)(-1)}}{2(1)}$$

$$w = \frac{\alpha_3 \pm \sqrt{\alpha_3^{\ 2} + 4}}{2}$$

$$w = \frac{\alpha_3 \pm 2\sqrt{\frac{\alpha_3^{\ 2}}{4} + 1}}{2}$$

$$w = \frac{2\left(\frac{\alpha_3}{2} \pm \sqrt{\frac{\alpha_3^{\ 2}}{4} + 1}\right)}{2}$$

$$w=\frac{\alpha_3}{2}\pm\sqrt{\frac{\alpha_3{}^2}{4}+1}$$

$$w=0.5\alpha_3\pm\sqrt{0.25\alpha_3{}^2+1}$$

$$w_1=0.5\alpha_3-\sqrt{0.25\alpha_3{}^2+1}\ ,\mathrm{w}_2=0.5\alpha_3+\sqrt{0.25\alpha_3{}^2+1}$$

$$put\ back\ \text{into}\ \ y^3=w$$

$$y=w^{1/3}$$

so

$$y=\left(0.5\alpha_3-\sqrt{0.25\alpha_3{}^2+1}\right)^{1/3}$$

$$y=\left(0.5\alpha_3+\sqrt{0.25\alpha_3{}^2+1}\right)^{1/3}$$

The trial solution accomplishing this miracle turns out to be the symmetrical expression as

$$f=\left(\sqrt{0.25\alpha_3{}^2+1}-0.5\alpha_3\right)^{1/3}-\left(\sqrt{0.25\alpha_3{}^2+1}-0.5\alpha_3\right)^{1/3} \tag{2.2.9}$$

Blest's coefficient of skewness is derived for Power Function Distribution by putting the eq. (2.2.1) into eq. (2.2.9) as

$$f=\left(\sqrt{0.25\left(\frac{2(1-c)(c+2)^{1/2}}{c^{1/2}(c+3)}\right)^2+1}+0.5\left(\frac{2(1-c)(c+2)^{1/2}}{c^{1/2}(c+3)}\right)\right)^{\frac{1}{3}}$$
$$-\left(\sqrt{0.25\left(\frac{2(1-c)(c+2)^{1/2}}{c^{1/2}(c+3)}\right)^2+1}-0.5\left(\frac{2(1-c)(c+2)^{1/2}}{c^{1/2}(c+3)}\right)\right)^{\frac{1}{3}} \tag{2.2.10}$$

is the function of only shape parameter.

And coefficient of kurtosis is adjusted for skewness, as the standardized fourth moment about the Meson is obtained by taking the eq. (2.2.4).

$$\mu_4^* = \mu_4 - 6\sigma^4 f^2 - 3\sigma^4 f^4$$

$$\frac{\mu_4^*}{\sigma^4} = \frac{\mu_4}{\sigma^4} - \frac{6\sigma^4 f^2}{\sigma^4} - \frac{3\sigma^4 f^4}{\sigma^4}$$

$$\alpha_4^* = \alpha_4 - 6 f^2 - 3 f^4$$

$$\alpha_4^* = \alpha_4 - 3(f^4 + 2 f^2)$$

$$\alpha_4^* = \alpha_4 - 3((f^2)^2 + 2(f^2))$$

$$\alpha_4^* = \alpha_4 - 3\{(f^2)^2 + 2(f^2) + (1)^2 - (1)^2\}$$

$$\alpha_4^* = \alpha_4 - 3\{(1+ f^2)^2 - (1)^2\}$$

$$\alpha_4^* = \alpha_4 - 3((1+ f^2)^2 - 1) \qquad (2.2.11)$$

From Figure 3, it is clear that both measures have same interpretation for symmetric Distribution. We can see that how Karl Pearson coefficient of skewness gives false impression about skewness for Case II and III.

Figure 2 shows that for symmetric Distribution both measures of kurtosis are same but for skewed Distribution Blest measures always less than the Pearson kurtosis. The gap between two is smaller in Case II and it increased in Case III.

2.3 Relation among Meson, Mean and Median: The gap between median and meson is another way measuring the degree of kurtosis.

$$\text{Mean} = \frac{bc}{c+1} \qquad (2.3.1)$$

$$Median = \frac{b}{2^{1/c}} \qquad (2.3.2)$$

$$\sigma = \frac{bc^{1/2}}{(c+1)(c+2)^{1/2}} \qquad (2.3.3)$$

$$Meson = \zeta \Rightarrow \mu + \sigma f$$

$$\zeta = \frac{bc}{c+1} + \frac{bc^{1/2}}{(c+1)(c+2)^{1/2}} \left[\left(\sqrt{0.25\left(\frac{2(1-c)(c+2)^{1/2}}{c^{1/2}(c+3)} \right)^2 + 1} + 0.5\left(\frac{2(1-c)(c+2)^{1/2}}{c^{1/2}(c+3)} \right) \right)^{\frac{1}{3}} - \left(\sqrt{0.25\left(\frac{2(1-c)(c+2)^{1/2}}{c^{1/2}(c+3)} \right)^2 + 1} - 0.5\left(\frac{2(1-c)(c+2)^{1/2}}{c^{1/2}(c+3)} \right) \right)^{\frac{1}{3}} \right]$$

(2.3.4)

So the Figure 4 shows the relation between three measure of central tendency for b=1

It is clear from the Figure 4 that

- Meson = Mean = Median for Case I
- Meson >Mean> Median for Case II
- Meson <Mean< Median for Case III

So the Distributions for Case III are far flatter than Distributions in Case II.

2.4 L-kurtosis: L-moments are expectations of certain linear combinations of Order Statistics. L-moment exist for a real valued random variable *X*, iff *X* has a finite mean. A Distribution whose mean exists is characterized by its L-moments (Hosking, 1992).

The first four L-moment for random variable *X* are defined by

$$\lambda_1 = E(X) = \int x(F)dF$$

$$\lambda_2 = \frac{1}{2}E(X_{2,2} - X_{1,2}) = \int x(F)(2F-1)dF$$

$$\lambda_3 = \frac{1}{3}E(X_{3,3} - 2X_{2,3} + X_{1,3}) = \int x(F)(6F^2 - 6F + 1)dF$$

$$\lambda_4 = \frac{1}{4}E(X_{4,4} - 3X_{3,4} + 3X_{2,4} + X_{1,4}) = \int x(F)(20F^3 - 30F^2 + 12F - 1)dF$$

where,

$X_{k,n}$ is the k^{th} Order statistic for sample size *n*, the limits on the integral are 0 to 1. L-skewness and L-kurtosis is defined as $\tau_3 = \frac{\lambda_3}{\lambda_2}$ and $\tau_4 = \frac{\lambda_4}{\lambda_2}$, respectively.

K^{th} moment of Order Statistics for sample size *n* from Power Function Distribution is defined as

$$\alpha_{(k)}^{n} = \frac{n!}{(k-1)!(n-k)!}\int_0^b x\left[\frac{x^c}{b^c}\right]^{k-1}\left[1-\frac{x^c}{b^c}\right]^{n-k}\left[\frac{cx^{c-1}}{b^c}\right]dx$$

$$\alpha_{(k)}^{n} = \frac{bn!}{(k-1)!(n-k)!}\frac{Gamma\left(k+\frac{1}{c}\right)Gamma\left(n-k+1\right)}{Gamma\left(n+\frac{1}{c}+1\right)} \qquad (2.4.1)$$

$$\lambda_2 = \frac{1}{2}\left[\alpha_{(2)}^{(2)} - \alpha_{(1)}^{(2)}\right]$$

$$= \frac{1}{2}\left[\frac{2bc}{(1+2c)} - \frac{4bc^2}{(1+2c)(1+c)}\right]$$

$$\lambda_2 = \frac{bc}{(1+2c)(1+c)} \tag{2.4.2}$$

$$\lambda_3 = \frac{1}{3}\left[\alpha_{(3)}^{(3)} - 2\alpha_{(2)}^{(3)} + \alpha_{(1)}^{(3)}\right]$$

$$= \frac{1}{3}\left[\frac{3bc}{(1+3c)} - \frac{12bc^2}{(1+3c)(1+2c)} + \frac{6bc^3}{(1+3c)(1+2c)(1+c)}\right]$$

$$= \frac{bc}{(1+3c)}\left[\frac{(1+2c)(1+c) - 4c(1+c) + 2c^2}{(1+2c)(1+c)}\right]$$

$$\lambda_3 = \frac{bc(1-c)}{(1+3c)(1+2c)(1+c)} \tag{2.4.3}$$

$$\lambda_4 = \frac{1}{4}\left[\alpha_{(4)}^{(4)} - 3\alpha_{(3)}^{(4)} + 3\alpha_{(2)}^{(4)} - \alpha_{(1)}^{(4)}\right]$$

$$= \frac{1}{4}\left[\begin{array}{l} \dfrac{16bc}{(1+4c)} - 3\dfrac{(12bc^2)}{(1+3c)(1+4c)} + 3\dfrac{(24bc^3)}{(1+4c)(1+3c)(1+2c)} \\ -\dfrac{24bc^4}{(1+4c)(1+3c)(1+2c)(1+c)} \end{array}\right]$$

$$= \frac{bc}{(1+4c)}\left[\frac{(1+3c)(1+2c)(1+c) - 9c(1+c)(1+2c) + 18c^2(1+c) - 6c^3}{(1+c)(1+2c)(1+3c)}\right]$$

$$\lambda_4 = \frac{bc(2c-1)(c-1)}{(1+4c)(1+3c)(1+2c)(1+c)} \tag{2.4.4}$$

L-skewness is defined as $\tau_3 = \dfrac{\lambda_3}{\lambda_2}$

$$\tau_3 = \frac{1-c}{1+3c} \tag{2.4.5}$$

L-kurtosis is defined as $\tau_4 = \dfrac{\lambda_4}{\lambda_2}$

$$\tau_4 = \frac{(2c-1)(c-1)}{(4c+1)(3c+1)} \tag{2.4.6}$$

2.5 Comparison Among Skewness and Kurtosis Statistics: It is interesting to note that all three measures of skewness have same interpretation for symmetric Distribution. For Case II, Karl Pearson showed the poor performance in measuring the skewness but Blest's and L-skewness gave similar result for this case. And for Case III L-skewness is less than the Blest's measure and Karl Pearson. Figure 6 indicates that L-kurtosis gives less weight to extreme tail distribution as compared to Blest's and Pearson measure of kurtosis.

2.6 Trimmed L-Moments: TL-moments defined by Elamir and Seheult (2003) are generalization of L-moments that do not require the mean of underlying Distribution to exist. They are defined by

$$\lambda_r^{(s,t)} = \frac{1}{r}\sum_{j=0}^{r-1}(-1)^j \binom{r-1}{j} E\left(X_{r+s-j:r+s+t}\right) \qquad (2.6.1)$$

Here, *s* and *t* are positive integers. The case s = t = 0 yields the original L-moments defined by Hosking(1990). The term " trimmed" is appropriate because the definition of $\lambda_r^{(s,t)}$ does not involve the expectation of the *s* smallest or the *t* largest Order Statistics of sample of size r+s+t. The (r+s-j)th Order Statistics from Power Function Distribution defined as

$$E(X_{r+s-j:r+s+t}) = \frac{(r+s+t)!}{(r+s-j-1)!(t+j)!}\int_0^b x\left[\frac{x^c}{b^c}\right]^{r+s-j-1}\left[1-\frac{x^c}{b^c}\right]^{t+j}\left[\frac{cx^{c-1}}{b^c}\right]dx$$

$$put\ \ y = \frac{x^c}{b^c}\ in\, eq(33)\ then\, x = by^{1/c}$$

$$when\ \ x \to 0\, then\, y \to 0$$

$$when\ \ x \to b\, then\, y \to 1$$

$$= \frac{(r+s+t)!}{(r+s-j-1)!(t+j)!}\int_0^1 by^{1/c}\left[y\right]^{r+s-j-1}\left[1-y\right]^{t+j} dy$$

$$E(X_{r+s-j:r+s+t}) = \frac{b(r+s+t)!}{(r+s-j-1)!(t+j)!}\frac{Gamma(r+s+1/c-j)}{Gamma(r+s+t+1/c+1)}$$

Trimmed L-moments ratio are defined as $\tau_r^{(s,t)} = \dfrac{\lambda_r^{(s,t)}}{\lambda_2^{(s,t)}}$ are dimensionless measures of the shape of a Distribution. The close form of the first four TL-moments of Power Function Distribution with various choice of trimming i.e.

(0,1), (1,0) and (1,1) are obtained and then evaluated the TL-skewness and TL-kurtosis as r = 3,4

$$\tau_r^{(0,1)} = \frac{\lambda_r^{(0,1)}}{\lambda_2^{(0,1)}} \tag{2.6.2}$$

$$\tau_r^{(1,0)} = \frac{\lambda_r^{(1,0)}}{\lambda_2^{(1,0)}} \tag{2.6.3}$$

$$\tau_r^{(1,1)} = \frac{\lambda_r^{(1,1)}}{\lambda_2^{(1,1)}} \tag{2.6.4}$$

2.7 Comparison between L and TL Moment Ratios: Interesting to note that all measures have same interpretation for symmetric Distribution. For positively skewed Distribution, the size of trimming affect the amount of skewness as well as kurtosis. There is less skewness and peakednes for (1,0) as compared to other choices of trimming. For negatively skewed Distribution, all measures are relatively equal except for choice (1,0).

3. General Conclusion

Moments are used to provide parameter estimation, fitting of Distribution and empirical description of data. In this paper, we are focusing the objective of measuring numerical description of Distribution. For this, we evaluate Karl Pearson Moment Ratio for Power Function Distribution. Karl Pearson Moment Ratio has accurate interpretation for only symmetric Distribution. It does not provide true amount of skewness and peakedness for heavy tailed Distributions, measuring the true amount we evaluate the alternate measures i.e Blest's measures, L-moments and TL-moments. Comparing Blest's measure with Karl Pearson, founded that Blest's coincide with Karl Pearson when Distribution is symmetric but as the amount of skewness (positive or negative) increases Blest's picture of existing peakedness of a Distribution becomes clearer as it removes the effect of asymmetry. Pearson's measure in this regard is found to be over pronouncing the peakedness.

The gap between median and meson is another way measuring the degree of kurtosis for a negatively (positively) skewed Distribution. Positively skewed Distributions of Power Function Distribution covers higher areas between meson and median and so, they are more peaked than its negatively skewed Distributions.

Both Blest's and Karl Pearson measures are based on higher moments of a distribution. So these measures can't be evaluated for those Distributions like Cauchy and Inver Rayleigh Distribution whose higher moments do not exists. So how can we study the description of a such Distribution? Answer is obtained by evaluating another alternate measures the L-moments. Due to advantages of L-moments over the convention moments many Distribution are analyzed by these moments. Linear combination of Order Statistics of Power Function Distribution are used to compute the L-moments. Furthermore, these moments are less sensitive in the case of Outlier (Vogel and Fennenscy 1993).

Comparison with Karl Pearson and Blest's, L-Moment Ratio always give less weight to heavy tailed Distributions. L-moments cannot be defined for the Distributions whose mean do not exist. So we seek another alternative measures for such Distribution i.e TL-moments. TL-moments as a generalization of the L-moments and with more advantages over L-moments and conventional moments. TL-moments assign zero weight to extreme observations, they are more Robust than L-moments when used to estimate from a sample containing Outliers. Like L-moments, TL-moments also completely determine the distribution. Different choices for the amount of trimming give different amount of skewness and peakedness for Power Function Distribution. We evaluate the TL-skewness and TL-kurtosis for trimming choices (1,0), (0,1) and (1,1). And compare with the L-skewness and kurtosis. Our all measures are coincide for symmetric Distributions. For positively skewed Distribution the TL-skewness and kurtosis for choice (1,0) give less weight as compared to other choices. The gap among the TL-skewness, kurtosis statistics and L-skewness and L-kurtosis becomes smaller and smaller as value of shape parameter increases except for choice (1,0).

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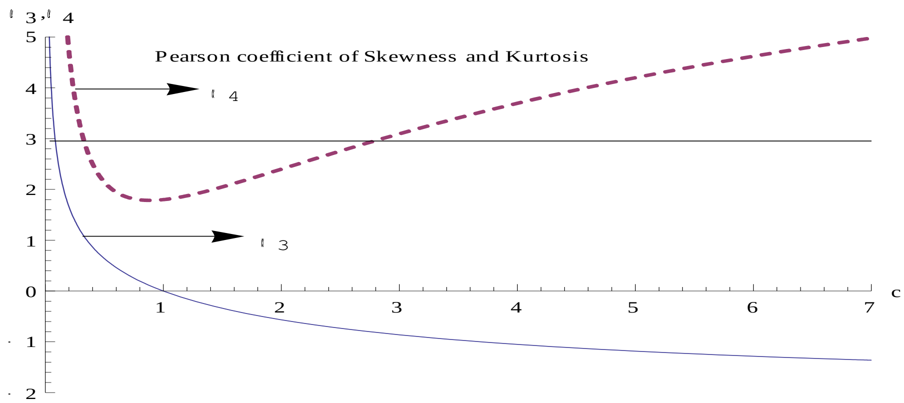

Figure 1: Skewness and kurtosis for Power Function Distribution

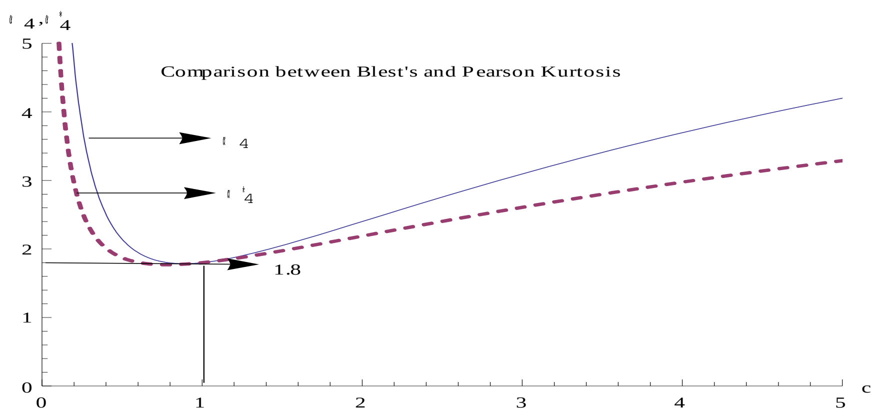

Figure 2: Kurtosis of Karl Pearson and Blest

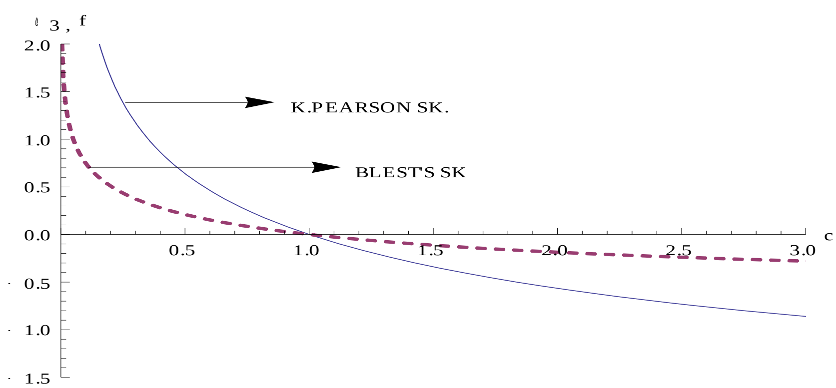

Figure 3: Skewness of Karl Pearson and Blest

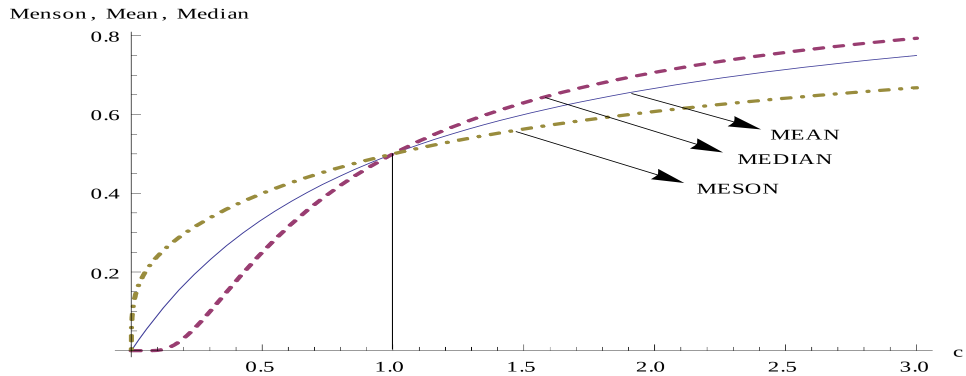

Figure 4: Relation between Meson, Mean and Median for b=1

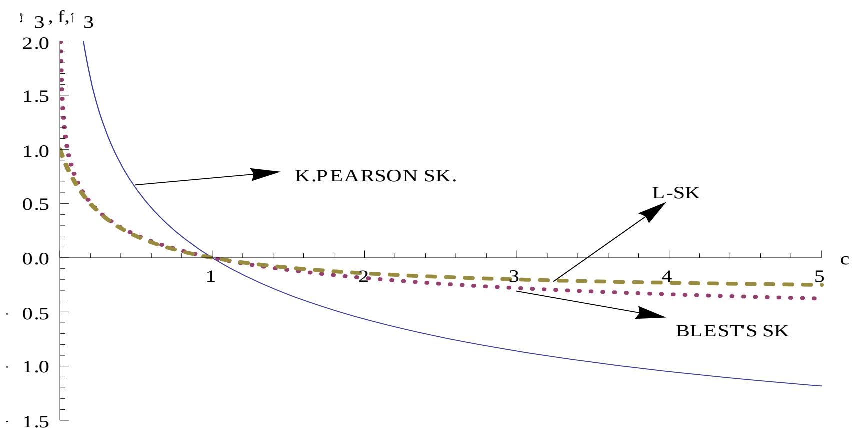

Figure 5: Skewness statistics.

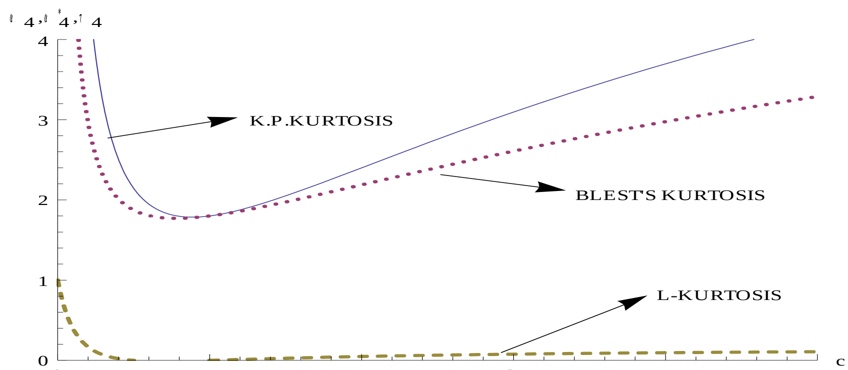

Figure 6: Relation among kurtosis statistics

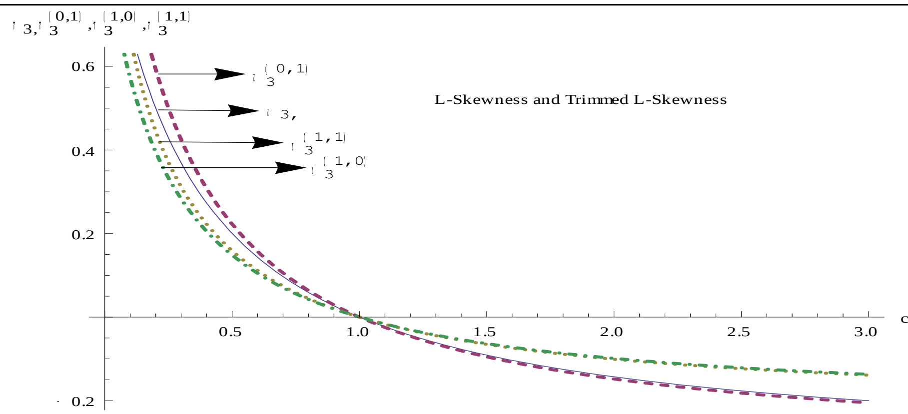

Figure 7: L-skewness and TL-skewness

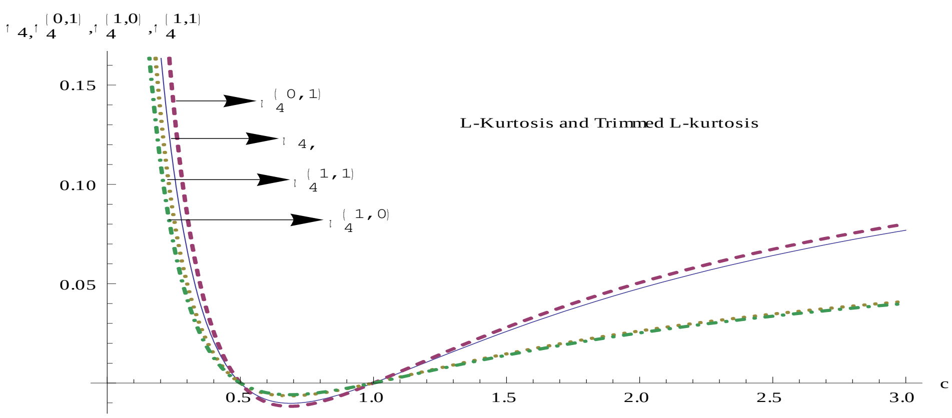

Figure 8: L-kurtosis and TL-kurtosis

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