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Some Infinite Series of Neighbor Designs in Circular Blocks of Small Sizes

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Abstract

In field experiments, blocks of small sizes have their own importance and if Neighbor Effects exist there then Neighbor Designs are useful to control these Effects. In literature, Neighbor Designs in blocks of size three are available almost for every case. Therefore, in this article, five infinite series of Neighbor Designs are developed in circular blocks of four units.

Keywords

Circular blocks, Neighbor designs, Neighbor effects, Equi-neighbored designs, Method of differences

1. Introduction

A design for v treatments in b circular blocks (each with k plots) in which each treatment is a Neighbor of every other treatment exactly λ^t times is said to be Neighbor Design. Neighbor Designs were initially used in serology. Rees (1967) presented a technique used in virus research, which requires the arrangement in circles of samples from a number of virus preparations such that over the whole set, a sample from each virus preparation appears next to a sample from every other virus preparation. Experiments in agriculture, horticulture and forestry often show Neighbor Effects (see Azais et al. 1993).

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Neighbor Designs ensure that treatment comparisons will be as little affected by competition/ Neighbor Effects as possible. In plants with an important root system, such as potatoes, varieties which germinate earlier will establish their roots and take nutrients from adjoining plots on both sides.

Ahmed and Akhtar (2008), Azais et al. (1993), Hwang (1973), Iqbal et al. (2009), Lawless (1971), and Rees (1967) developed several series of Neighbor Balanced Designs. Jacroux (1998) constructed Equi-neighbored Designs in linear blocks of size three for all *v*. Yasmin et al. (2013) presented a catalogue of Neighbor Designs in circular blocks of size 4. Akhtar et al. (2010) presented Neighbor Designs (ND) in circular blocks of size five. Ahmed and Akhtar (2011) constructed Neighbor Designs in circular blocks of size six. In this article, efforts are made to develop some infinite series to construct the Neighbor Designs in circular blocks of size four.

Iqbal et al. (2009) suggested the following model for the analysis of Neighbor Designs:

$$Y = X_0\mu + X_1\tau + X_2\beta + X_3\eta + \varepsilon$$

where,

Y is the b_k x 1 vector of response.
X_0 is the b_k x 1 vector of 1's.
X_1 is the b_k x v incidence matrix for treatment effects.
X_2 is the b_k x b incidence matrix for block effects.
X_3 is the b_k x v incidence matrix for neighbor (backward and forward) effects.
ε is the b_k x 1 vector of random errors.

For a Neighbor Balanced, all off-diagonal elements of $X_1'X_3$ must be equal, say λ^t.

2. Construction of Neighbor Design in Circular Blocks of our Units

The proposed designs are constructed using well known method of differences. By this method, a design is Neighbor Design if the differences between each number and its first order Neighbors (including the difference between first and last numbers) take all values from 1 to v-1 same number of time, say λ^t.

2.1 Neighbor Design for v = 8t+1: (Series 1) ND for v = 8t+1, where t integer can be generated in blocks of 4 units with $\lambda^t = 1$ by developing the following t initial blocks cyclically mod v:

$I_j = (0, v-(4j-3), 1, 4j) \bmod v$; $j = 1, 2, \dots, t$.

Proof: The combined set of forward and backward differences between Neighboring elements takes all the values from 1 to v-1 once. It is, therefore, Neighbor Design with $\lambda^t = 1$.

Example 1: Neighbor Design is generated for v = 33 and k = 4 by developing the following four initial blocks:

$I_1 = (0, 32, 1, 4)$, $I_2 = (0, 28, 1, 8)$,
$I_3 = (0, 24, 1, 12)$, $I_4 = (0, 20, 1, 16)$.

2.2 Neighbor Design for v = 4t+1: (Series 2) Neighbor Design for v = 4t+1, where t = 2s+1 and s integer, can be generated in blocks of 4 units with $\lambda^t = 2$ by developing the following t initial blocks cyclically mod v:

$I_j = (0, v-(4j-3), 1, 4j)$; $j = 1, 2, \dots, s$.
$I_{j+s} = (0, v-(4j-3), 1, 4j)$; $j = 1, 2, \dots, s$.
$I_t = (0, (v-3)/2, (v-2), (v-1)/2)$

***Example 2*:** Neighbor Design is generated for v = 21 and k = 4 by developing the following five initial blocks cyclically mod 21:

$I_1 = (0, 20, 1, 4)$, $I_2 = (0, 16, 1, 8)$, $I_3 = (0, 20, 1, 4)$,
$I_4 = (0, 16, 1, 8)$, $I_5 = (0, 9, 19, 10)$

2.3 Neighbor Design for v = 4t+3: (Series 3) Neighbor Design for v = 4t+3, where t (>1) integer can be generated in blocks of 4 units with $\lambda^t = 4$ by developing the following m initial blocks cyclically mod v:

$I_{j+1} = I_{j+w+2} = (0, v-(2j+1), 1, 2j+2)$; $j = 0, 1, 2, \dots, t-2$.
$I_{m-2} = I_{m-1} = (0, m-2, 2m, m-1)$,
$I_m = (0, m+3, 2, m)$, where $m = (v-1)/2$ and $\infty = v-1$.

***Example 3*:** Neighbor Design is generated for v = 27 and k = 4 by developing following 13 initial blocks cyclically mod 27:

$I_1 = (0,11,23,12)$, $I_2 = (0,11,23,12)$, $I_3 = (0,11,15,12)$,
$I_4 = (0,1,3,2)$, $I_5 = (0,3,7,4)$, $I_6 = (0,5,11,6)$,
$I_7 = (0,7,15,8)$, $I_8 = (0,9,19,10)$, $I_9 = (0,1,3,2)$,
$I_{10} = (0,3,7,4)$, $I_{11} = (0,5,11,6)$, $I_{12} = (0,7,15,8)$,

I_{13} = (0,9,19,10)

2.4 Neighbor Design for v = 4t: (Series 4) Neighbor Design for v = 4t+1, where t (>1) integer can be generated in blocks of 4 units with $\lambda^t = 2$ by developing the following t initial blocks cyclically mod (v-1):

$I_j = (0, 2j\text{-}1, 4j\text{-}1, 2j)$; $j = 1, 2, \ldots, t\text{-}1$.

$I_t = (0, m, 2m, \infty)$, where $\infty = v\text{-}1$ and $m = (v\text{-}1)/2$.

Example 4: Neighbor Design is generated for v = 12 and k = 4 by developing the following three initial blocks cyclically mod 11:

I_1 = (0,1,3,2), I_2 = (0,3,7,4), I_3 = (0,5,10,∞).

2.5 Neighbor Design for v = 4t+2: (Series 5) ND for v = 4t+2, where t (>1) integer, can be generated in blocks of 4 units with $\lambda^t = 4$ by developing the following v/2 initial blocks cyclically mod (v-1):

$I_1 = (0, m\text{-}1, 2m\text{-}1, \infty)$,

$I_2 = (0, m\text{-}1, 2m\text{-}1, \infty)$,

$I_3 = (0, m\text{-}1, 2m\text{-}1, m)$, where $\infty = v\text{-}1$ and $m = (v\text{-}2)/2$.

If m >3 then include the following initial blocks in the above three initial blocks:

$I_{j+3} = (0, 2j\text{-}1, 4j\text{-}1, 2j)$; $j = 1, 2, \ldots, \frac{1}{2}(m\text{-}2)$.

$I_{j+\frac{1}{2}(m+4)} = (0, 2j\text{-}1, 4j\text{-}1, 2j)$; $j = 1, 2, \ldots, \frac{1}{2}(m\text{-}2)$.

Example 5: Neighbor Design is generated for v = 26 and k = 4 by developing the following 13 initial blocks cyclically mod 13:

$I_1 = I_2 = I_3$ = (0,11,23,∞), $I_4 = I_9$ = (0,1,3,2),
$I_5 = I_{10}$ = (0,3,7,4), $I_6 = I_{11}$ = (0,5,11,6),
$I_7 = I_{12}$ = (0,7,15,8), $I_8 = I_{13}$ = (0,9,19,10).

3. Discussion

Yasmin et al. (2013) presented a catalogue of Neighbor Designs in circular blocks of size 4 but they did not develop the infinite series to generate the Neighbor Design in circular blocks of size four. This paper gives the Neighbor Design in circular blocks of size four for every number of treatments. Neighbor Balanced Designs are more useful to remove the Neighbor Effects in experiments because these designs are a tool for local control in biometrics, agriculture, horticulture and forestry. Therefore, our proposed designs may be used in (i) serology for the ouchterlony gel diffusion test, (ii) pesticide or fungicide experiments, (iii) agro-forestry experiments, and (iv) plant breeding experiments.

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