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# A Class of Modified Linear Regression Type Ratio Estimators for Estimation of Population Mean using Coefficient of Variation and Quartiles of an Auxiliary Variable

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## Abstract

The present paper deals with a class of Modified Linear Regression Type Ratio Estimators for estimation of population mean of the study variable when the population coefficient of variation and quartiles of the auxiliary variable are known. The Bias and the Mean Squared Error of the proposed Estimators are derived and are compared with that of Simple Random Sampling With-Out Replacement (SRSWOR) sample mean, the Ratio Estimator and the existing Modified Ratio Estimators. As a result we have derived the conditions for which the proposed Estimators perform better than the other existing Estimators. Further, the performance of the proposed Estimators with that of the existing Estimators are assessed for a natural population. From the numerical study, it is observed that the proposed Modified Ratio Estimators perform better than the existing Estimators.

## Keywords

First quartile, Inter-quartile range, Simple random sampling, Semi-quartile average, Semi-quartile range

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## 1. Introduction

Consider a finite population $U = \{U_1, U_2, \dots, U_N\}$ of $N$ distinct and identifiable units. Let, $Y$ be a study variable with value $Y_i$ measured on $U_i, i = 1, 2, 3, \dots, N$ giving a vector$Y = \{Y_1, Y_2, \dots, Y_N\}$. The problem is to estimate the population mean $\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ on the basis of a random sample selected from the population$U$. The SRSWOR sample mean is the simplest Estimator for estimating the population mean. If an auxiliary variable$X$, closely related to the study variable $Y$ is available then one can improve the performance of the Estimator of the study variable by using the known values of the population parameters of the auxiliary variable. That is, when the population parameters of the auxiliary variable $X$ such as population mean, coefficient of variation, coefficient of Kurtosis, coefficient of Skewness are known, a number of Estimators such as Ratio, Product and Linear Regression Estimators and their modifications are available in literature and are performing better than the SRSWORsample mean under certain conditions. Among these Estimators the Ratio Estimator and its modifications are widely attracted many researchers for the estimation of the mean of the study variable.

Recently, Al-Omari et al. (2009), Subramani and Kumarapandiyan (2012, 2013) and Yan and Tian (2010), have considered in modifying the Ratio Estimator for estimating population mean. In search of the best Estimators among the classes of Estimators, Hanif et al. (2009), Subramani (2013), Subramani et al. (2014) and Subramani and Kumarapandiyan (2014) and have suggested some Modified Ratio Estimators with known parameters of the auxiliary variable. The main objective of this study is to introduce some Modified Linear Regression Type Estimators which perform better than the Modified Estimators already existing in the literature. Further, it has been shown that the proposed Estimators perform better than existing Modified Ratio Estimators theoretically, analytically and numerically.

The structure of rest of this paper is as follows: In the next section the proposed Estimators are given in the two group format. The analytical results in this section demonstrate that the proposed Estimators are better than the Estimators suggested by various authors (refer Table 1 and Table 2). Section 3 shows the efficiency of the proposed Estimators while in Section 4 an empirical comparison of the proposed and various exiting Estimators has been presented.

Notations to be used in this paper has been given in Appendix A. Further, SRSWOR sample mean, the Ratio and the Linear Regression Estimator for estimating population mean have been discussed in Appendix B.

The Ratio Estimator and the Linear Regression Estimator are used for improving the precision of estimate of the population mean compared to SRSWOR when there is a positive correlation between X andY. It has been established; in general that the Linear Regression Estimator is more efficient than the Ratio Estimator whenever the Regression line of the study variable on the auxiliary variable does not passes through the neighborhood of the origin. Thus, the Linear Regression Estimator is more precise than the Ratio Estimator unless $\beta = R$ (See page 196 in Cochran (1977)). The Ratio Estimator and Modified Ratio Estimators can be applicable in the following practical situation.

- A national park is partitioned into N units.
  - Y = the number of animals in the $i^{th}$ unit
  - X = the size of the $i^{th}$ unit
- A certain city has N bookstores.
  - Y = the sales of a given book title at the $i^{th}$ bookstore
  - X = the size of the $i^{th}$ bookstore
- A forest that has N trees.
  - Y = the volume of the tree
  - X = the diameter of the tree

Kadilar and Cingi (2004) have suggested a class of Modified Ratio Estimators in the form of Linear Regression Estimator as given below:

$$\hat{\bar{Y}}_1 = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{\bar{X}}{\bar{x}}\right] \qquad (1.1)$$

Kadilar andCingi (2004) have shown that the Modified Linear Regression Type Ratio Estimator given in equation (1.1) perform well compare to the Ratio Estimator under certain conditions. Further Estimators are proposed by modifying the Estimator given in equation (1.1) by using the known coefficient of variation, coefficient of Kurtosis, coefficient of Skewness etc. together with some other Modified Ratio Estimators in Kadilar and Cingi (2004, 2006) and Yan and Tian (2010). The list of existing Modified Linear Regression Type Ratio Estimators together with the constant, the Bias and the Mean Squared Error are given in Table 1.

Al-Omari et al. (2009) have suggested Modified Ratio Estimators using the first quartile $Q_1$ and the third quartile $Q_3$ of the auxiliary variable. Subramani and Kumarapandiyan (2012) have suggested a class of Modified Ratio Estimators using the inter-quartile range $Q_r$, semi-quartile range $Q_d$ and semi-quartile average $Q_a$ of the auxiliary variable. The class of Modified Ratio Estimators together with the constant, the Bias and the Mean Squared Error are given in Table 2.

It is to be noted that "the existing Modified Ratio Estimators" means the list of Modified Ratio Estimators to be considered in this paper unless otherwise stated. It does not mean to the entire list of Modified Ratio Estimators available in the literature.

The list of Modified Linear Regression Type Ratio Estimators given in Table 1 uses the known values of the parameters like $\overline{X}$, $C_x$, $\beta_1$, $\beta_2$, $\rho$and their Linear combinations whereas the list given in Table 2 uses quartiles and their Linear combinations of an auxiliary variable like inter-quartile range, semi-quartile range and semi-quartile average to improve the Ratio Estimator in estimation of population mean. In this paper, an attempt is made to use the Linear combination of the known values of the coefficient of variation and quartiles of the auxiliary variable to introduce a class of Modified Linear Regression Type Ratio Estimators for estimating population mean in line with Kadilar and Cingi (2004).

## 2. Proposed Modified Linear Regression Type Ratio Estimators

In this section, a class of Modified Linear Regression Type Ratio Estimators is proposed for estimating the finite population mean and also derived the Bias and the Mean Squared Error of the proposed Estimators. The proposed Estimators are classified into two groups:Group 1 and Group 2.

***2.1 Proposed Estimators (Group 1):*** The proposed Estimators belong to Group 1 is represented as$\hat{\hat{Y}}_{p_j}$; $j = 1,2,3,4$ and 5 and it is defined as given below:

$$\hat{\hat{Y}}_{p_j} = [\bar{y} + b(\overline{X} - \bar{x})]\left[\frac{\overline{X} + Q_{(.)}}{\bar{x} + Q_{(.)}}\right]; j = 1,2,3,4 \text{ and } 5$$

where

$Q_{(.)} = Q_1, Q_3, Q_r, Q_d$ and $Q_a$

***2.2 Proposed Estimators (Group 2):*** The proposed Estimators belong to Group 2 is represented as $\hat{\hat{Y}}_{p_j}$ and is defined as given below:

$$\hat{\hat{Y}}_{p_j} = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{C_x\bar{X} + Q_{(.)}}{C_x\bar{x} + Q_{(.)}}\right]; j = 6,7,8,9 \text{ and } 10$$

where

$Q_{(.)} = Q_1, Q_3, Q_r, Q_d$ and $Q_a$

The Bias and Mean Squared Error of the proposed Estimators have been derived (see Appendix C). The proposed Estimators (Group 1 and Group 2) are combined into a single Class (say Class 3) and it is defined as $\hat{\hat{Y}}_{p_j}; j = 1,2,\ldots,10$ for estimating the population mean $\bar{Y}$ together with the constant, the Bias and the Mean Squared Error are presented in Table 3.

## 3. Efficiency of the proposed Estimators

The Variance of SRSWOR sample mean $\bar{y}_{srs}$ is given below:

$$V(\bar{y}_{srs}) = \frac{(1-f)}{n} S_y^2 \quad (3.1)$$

The Mean Squared Error of theRatio Estimator $\hat{\hat{Y}}_R$ to the first degree of approximation is given below:

$$MSE\left(\hat{\hat{Y}}_R\right) = \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + C_x^2 - 2\rho C_x C_y\right) \quad (3.2)$$

The Modified Ratio Estimators given in Table 1, Table 2 and the proposed Modified Ratio Estimators given in Table 3 are represented in three classes as given below:

***3.1 Class 1:*** The Mean Squared Error and the constant of the Modified Ratio Estimators $\hat{\hat{Y}}_1$ to $\hat{\hat{Y}}_{12}$ listed in the Table 1 are represented in a single class (say, Class 1), which will be very much useful for comparing with that of proposed Modified Ratio Estimators and are given below:

$$MSE\left(\hat{\hat{Y}}_i\right) = \frac{(1-f)}{n}\left(R_i^2 S_x^2 + S_y^2(1-\rho^2)\right); i = 1,2,3,\ldots,12 \quad (3.1.1)$$

***3.2 Class 2:*** The Mean Squared Error and the constant of the Modified Ratio Estimators $\hat{\hat{Y}}_{13}$ to $\hat{\hat{Y}}_{17}$ listed in Table 2 are represented in a single class (say, Class

2), which will be very much useful for comparing with that of proposed Modified Ratio Estimators and are given below:

$$\mathrm{MSE}\left(\hat{\hat{\bar{Y}}}_i\right) = \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_i^2 C_x^2 - 2\rho\theta_i C_x C_y\right); i = 13,14,15,16 \text{ and } 17 \quad (3.2.1)$$

***3.3 Class 3:*** The Mean Squared Error of the 10 proposed Modified Ratio Estimators $\hat{\hat{\bar{Y}}}_{p_1}$ to $\hat{\hat{\bar{Y}}}_{p_{10}}$ listed in the Table 3 are represented in a single class (say, Class 3), which will be very much useful for comparing with that of existing Modified Ratio Estimators (given in Class1 and Class 2) are given below:

$$\mathrm{MSE}\left(\hat{\hat{\bar{Y}}}_{p_j}\right) = \frac{(1-f)}{n}\left(R_{p_j}^2 S_x^2 + S_y^2(1-\rho^2)\right); j = 1,2,3,\ldots,10 \quad (3.3.1)$$

From the expressions given inequation (3.1), (3.2), (3.1.1), (3.2.1) and (3.3.1) we have derived the conditions (see AppendixD) for which the proposed Estimator $\hat{\hat{\bar{Y}}}_{p_j}$ are more efficient than the Simple Random Sampling With-Out Replacement (SRSWOR) sample mean $\bar{y}_{srs}$ and are given below:

$$\mathrm{MSE}\left(\hat{\hat{\bar{Y}}}_{p_j}\right) < V(\bar{y}_{srs}) \text{ if } R_{p_j} \leq \rho\frac{S_y}{S_x}; \ j = 1,2,3,\ldots,10 \quad (3.3.2)$$

From the expressions given in equation (3.2), (3.1.1), (3.2.1) and (3.3.1) we have derived the conditions (see Appendix E) for which the proposed Estimators $\hat{\hat{\bar{Y}}}_{p_j}$ are more efficient than the Ratio Estimator and are given below:

$$\mathrm{MSE}\left(\hat{\hat{\bar{Y}}}_{p_j}\right) \leq \mathrm{MSE}\left(\hat{\hat{\bar{Y}}}_R\right) \text{ if } \bar{Y}\left(\frac{C_x - \rho C_y}{S_x}\right) \leq R_{p_j} \leq \bar{Y}\left(\frac{\rho C_y - C_x}{S_x}\right)$$

or

$$\bar{Y}\left(\frac{\rho C_y - C_x}{S_x}\right) \leq R_{p_j} \leq \bar{Y}\left(\frac{C_x - \rho C_y}{S_x}\right) \quad (3.3.3)$$

From the expressions given in equation(3.1.1), (3.2.1) and (3.3.1) we have derived the conditions (see Appendix F) for which the proposed Estimators $\hat{\hat{\bar{Y}}}_{p_j}; j = 1,2,3,\ldots 10$ are more efficient than the existing Modified Ratio Estimators given in Class 1, $\hat{\hat{\bar{Y}}}_i; i = 1,2,3,\ldots,12$ and are given below:

$$\mathrm{MSE}\left(\hat{\hat{\bar{Y}}}_{p_j}\right) < \mathrm{MSE}\left(\hat{\hat{\bar{Y}}}_i\right) \text{ if } R_{p_j} < R_i; i = 1,2,3,\ldots,12 : j = 1,2,3,\ldots,10 \quad (3.3.3)$$

From the expressions given in equation (3.2.1) and (3.3.1) we have derived the conditions (see Appendix G) for which the proposed Estimators $\hat{\hat{\bar{Y}}}_{p_j}; j =$

$1,2,3,\ldots 10$ are more efficient than the existing Modified Ratio Estimators given in Class 2, $\hat{\bar{Y}}_i$ ; $i = 13, 14, 15, 16$ and $17$ and are given below:

$\text{MSE}\left(\hat{\bar{Y}}_{p_j}\right) \leq \text{MSE}\left(\hat{\bar{Y}}_i\right)$ if $\bar{Y}\left(\frac{\theta_i C_x - \rho C_y}{S_x}\right) \leq R_{p_j} \leq \bar{Y}\left(\frac{\rho C_y - \theta_i C_x}{S_x}\right)$ or

$$\bar{Y}\left(\frac{\rho C_y - \theta_i C_x}{S_x}\right) \leq R_{p_j} \leq \bar{Y}\left(\frac{\theta_i C_x - \rho C_y}{S_x}\right) \qquad (3.3.4)$$

## 4. Empirical study

The performance of the proposed Modified Linear Regression Type Ratio Estimators are assessed with that of the SRSWOR sample mean, the Ratio Estimator and the existing modified ratio Estimators for a natural population considered by Kadilar and Cingi (2004). The population consists of data on apple production amount (as study variable) and number of apple trees (as auxiliary variable) in 106 villages of Aegean Region in 1999. The parameters computed from the above population are given below:

| | | | |
|---|---|---|---|
| $N = 106$ | $n = 40$ | $\bar{Y} = 2212.5943$ | $\bar{X} = 27421.6981$ |
| $S_y = 11496.9102$ | $C_y = 5.1961$ | $S_x = 57188.9320$ | $C_x = 2.0855$ |
| $\rho = 0.8560$ | $\beta_{2(x)} = 34.5723$ | $\beta_{1(x)} = 5.1238$ | $b = 0.1721$ |
| $Q_1 = 2387.5$ | $Q_3 = 26700$ | $Q_r = 24312.5$ | $Q_d = 12156.25$ |
| $Q_a = 14543.75$ | | | |

The constant, the Bias and the Mean Squared Error of the existing and proposed Modified Ratio Estimators are given in Table 4. From the values of Table 4, it is observed that Mean Squared Error of the proposed Modified Ratio Estimators are less than the variance of SRSWOR sample mean, Mean Squared Error of the Ratio Estimator and all the 17 existing Modified Ratio Estimators. The Percent Relative Efficiencies (PRE's) of the proposed Estimators with respect to the existing Estimators computed by the formula as given below:

$\text{PRE}\left(\hat{\bar{Y}}_{p_j}\right) = \frac{\text{MSE}\,(.)}{\text{MSE}\left(\hat{\bar{Y}}_{p_j}\right)} * 100; j = 1,2,3,\ldots,10$ and are presented in Table 5.

From the values of Table 5, it is observed that the PRE's of the proposed Estimators with respect to the existing Estimators are ranging from 102.93 to 324.02. This shows that proposed Estimators are more efficient and perform better than the SRSWOR sample mean, the Ratio Estimator and the existing Modified Ratio Estimators. It has been shown that all the ten proposed Estimators perform better than the existing Estimators. We want to know that among the 10 proposed Estimators which proposed Estimator is more efficient. To find that, PRE's of each proposed Estimator with respect to remaining 9 proposed Estimators respectively are computed and presented in Table 6.

From the values of Table 6, it has been observed that PRE's of proposed Estimator$\hat{\hat{\mathrm{Y}}}_{p_2}$ with respect to remaining nine proposed Estimators is more than 100 and is ranging from 101.27 to 130.77. This shows that the proposed Estimator$\hat{\hat{\mathrm{Y}}}_{p_2}$ is more efficient among the all 10 proposed Estimators.

## 5. Conclusion

In this paper, we have proposed a class of Modified Ratio Estimators using the Linear combination of the coefficient of variation, first quartile, third quartile, inter-quartile range, semi-quartile range and semi-quartile average of the auxiliary variable. The Bias and Mean Squared Error of the proposed Estimators are obtained and compared with that of the existing Estimators. Further, we have derived the conditions for which the proposed Estimators are more efficient than the SRSWOR sample mean, the Ratio Estimator and the existing Modified Ratio Estimators. We have also assessed the performance of the proposed Estimators with that of the existing Estimators for a natural population. From the numerical study it is observed that the Mean Squared Error of the proposed Estimators are less than variance of the SRSWOR sample mean and theMean Squared Error of the Ratio Estimator and the existing Modified Ratio Estimators. Hence, we strongly recommend that the proposed Modified Ratio Estimators may be preferred over the existing Estimators for the use of practical applications.

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**Table 1:** Existing Modified Ratio Estimators (Class 1) together with the constant, Bias and Mean Squared Error

| **Estimators** | **Constant $R_i$** | **Bias-B(.)** | **MSE(.)** |
|---|---|---|---|
| $\hat{\bar{Y}}_1 = [\bar{y} + b(\bar{X} - \bar{x})]\frac{\bar{X}}{\bar{x}}$ Kadilar and Cingi (2004) | $R_1 = \frac{\bar{Y}}{\bar{X}}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_1^2$ | $\frac{(1-f)}{n}\left(R_1^2S_x^2 + S_y^2(1-\rho^2)\right)$ |
| $\hat{\bar{Y}}_2 = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{\bar{X} + C_x}{\bar{x} + C_x}\right]$ Kadilar and Cingi (2004) | $R_2 = \frac{\bar{Y}}{\bar{X} + C_x}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_2^2$ | $\frac{(1-f)}{n}\left(R_2^2S_x^2 + S_y^2(1-\rho^2)\right)$ |
| $\hat{\bar{Y}}_3 = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{\bar{X} + \beta_2}{\bar{x} + \beta_2}\right]$ Kadilar and Cingi (2004) | $R_3 = \frac{\bar{Y}}{\bar{X} + \beta_2}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_3^2$ | $\frac{(1-f)}{n}\left(R_3^2S_x^2 + S_y^2(1-\rho^2)\right)$ |
| $\hat{\bar{Y}}_4 = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{\beta_2\bar{X} + C_x}{\beta_2\bar{x} + C_x}\right]$ Kadilar and Cingi (2004) | $R_4 = \frac{\beta_2\bar{Y}}{\beta_2\bar{X} + C_x}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_4^2$ | $\frac{(1-f)}{n}\left(R_4^2S_x^2 + S_y^2(1-\rho^2)\right)$ |
| $\hat{\bar{Y}}_5 = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{C_x\bar{X} + \beta_2}{C_x\bar{x} + \beta_2}\right]$ Kadilar and Cingi (2004) | $R_5 = \frac{C_x\bar{Y}}{C_x\bar{X} + \beta_2}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_5^2$ | $\frac{(1-f)}{n}\left(R_5^2S_x^2 + S_y^2(1-\rho^2)\right)$ |
| $\hat{\bar{Y}}_6 = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{\bar{X} + \beta_1}{\bar{x} + \beta_1}\right]$ Yan and Tian (2010) | $R_6 = \frac{\bar{Y}}{\bar{X} + \beta_1}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_6^2$ | $\frac{(1-f)}{n}\left(R_6^2S_x^2 + S_y^2(1-\rho^2)\right)$ |
| $\hat{\bar{Y}}_7 = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{\beta_1\bar{X} + \beta_2}{\beta_1\bar{x} + \beta_2}\right]$ Yan and Tian (2010) | $R_7 = \frac{\beta_1\bar{Y}}{\beta_1\bar{X} + \beta_2}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_7^2$ | $\frac{(1-f)}{n}\left(R_7^2S_x^2 + S_y^2(1-\rho^2)\right)$ |
| $\hat{\bar{Y}}_8 = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{\bar{X} + \rho}{\bar{x} + \rho}\right]$ Kadilar and Cingi (2006) | $R_8 = \frac{\bar{Y}}{\bar{X} + \rho}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_8^2$ | $\frac{(1-f)}{n}\left(R_8^2S_x^2 + S_y^2(1-\rho^2)\right)$ |
| $\hat{\bar{Y}}_9 = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{C_x\bar{X} + \rho}{C_x\bar{x} + \rho}\right]$ Kadilar and Cingi (2006) | $R_9 = \frac{C_x\bar{Y}}{C_x\bar{X} + \rho}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_9^2$ | $\frac{(1-f)}{n}\left(R_9^2S_x^2 + S_y^2(1-\rho^2)\right)$ |
| $\hat{\bar{Y}}_{10} = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{\rho\bar{X} + C_x}{\rho\bar{x} + C_x}\right]$ Kadilar and Cingi (2006) | $R_{10} = \frac{\rho\bar{Y}}{\rho\bar{X} + C_x}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{10}^2$ | $\frac{(1-f)}{n}\left(R_{10}^2S_x^2 + S_y^2(1-\rho^2)\right)$ |
| $\hat{\bar{Y}}_{11} = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{\beta_2\bar{X} + \rho}{\beta_2\bar{x} + \rho}\right]$ Kadilar and Cingi (2006) | $R_{11} = \frac{\beta_2\bar{Y}}{\beta_2\bar{X} + \rho}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{11}^2$ | $\frac{(1-f)}{n}\left(R_{11}^2S_x^2 + S_y^2(1-\rho^2)\right)$ |

| Estimators | Constant $R_i$ | Bias-B(.) | MSE(.) |
|---|---|---|---|
| $\hat{\bar{Y}}_{12} = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{\rho\bar{X} + \beta_2}{\rho\bar{x} + \beta_2}\right]$ Kadilar and Cingi (2006) | $R_{12} = \frac{\rho\bar{Y}}{\rho\bar{X} + \beta_2}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{12}^2$ | $\frac{(1-f)}{n}\left(R_{12}^2 S_x^2 + S_y^2(1-\rho^2)\right)$ |

**Table 2:** Existing Modified Ratio Estimators (Class 2) together with the constant, the Bias and the Mean Squared Errors

| Estimators | Constant $\theta_i$ | Bias - B(.) | MSE(.) |
|---|---|---|---|
| $\hat{\bar{Y}}_{13} = \bar{y}\left[\frac{\bar{X} + Q_1}{\bar{x} + Q_1}\right]$ | $\theta_{13} = \frac{\bar{X}}{\bar{X} + Q_1}$ | $\frac{(1-f)}{n}\bar{Y}(\theta_{13}^2 C_x^2 - \theta_{13} C_x C_y \rho)$ | $\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_{13}^2 C_x^2 - 2\theta_{13} C_x C_y \rho)$ |
| $\hat{\bar{Y}}_{14} = \bar{y}\left[\frac{\bar{X} + Q_3}{\bar{x} + Q_3}\right]$ | $\theta_{14} = \frac{\bar{X}}{\bar{X} + Q_3}$ | $\frac{(1-f)}{n}\bar{Y}(\theta_{14}^2 C_x^2 - \theta_{14} C_x C_y \rho)$ | $\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_{14}^2 C_x^2 - 2\theta_{14} C_x C_y \rho)$ |
| $\hat{\bar{Y}}_{15} = \bar{y}\left[\frac{\bar{X} + Q_r}{\bar{x} + Q_r}\right]$ | $\theta_{15} = \frac{\bar{X}}{\bar{X} + Q_r}$ | $\frac{(1-f)}{n}\bar{Y}(\theta_{15}^2 C_x^2 - \theta_{15} C_x C_y \rho)$ | $\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_{15}^2 C_x^2 - 2\theta_{15} C_x C_y \rho)$ |
| $\hat{\bar{Y}}_{16} = \bar{y}\left[\frac{\bar{X} + Q_d}{\bar{x} + Q_d}\right]$ | $\theta_{16} = \frac{\bar{X}}{\bar{X} + Q_d}$ | $\frac{(1-f)}{n}\bar{Y}(\theta_{16}^2 C_x^2 - \theta_{16} C_x C_y \rho)$ | $\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_{16}^2 C_x^2 - 2\theta_{16} C_x C_y \rho)$ |
| $\hat{\bar{Y}}_{17} = \bar{y}\left[\frac{\bar{X} + Q_a}{\bar{x} + Q_a}\right]$ | $\theta_{17} = \frac{\bar{X}}{\bar{X} + Q_a}$ | $\frac{(1-f)}{n}\bar{Y}(\theta_{17}^2 C_x^2 - \theta_{17} C_x C_y \rho)$ | $\frac{(1-f)}{n}\bar{Y}^2(C_y^2 + \theta_{17}^2 C_x^2 - 2\theta_{17} C_x C_y \rho)$ |

**Table 3:** Proposed Modified Ratio Estimators with the constant, Bias and Mean Squared Error

| Estimators | Constant $R_{p_j}$ | Bias-B(.) | MSE(.) |
|---|---|---|---|
| $\hat{\bar{Y}}_{p_1} = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{\bar{X} + Q_1}{\bar{x} + Q_1}\right]$ | $R_{p_1} = \frac{\bar{Y}}{\bar{X} + Q_1}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p_1}^2$ | $\frac{(1-f)}{n}\left(R_{p_1}^2 S_x^2 + S_y^2(1-\rho^2)\right)$ |
| $\hat{\bar{Y}}_{p_2} = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{\bar{X} + Q_3}{\bar{x} + Q_3}\right]$ | $R_{p_2} = \frac{\bar{Y}}{\bar{X} + Q_3}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p_2}^2$ | $\frac{(1-f)}{n}\left(R_{p_2}^2 S_x^2 + S_y^2(1-\rho^2)\right)$ |
| $\hat{\bar{Y}}_{p_3} = [\bar{y} + b(\bar{X} - \bar{x})]\left[\frac{\bar{X} + Q_r}{\bar{x} + Q_r}\right]$ | $R_{p_3} = \frac{\bar{Y}}{\bar{X} + Q_r}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p_3}^2$ | $\frac{(1-f)}{n}\left(R_{p_3}^2 S_x^2 + S_y^2(1-\rho^2)\right)$ |

| Estimators | Constant $R_{p_j}$ | Bias-B(.) | MSE(.) |
|---|---|---|---|
| $\hat{\bar{Y}}_{p_4} = [\bar{y} + b(\bar{X}-\bar{x})]\left[\frac{\bar{X}+Q_d}{\bar{x}+Q_d}\right]$ | $R_{p_4} = \frac{\bar{Y}}{\bar{X}+Q_d}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p_4}^2$ | $\frac{(1-f)}{n}\left(R_{p_4}^2 S_x^2 + S_y^2(1-\rho^2)\right)$ |
| $\hat{\bar{Y}}_{p_5} = [\bar{y} + b(\bar{X}-\bar{x})]\left[\frac{\bar{X}+Q_a}{\bar{x}+Q_a}\right]$ | $R_{p_5} = \frac{\bar{Y}}{\bar{X}+Q_a}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p_5}^2$ | $\frac{(1-f)}{n}\left(R_{p_5}^2 S_x^2 + S_y^2(1-\rho^2)\right)$ |
| $\hat{\bar{Y}}_{p_6} = [\bar{y} + b(\bar{X}-\bar{x})]\left[\frac{C_x\bar{X}+Q_1}{C_x\bar{x}+Q_1}\right]$ | $R_{p_6} = \frac{C_x\bar{Y}}{C_x\bar{X}+Q_1}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p_6}^2$ | $\frac{(1-f)}{n}\left(R_{p_6}^2 S_x^2 + S_y^2(1-\rho^2)\right)$ |
| $\hat{\bar{Y}}_{p_7} = [\bar{y} + b(\bar{X}-\bar{x})]\left[\frac{C_x\bar{X}+Q_3}{C_x\bar{x}+Q_3}\right]$ | $R_{p_7} = \frac{C_x\bar{Y}}{C_x\bar{X}+Q_3}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p_7}^2$ | $\frac{(1-f)}{n}\left(R_{p_7}^2 S_x^2 + S_y^2(1-\rho^2)\right)$ |
| $\hat{\bar{Y}}_{p_8} = [\bar{y} + b(\bar{X}-\bar{x})]\left[\frac{C_x\bar{X}+Q_r}{C_x\bar{x}+Q_r}\right]$ | $R_{p_8} = \frac{C_x\bar{Y}}{C_x\bar{X}+Q_r}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p_8}^2$ | $\frac{(1-f)}{n}\left(R_{p_8}^2 S_x^2 + S_y^2(1-\rho^2)\right)$ |
| $\hat{\bar{Y}}_{p_9} = [\bar{y} + b(\bar{X}-\bar{x})]\left[\frac{C_x\bar{X}+Q_d}{C_x\bar{x}+Q_d}\right]$ | $R_{p_9} = \frac{C_x\bar{Y}}{C_x\bar{X}+Q_d}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p_9}^2$ | $\frac{(1-f)}{n}\left(R_{p_9}^2 S_x^2 + S_y^2(1-\rho^2)\right)$ |
| $\hat{\bar{Y}}_{p_{10}} = [\bar{y} + b(\bar{X}-\bar{x})]\left[\frac{C_x\bar{X}+Q_a}{C_x\bar{x}+Q_a}\right]$ | $R_{p_{10}} = \frac{C_x\bar{Y}}{C_x\bar{X}+Q_a}$ | $\frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}}R_{p_{10}}^2$ | $\frac{(1-f)}{n}\left(R_{p_{10}}^2 S_x^2 + S_y^2(1-\rho^2)\right)$ |

**Table 4:** Constant, Bias and Mean Squared Error of the existing and proposed Estimators

| Estimators | | Constant | B(.) | MSE(.) |
|---|---|---|---|---|
| **SRSWOR Sample mean** | $\bar{y}_{srs}$ | - | - | 2057484.9094 |
| **Ratio Estimator** | $\hat{\bar{Y}}_R$ | 0.0807 | 169.6823 | 975171.1057 |
| **Linear Regression Estimator** | $\hat{\bar{Y}}_{lr}$ | - | - | 549896.3301 |
| **Existing Modified Ratio Estimators (Class 1)** | $\hat{\bar{Y}}_1$ | 0.0807 | 149.8012 | 881345.5029 |
| | $\hat{\bar{Y}}_2$ | 0.0807 | 149.7784 | 881295.0933 |
| | $\hat{\bar{Y}}_3$ | 0.0806 | 149.4241 | 880511.3222 |
| | $\hat{\bar{Y}}_4$ | 0.0807 | 149.8005 | 881344.0447 |
| | $\hat{\bar{Y}}_5$ | 0.0806 | 149.6202 | 880945.1187 |
| | $\hat{\bar{Y}}_6$ | 0.0807 | 149.7452 | 881221.6738 |
| | $\hat{\bar{Y}}_7$ | 0.0807 | 149.7275 | 881182.4501 |
| | $\hat{\bar{Y}}_8$ | 0.0807 | 149.7918 | 881324.8108 |

| | | | | |
|---|---|---|---|---|
| | $\hat{\bar{Y}}_9$ | 0.0807 | 149.7967 | 881335.5808 |
| | $\hat{\bar{Y}}_{10}$ | 0.0807 | 149.7745 | 881286.6144 |
| | $\hat{\bar{Y}}_{11}$ | 0.0807 | 149.8009 | 881344.9044 |
| | $\hat{\bar{Y}}_{12}$ | 0.0806 | 149.3609 | 880371.3022 |
| **Existing Modified Ratio Estimators (Class 2)** | $\hat{\bar{Y}}_{13}$ | 0.9199 | 167.1286 | 1037436.9350 |
| | $\hat{\bar{Y}}_{14}$ | 0.5067 | 123.4148 | 1426267.1098 |
| | $\hat{\bar{Y}}_{15}$ | 0.5300 | 127.2538 | 1401244.5947 |
| | $\hat{\bar{Y}}_{16}$ | 0.6929 | 149.4426 | 1237068.0672 |
| | $\hat{\bar{Y}}_{17}$ | 0.6534 | 144.7988 | 1275206.4165 |
| **Proposed Modified Ratio Estimators (Class 3)** | $\hat{\bar{Y}}_{p_1}$ | 0.0742 | 126.7661 | 830378.3611 |
| | $\hat{\bar{Y}}_{p_2}$ | 0.0409 | 38.4557 | 634983.2497 |
| | $\hat{\bar{Y}}_{p_3}$ | 0.0428 | 42.0870 | 643017.8780 |
| | $\hat{\bar{Y}}_{p_4}$ | 0.0559 | 71.9113 | 709006.9180 |
| | $\hat{\bar{Y}}_{p_5}$ | 0.0527 | 63.9617 | 691417.6597 |
| | $\hat{\bar{Y}}_{p_6}$ | 0.0775 | 138.0351 | 855312.0137 |
| | $\hat{\bar{Y}}_{p_7}$ | 0.0550 | 69.6186 | 703933.9646 |
| | $\hat{\bar{Y}}_{p_8}$ | 0.0566 | 73.7572 | 713091.0075 |
| | $\hat{\bar{Y}}_{p_9}$ | 0.0665 | 101.8835 | 775323.2185 |
| | $\hat{\bar{Y}}_{p_{10}}$ | 0.0643 | 95.2143 | 760566.8439 |

**Table 5:** PREs of the proposed Estimators

| **Estimators** | $\hat{\bar{Y}}_{p_1}$ | $\hat{\bar{Y}}_{p_2}$ | $\hat{\bar{Y}}_{p_3}$ | $\hat{\bar{Y}}_{p_4}$ | $\hat{\bar{Y}}_{p_5}$ | $\hat{\bar{Y}}_{p_6}$ | $\hat{\bar{Y}}_{p_7}$ | $\hat{\bar{Y}}_{p_8}$ | $\hat{\bar{Y}}_{p_9}$ | $\hat{\bar{Y}}_{p_{10}}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $\bar{y}_{srs}$ | 253.83 | 247.78 | 324.02 | 319.97 | 290.19 | 297.57 | 240.55 | 292.28 | 288.53 | 265.37 |
| $\hat{\bar{Y}}_R$ | 120.31 | 117.44 | 153.57 | 151.66 | 137.54 | 141.04 | 114.01 | 138.53 | 136.75 | 125.78 |
| $\hat{\bar{Y}}_1$ | 106.14 | 138.80 | 137.06 | 124.31 | 127.47 | 103.04 | 125.20 | 123.60 | 113.67 | 115.88 |
| $\hat{\bar{Y}}_2$ | 106.13 | 138.79 | 137.06 | 124.30 | 127.46 | 103.04 | 125.20 | 123.59 | 113.67 | 115.87 |
| $\hat{\bar{Y}}_3$ | 106.04 | 138.67 | 136.93 | 124.19 | 127.35 | 102.95 | 125.08 | 123.48 | 113.57 | 115.77 |
| $\hat{\bar{Y}}_4$ | 106.14 | 138.80 | 137.06 | 124.31 | 127.47 | 103.04 | 125.20 | 123.59 | 113.67 | 115.88 |
| $\hat{\bar{Y}}_5$ | 106.09 | 138.74 | 137.00 | 124.25 | 127.41 | 103.00 | 125.15 | 123.54 | 113.62 | 115.83 |
| $\hat{\bar{Y}}_6$ | 106.12 | 138.78 | 137.04 | 124.29 | 127.45 | 103.03 | 125.19 | 123.58 | 113.66 | 115.86 |
| $\hat{\bar{Y}}_7$ | 106.12 | 138.77 | 137.04 | 124.28 | 127.45 | 103.02 | 125.18 | 123.57 | 113.65 | 115.86 |
| $\hat{\bar{Y}}_8$ | 106.14 | 138.79 | 137.06 | 124.30 | 127.47 | 103.04 | 125.20 | 123.59 | 113.67 | 115.88 |
| $\hat{\bar{Y}}_9$ | 106.14 | 138.80 | 137.06 | 124.31 | 127.47 | 103.04 | 125.20 | 123.59 | 113.67 | 115.88 |
| $\hat{\bar{Y}}_{10}$ | 106.13 | 138.79 | 137.05 | 124.30 | 127.46 | 103.04 | 125.19 | 123.59 | 113.67 | 115.87 |
| $\hat{\bar{Y}}_{11}$ | 106.14 | 138.80 | 137.06 | 124.31 | 127.47 | 103.04 | 125.20 | 123.60 | 113.67 | 115.88 |
| $\hat{\bar{Y}}_{12}$ | 106.02 | 138.64 | 136.91 | 124.17 | 127.33 | 102.93 | 125.06 | 123.46 | 113.55 | 115.75 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\hat{\bar{Y}}_{13}$ | 124.94 | 163.38 | 161.34 | 146.32 | 150.04 | 121.29 | 147.38 | 145.48 | 133.81 | 136.40 |
| $\hat{\bar{Y}}_{14}$ | 171.76 | 224.61 | 221.81 | 201.16 | 206.28 | 166.75 | 202.61 | 200.01 | 183.96 | 187.53 |
| $\hat{\bar{Y}}_{15}$ | 168.75 | 220.67 | 217.92 | 197.63 | 202.66 | 163.83 | 199.06 | 196.50 | 180.73 | 184.24 |
| $\hat{\bar{Y}}_{16}$ | 148.98 | 194.82 | 192.38 | 174.48 | 178.92 | 144.63 | 175.74 | 173.48 | 159.56 | 162.65 |
| $\hat{\bar{Y}}_{17}$ | 153.57 | 200.83 | 198.32 | 179.86 | 184.43 | 149.09 | 181.15 | 178.83 | 164.47 | 167.67 |

**Table 6:** PREs among the proposed Estimators

| **Estimators** | $\hat{\bar{Y}}_{p_1}$ | $\hat{\bar{Y}}_{p_2}$ | $\hat{\bar{Y}}_{p_3}$ | $\hat{\bar{Y}}_{p_4}$ | $\hat{\bar{Y}}_{p_5}$ | $\hat{\bar{Y}}_{p_6}$ | $\hat{\bar{Y}}_{p_7}$ | $\hat{\bar{Y}}_{p_8}$ | $\hat{\bar{Y}}_{p_9}$ | $\hat{\bar{Y}}_{p_{10}}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $\hat{\bar{Y}}_{p_1}$ | 100.0 | 130.77* | 129.14 | 117.12 | 120.10 | 97.08 | 117.96 | 116.45 | 107.10 | 109.18 |
| $\hat{\bar{Y}}_{p_2}$ | 76.47 | 100.00* | 98.75 | 89.56 | 91.84 | 74.24 | 90.20 | 89.05 | 81.90 | 83.49 |
| $\hat{\bar{Y}}_{p_3}$ | 77.44 | 101.27* | 100.00 | 90.69 | 93.00 | 75.18 | 91.35 | 90.17 | 82.94 | 84.54 |
| $\hat{\bar{Y}}_{p_4}$ | 85.38 | 111.66* | 110.26 | 100.00 | 102.54 | 82.89 | 100.72 | 99.43 | 91.45 | 93.22 |
| $\hat{\bar{Y}}_{p_5}$ | 83.27 | 108.89* | 107.53 | 97.52 | 100.00 | 80.84 | 98.22 | 96.96 | 89.18 | 90.91 |
| $\hat{\bar{Y}}_{p_6}$ | 103.0 | 134.70* | 133.02 | 120.64 | 123.70 | 100.00 | 121.50 | 119.94 | 110.32 | 112.46 |
| $\hat{\bar{Y}}_{p_7}$ | 84.77 | 110.86* | 109.47 | 99.28 | 101.81 | 82.30 | 100.00 | 98.72 | 90.79 | 92.55 |
| $\hat{\bar{Y}}_{p_8}$ | 85.88 | 112.30* | 110.90 | 100.58 | 103.13 | 83.37 | 101.30 | 100.00 | 91.97 | 93.76 |
| $\hat{\bar{Y}}_{p_9}$ | 93.37 | 122.10* | 120.58 | 109.35 | 112.14 | 90.65 | 110.14 | 108.73 | 100.00 | 101.94 |
| $\hat{\bar{Y}}_{p_{10}}$ | 91.59 | 119.78* | 118.28 | 107.27 | 110.00 | 88.92 | 108.05 | 106.66 | 98.10 | 100.00 |

*indicates the minimum Mean squared error

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## Appendix A

Notations used in this paper are given below:

| **Symbol** | **Explanation** |
|---|---|
| $N$ | Population size |
| $n$ | Sample size |
| $f = n/N$ | Sampling fraction |
| $Y$ | Study variable |
| $X$ | Auxiliary variable |
| $x, y$ | Sample totals |
| $\bar{X}, \bar{Y}$ | Population means |
| $\bar{x}, \bar{y}$ | Sample means |
| $S_x$ , $S_y$ | Population standard deviations |
| $S_{xy}$ | Population covariance between X and Y |
| $C_x$ , $C_y$ | Coefficient of variations |
| $\rho$ | Coefficient of correlation between X and Y |

| | |
|---|---|
| $\beta_1 = \frac{\mu_3^2}{\mu_2^3}$ | Coefficient of skewness of the auxiliary variable |
| $\beta_2 = \frac{\mu_4}{\mu_2^2}$ | Coefficient of kurtosis of the auxiliary variable |
| $b = \beta = \frac{S_{xy}}{S_x^2}$ | Regression coefficient of Y on X |
| $Q_1$ | First (lower) quartile of Auxiliary Variable |
| $Q_3$ | Third (upper) quartile of Auxiliary Variable |
| $Q_r = (Q_3 - Q_1)$ | Inter-quartile range of Auxiliary Variable |
| $Q_d = \frac{(Q_3 - Q_1)}{2}$ | Semi-quartile range of Auxiliary Variable |
| $Q_a = \frac{(Q_3 + Q_1)}{2}$ | Semi-quartile average of Auxiliary Variable |
| $B(.)$ | Bias of the Estimator |
| $MSE(.)$ | Mean squared error of the Estimator |
| $\hat{\bar{Y}}_i(\hat{\bar{Y}}_{p_j})$ | $i^{th}$ Existing ($j^{th}$ proposed) modified Ratio Estimator of $\bar{Y}$ |

## Appendix B

In case of Simple Random Sampling With-Out Replacement (SRSWOR), the sample mean $\bar{y}_{srs}$ is used to estimate population mean $\bar{Y}$ which is an unbiased Estimator and its variance is given below:

$$V(\bar{y}_{srs}) = \frac{(1-f)}{n} S_y^2$$

The Ratio Estimator for estimating the population mean $\bar{Y}$ of the study variable Y is defined as:

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}} \bar{X}$$

The Bias and Mean Squared Error of $\hat{\bar{Y}}_R$ to the first order of approximation are given below:

$$B\left(\hat{\bar{Y}}_R\right) = \frac{(1-f)}{n} \bar{Y}\left(C_x^2 - \rho C_x C_y\right)$$

$$MSE\left(\hat{\bar{Y}}_R\right) = \frac{(1-f)}{n} \bar{Y}^2\left(C_y^2 + C_x^2 - 2\rho C_x C_y\right)$$

The Linear Regression Estimator together with its variance is given below:

$$\hat{\bar{Y}}_{lr} = \bar{y} + \beta(\bar{X} - \bar{x})$$

$$V\left(\hat{\bar{Y}}_{lr}\right) = \frac{(1-f)}{n} S_y^2 (1 - \rho^2)$$

## Appendix C

We have derived the Bias and Mean Squared Error of the proposed Estimator $\hat{\hat{\bar{Y}}}_{p_j}$; $j = 1,2,3,\ldots,10$ to first order of approximation and are given below:

Let $e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}}$ and $e_1 = \frac{\bar{x} - \bar{X}}{\bar{X}}$.

Further, we can write $\bar{y} = \bar{Y}(1 + e_0)$ and $\bar{x} = \bar{X}(1 + e_1)$ and from the definition of $e_0$ and $e_1$ we obtain:

$E[e_0] = E[e_1] = 0$

$E[e_0^2] = \frac{(1-f)}{n} C_y^2$; $E[e_1^2] = \frac{(1-f)}{n} C_x^2$; $E[e_0 e_1] = \frac{(1-f)}{n} \rho C_y C_x$

The proposed Estimators $\hat{\hat{\bar{Y}}}_{p_j}$ in the form of $e_0$ and $e_1$ is given below:

$$\hat{\hat{\bar{Y}}}_{p_j} = \bar{Y}(1 + e_0) + b(\bar{X} - \bar{x}) \left[\frac{(\bar{X} + \lambda_j)}{(\bar{X}(1 + e_1) + \lambda_j)}\right]; j = 1,2,3,\ldots,10$$

where

$$\lambda_1 = Q_1, \lambda_2 = Q_3, \lambda_3 = Q_r, \lambda_4 = Q_d, \lambda_5 = Q_a, \lambda_6 = \frac{Q_1}{C_x}, \lambda_7 = \frac{Q_3}{C_x}, \lambda_8 = \frac{Q_r}{C_x}$$

$\lambda_9 = \frac{Q_d}{C_x}$ and $\lambda_{10} = \frac{Q_a}{C_x}$

$$\hat{\hat{\bar{Y}}}_{p_j} = \bar{Y}(1 + e_0) + b(\bar{X} - \bar{x}) \left[\frac{(\bar{X} + \lambda_j)}{(\bar{X} + \lambda_j)\left(1 + \frac{e_1 \bar{X}}{\bar{X} + \lambda_j}\right)}\right]$$

$$\Longrightarrow \hat{\hat{\bar{Y}}}_{p_j} = \frac{\bar{Y}(1 + e_0) + b(\bar{X} - \bar{x})}{\left(1 + \theta_{p_j} e_1\right)}$$

where

$$\theta_{p_j} = \frac{\bar{X}}{\bar{X} + \lambda_j}; j = 1,2,3,\ldots,10$$

$$\Longrightarrow \hat{\hat{\bar{Y}}}_{p_j} = \bar{Y}(1 + e_0) + b(\bar{X} - \bar{x}) \left(1 + \theta_{p_j} e_1\right)^{-1}$$

$$\Longrightarrow \hat{\hat{\bar{Y}}}_{p_j} = \bar{Y}(1 + e_0) + b(\bar{X} - \bar{x}) \left(1 - \theta_{p_j} e_1 + \theta_{p_j}^2 e_1^2 - \theta_{p_j}^3 e_1^3 + \cdots\right)$$

Neglecting the terms higher than third order, we will get:

$$\hat{\hat{\bar{Y}}}_{p_j} = \bar{Y} + \bar{Y}e_0 - b\bar{X}e_1 - \bar{Y}\theta_{p_j}e_1 - \bar{Y}\theta_{p_j}e_0e_1 + b\bar{X}\theta_{p_j}e_1^2 + \bar{Y}\theta_{p_j}^2e_1^2$$

$$\Longrightarrow \hat{\hat{\bar{Y}}}_{p_j} - \bar{Y} = \bar{Y}e_0 - b\bar{X}e_1 - \bar{Y}\theta_{p_j}e_1 - \bar{Y}\theta_{p_j}e_0e_1 + b\bar{X}\theta_{p_j}e_1^2 + \bar{Y}\theta_{p_j}^2e_1^2 \quad \text{(A)}$$

Taking expectation on both sides of (A), we get:

$$E\left(\hat{\hat{\bar{Y}}}_{p_j} - \bar{Y}\right) = \bar{Y}E(e_0) + b\bar{X}E(e_1) - \bar{Y}\theta_{p_j}E(e_1) - \bar{Y}\theta_{p_j}E(e_0e_1) + b\bar{X}\theta_{p_j}E(e_1^2) + \bar{Y}\theta_{p_j}^2E(e_1^2)$$

$$\text{Bias}\left(\hat{\hat{\bar{Y}}}_{p_j}\right) = \frac{(1-f)}{n}\left(\bar{Y}\theta_{p_j}^2C_x^2 - \theta_{p_j}\frac{S_{xy}}{S_x}C_x + \theta_{p_j}\frac{S_{xy}}{S_x}C_x\right)$$

$$\text{Bias}\left(\hat{\hat{\bar{Y}}}_{p_j}\right) = \frac{(1-f)}{n}\left(\bar{Y}\theta_{p_j}^2C_x^2\right); j = 1, 2, 3, \ldots, 10 \quad \text{(B)}$$

Substitute the value of $\theta_{p_j}$ in (B), we get the Bias of the proposed Estimator$\hat{\hat{\bar{Y}}}_{p_j}; j = 1, 2, 3, \ldots, 10$ as given below:

$$\text{Bias}\left(\hat{\hat{\bar{Y}}}_{p_j}\right) = \frac{(1-f)}{n}\left(\bar{Y}\frac{\bar{X}^2}{\left(\bar{X}+\lambda_j\right)^2}C_x^2\right) \quad \text{(C)}$$

Multiply and divide by $\bar{Y}^2$ in (3.19), we get:

$$\text{Bias}\left(\hat{\hat{\bar{Y}}}_{p_j}\right) = \frac{(1-f)}{n}\left(\frac{S_x^2}{\bar{Y}}R_{p_j}^2\right) \text{ where } R_{p_j} = \frac{\bar{Y}}{\bar{X}+\lambda_j};\ j = 1, 2, 3, \ldots, 10 \quad \text{(D)}$$

Squaring both sides of (A), neglecting the terms more than 2nd order and taking expectation on both sides, we get:

$$E\left(\hat{\hat{\bar{Y}}}_{p_j} - \bar{Y}\right)^2 = E(\bar{Y}^2e_0^2) + b^2\bar{X}^2E(e_1^2) + \bar{Y}^2\theta_{p_j}^2E(e_1^2) - 2b\overline{YX}E(e_0e_1) - 2\bar{Y}^2\theta_{p_j}E(e_0e_1) + 2b\overline{XY}\theta_{p_j}E(e_1^2)$$

$$\text{MSE}\left(\hat{\hat{\bar{Y}}}_{p_j}\right) = \frac{(1-f)}{n}\left(\bar{Y}^2C_y^2 + b^2\bar{X}^2C_x^2 + \bar{Y}^2\theta_{p_j}^2C_x^2 - 2b\overline{YX}\rho C_yC_x - 2\bar{Y}^2\theta_{p_j}\rho C_yC_x + 2b\overline{XY}\theta_jC_x^2\right)$$

$$\text{MSE}\left(\hat{\hat{\bar{Y}}}_{p_j}\right) = \frac{(1-f)}{n}\left(\begin{array}{c}\bar{Y}^2C_y^2 + \frac{S_{xy}{}^2}{(S_x^2)^2}S_x^2 + \bar{Y}^2\theta_{p_j}^2C_x^2 - \\ 2\frac{S_{xy}}{S_x^2}\frac{S_{xy}}{S_xS_y}S_yS_x - 2\bar{Y}\theta_{p_j}\frac{S_{xy}}{S_xS_y}S_yC_x + 2\frac{S_{xy}}{S_x^2}S_x\bar{Y}\theta_{p_j}C_x\end{array}\right)$$

$$\text{MSE}\left(\hat{\hat{\bar{Y}}}_{p_j}\right) = \frac{(1-f)}{n}\left(\bar{Y}^2 C_y^2 + \bar{Y}^2 \theta_{p_j}^2 C_x^2 + \frac{S_{xy}^2}{S_x^2} - 2\frac{S_{xy}^2}{S_x^2} - 2\bar{Y}\theta_j \frac{S_{xy}}{S_x} C_x + 2\bar{Y}\theta_{p_j}\frac{S_{xy}}{S_x} C_x\right)$$

$$\text{MSE}\left(\hat{\hat{\bar{Y}}}_{p_j}\right) = \frac{(1-f)}{n}\left(\bar{Y}^2 \theta_{p_j}^2 C_x^2 + \bar{Y}^2 C_y^2 - \frac{S_{xy}^2}{S_x^2}\right)$$

$$\text{MSE}\left(\hat{\hat{\bar{Y}}}_{p_j}\right) = \frac{(1-f)}{n}\left(\bar{Y}^2 \theta_{p_j}^2 C_x^2 + S_y^2 - \frac{\rho^2 S_x^2 S_y^2}{S_x^2}\right) \text{ since } \rho = \frac{S_{xy}}{S_x S_y}$$

$$\text{MSE}\left(\hat{\hat{\bar{Y}}}_{p_j}\right) = \frac{(1-f)}{n}\left(\bar{Y}^2 \theta_{p_j}^2 C_x^2 + S_y^2 - \rho^2 S_y^2\right)$$

$$\text{MSE}\left(\hat{\hat{\bar{Y}}}_{p_j}\right) = \frac{(1-f)}{n}\left(\bar{Y}^2 \theta_{p_j}^2 C_x^2 + S_y^2(1-\rho^2)\right)$$

Substitute the value of $\theta_{p_j}$ in the above expression, we will get:

$$\text{MSE}\left(\hat{\hat{\bar{Y}}}_{p_j}\right) = \frac{(1-f)}{n}\left(\bar{Y}^2 \frac{\bar{X}^2}{\left(\bar{X}+\lambda_j\right)^2} C_x^2 + S_y^2(1-\rho^2)\right)$$

$$\Longrightarrow \text{MSE}\left(\hat{\hat{\bar{Y}}}_{p_j}\right) = \frac{(1-f)}{n}\left(\frac{\bar{Y}^2}{\left(\bar{X}+\lambda_j\right)^2} \bar{X}^2 C_x^2 + S_y^2(1-\rho^2)\right)$$

The Mean Squared Error of the proposed Estimators $\hat{\hat{\bar{Y}}}_{p_j}$ is given below:

$$\text{MSE}\left(\hat{\hat{\bar{Y}}}_{p_j}\right) = \frac{(1-f)}{n}\left(R_{p_j}^2 S_x^2 + S_y^2(1-\rho^2)\right) \text{where } R_{p_j} = \frac{\bar{Y}}{\bar{X}+\lambda_j}; j = 1,2,3,\ldots,10 \quad \text{(E)}$$

## Appendix D

The conditions for which the proposed Modified Ratio Estimators (Class 3) perform better than the SRSWOR sample mean are derived and are given below:

For $\text{MSE}\left(\hat{\hat{\bar{Y}}}_{p_j}\right) \leq V(\bar{y}_{srs})$

$$\frac{(1-f)}{n}\left(R_{p_j}^2 S_x^2 + S_y^2(1-\rho^2)\right) \le \frac{(1-f)}{n} S_y^2$$
$$\Rightarrow \left(R_{p_j}^2 S_x^2 + S_y^2(1-\rho^2)\right) \le S_y^2$$
$$\Rightarrow R_{p_j}^2 S_x^2 + S_y^2 - S_y^2\rho^2 - S_y^2 \le 0$$
$$\Rightarrow R_{p_j}^2 S_x^2 - S_y^2\rho^2 \le 0$$
$$\Rightarrow R_{p_j}^2 S_x^2 \le S_y^2\rho^2$$
$$\Rightarrow R_{p_j} S_x \le S_y \rho$$
$$\Rightarrow \rho \ge R_{p_j} \frac{S_x}{S_y}$$
$$\Rightarrow R_{p_j} \le \rho \frac{S_y}{S_x}$$

That is $MSE\left(\hat{\hat{\bar{Y}}}_{p_j}\right) \le V(\bar{y}_{srs})$ if $R_{p_j} \le \rho \frac{S_y}{S_x}$

## Appendix E

The conditions for which the proposed Modified Ratio Estimators (Class 3) perform better than the Ratio Estimator are derived and are given below:

For $MSE\left(\hat{\hat{\bar{Y}}}_{p_j}\right) \le MSE\left(\hat{\hat{\bar{Y}}}_R\right)$

$$\frac{(1-f)}{n}\left(R_{p_j}^2 S_x^2 + S_y^2(1-\rho^2)\right) \le \frac{(1-f)}{n} \bar{Y}^2\left(C_y^2 + C_x^2 - 2\rho C_x C_y\right)$$
$$\Rightarrow \bar{Y}^2\left(R_{p_j}^{*2} S_x^2 + C_y^2(1-\rho^2)\right) \le \bar{Y}^2\left(C_y^2 + C_x^2 - 2\rho C_x C_y\right)$$
where $R_{p_j}^* = \frac{R_{p_j}}{\bar{Y}}$
$$\Rightarrow R_{p_j}^{*2} S_x^2 + C_y^2(1-\rho^2) - C_y^2 - C_x^2 + 2\rho C_x C_y \le 0$$
$$\Rightarrow R_{p_j}^{*2} S_x^2 + C_y^2 - \rho^2 C_y^2 - C_y^2 - C_x^2 + 2\rho C_x C_y \le 0$$
$$\Rightarrow R_{p_j}^{*2} S_x^2 - \rho^2 C_y^2 - C_x^2 + 2\rho C_x C_y \le 0$$
$$\Rightarrow \left(C_x - \rho C_y\right)^2 - R_{p_j}^{*2} S_x^2 \ge 0$$
$$\Rightarrow \left(C_x - \rho C_y + R_{p_j}^* S_x\right)\left(C_x - \rho C_y - R_{p_j}^* S_x\right) \ge 0$$

**Condition 1:** $\left(C_x - \rho C_y + R^*_{p_j} S_x\right) \leq 0$ and $\left(C_x - \rho C_y - R^*_{p_j} S_x\right) \leq 0$

$$C_x + R^*_{p_j} S_x \leq \rho C_y \text{ and } C_x - R^*_{p_j} S_x \leq \rho C_y$$

$$\Rightarrow R^*_{p_j} \leq \frac{\rho C_y - C_x}{S_x} \text{ and } R^*_{p_j} \geq \frac{C_x - \rho C_y}{S_x}$$

$$\Rightarrow \frac{C_x - \rho C_y}{S_x} \leq R^*_{p_j} \leq \frac{\rho C_y - C_x}{S_x}$$

where $R^*_{p_j} = \dfrac{R_{p_j}}{\overline{Y}}$

$$\Rightarrow \overline{Y}\left(\frac{C_x - \rho C_y}{S_x}\right) \leq R_{p_j} \leq \overline{Y}\left(\frac{\rho C_y - C_x}{S_x}\right)$$

**Condition 2:** $\left(C_x - \rho C_y + R^*_{p_j} S_x\right) \geq 0$ and $\left(C_x - \rho C_y - R^*_{p_j} S_x\right) \geq 0$

$$C_x + R^*_{p_j} S_x \geq \rho C_y \text{ and } C_x - R^*_{p_j} S_x \geq \rho C_y$$

$$\Rightarrow R^*_{p_j} \geq \frac{\rho C_y - C_x}{S_x} \text{ and } R^*_{p_j} \leq \frac{C_x - \rho C_y}{S_x}$$

$$\Rightarrow \frac{\rho C_y - C_x}{S_x} \leq R^*_{p_j} \leq \frac{C_x - \rho C_y}{S_x}$$

$$\Rightarrow \overline{Y}\left(\frac{\rho C_y - C_x}{S_x}\right) \leq R_{p_j} \leq \overline{Y}\left(\frac{C_x - \rho C_y}{S_x}\right)$$

That is $MSE\left(\widehat{\overline{Y}}_{p_j}\right) \leq MSE\left(\widehat{\overline{Y}}_R\right)$ if $\overline{Y}\left(\frac{C_x - \rho C_y}{S_x}\right) \leq R_{p_j} \leq \overline{Y}\left(\frac{\rho C_y - C_x}{S_x}\right)$

or

$$\overline{Y}\left(\frac{\rho C_y - C_x}{S_x}\right) \leq R_{p_j} \leq \overline{Y}\left(\frac{C_x - \rho C_y}{S_x}\right)$$

## Appendix F

The conditions for which the proposed Modified Ratio Estimators (Class 3) perform better than the existing Modified Ratio Estimators (Class 1) are derived and are given below:

For $MSE\left(\widehat{\overline{Y}}_{p_j}\right) \leq MSE\left(\widehat{\overline{Y}}_i\right)$; $i = 1,2,3, \ldots 12$ and $j = 1,2,3, \ldots 10$

$$\frac{(1-f)}{n}\left(R^2_{p_j} S^2_x + S^2_y (1-\rho^2)\right) \leq \frac{(1-f)}{n}\left(R^2_i S^2_x + S^2_y (1-\rho^2)\right)$$

$$\Rightarrow R^2_{p_j} S^2_x + S^2_y (1-\rho^2) \leq R^2_i S^2_x + S^2_y (1-\rho^2)$$

$$\Rightarrow R^2_{p_j} S^2_x \leq R^2_i S^2_x$$

$$\Rightarrow R_{p_j}^2 \le R_i^2$$
$$\Rightarrow R_{p_j} \le R_i$$

That is $MSE\left(\hat{\hat{\bar{Y}}}_{p_j}\right) < MSE\left(\hat{\hat{\bar{Y}}}_i\right)$ if $R_{p_j} < R_i$ ; $i = 1, 2, 3, \dots, 12$: $j = 1, 2, 3, \dots, 10$

## Appendix G

The conditions for which the proposed Modified Ratio Estimators (Class 3) perform better than the existing Modified Ratio Estimators (Class 2) are derived and are given below:

For $MSE\left(\hat{\hat{\bar{Y}}}_{p_j}\right) \le MSE\left(\hat{\hat{\bar{Y}}}_i\right)$; $i = 13,14,15,16$ and $17$

$$\frac{(1-f)}{n}\left(R_{p_j}^2 S_x^2 + S_y^2(1-\rho^2)\right) \le \frac{(1-f)}{n}\bar{Y}^2\left(C_y^2 + \theta_i^2 C_x^2 - 2\theta_i \rho C_x C_y\right)$$
$$\Rightarrow \bar{Y}^2\left(R_{p_j}^{*2} S_x^2 + C_y^2(1-\rho^2)\right) \le \bar{Y}^2\left(C_y^2 + \theta_i^2 C_x^2 - 2\theta_i \rho C_x C_y\right)$$

where $R_{p_j}^* = \dfrac{R_{p_j}}{\bar{Y}}$

$$\Rightarrow R_{p_j}^{*2} S_x^2 + C_y^2(1-\rho^2) - C_y^2 - \theta_i^2 C_x^2 + 2\theta_i \rho C_x C_y \le 0$$
$$\Rightarrow R_{p_j}^{*2} S_x^2 + C_y^2 - \rho^2 C_y^2 - C_y^2 - \theta_i^2 C_x^2 + 2\theta_i \rho C_x C_y \le 0$$
$$\Rightarrow R_{p_j}^{*2} S_x^2 - \rho^2 C_y^2 - \theta_i^2 C_x^2 + 2\theta_i \rho C_x C_y \le 0$$
$$\Rightarrow \left(\theta_i C_x - \rho C_y\right)^2 - R_{p_j}^{*2} S_x^2 \ge 0$$
$$\Rightarrow \left(\theta_i C_x - \rho C_y + R_{p_j}^* S_x\right)\left(\theta_i C_x - \rho C_y - R_{p_j}^* S_x\right) \ge 0$$

**Condition 1:** $\left(\theta_i C_x - \rho C_y + R_{p_j}^* S_x\right) \le 0$ and $\left(\theta_i C_x - \rho C_y - R_{p_j}^* S_x\right) \le 0$

$$\theta_i C_x + R_{p_j}^* S_x \le \rho C_y \text{ and } \theta_i C_x - R_{p_j}^* S_x \le \rho C_y$$
$$\Rightarrow R_{p_j}^* \le \frac{\rho C_y - \theta_i C_x}{S_x} \text{ and } R_{p_j}^* \ge \frac{\theta_i C_x - \rho C_y}{S_x}$$
$$\Rightarrow \frac{\theta_i C_x - \rho C_y}{S_x} \le R_{p_j}^* \le \frac{\rho C_y - \theta_i C_x}{S_x}$$

where $R_{p_j}^* = \dfrac{R_{p_j}}{\bar{Y}}$

$$\Rightarrow \bar{Y}\left(\frac{\theta_i C_x - \rho C_y}{S_x}\right) \le R_{p_j} \le \bar{Y}\left(\frac{\rho C_y - \theta_i C_x}{S_x}\right)$$

**Condition 2:** $\left(\theta_i C_x - \rho C_y + R^*_{p_j} S_x\right) \geq 0$ and $\left(\theta_i C_x - \rho C_y - R^*_{p_j} S_x\right) \geq 0$

$$\theta_i C_x + R^*_{p_j} S_x \geq \rho C_y \text{ and } \theta_i C_x - R^*_{p_j} S_x \geq \rho C_y$$

$$\Longrightarrow R^*_{p_j} \geq \frac{\rho C_y - \theta_i C_x}{S_x} \text{ and } R^*_{p_j} \leq \frac{\theta_i C_x - \rho C_y}{S_x}$$

$$\Longrightarrow \frac{\rho C_y - \theta_i C_x}{S_x} \leq R^*_{p_j} \leq \frac{\theta_i C_x - \rho C_y}{S_x}$$

$$\Longrightarrow \overline{Y}\left(\frac{\rho C_y - \theta_i C_x}{S_x}\right) \leq R_{p_j} \leq \overline{Y}\left(\frac{\theta_i C_x - \rho C_y}{S_x}\right)$$

That is $MSE\left(\widehat{\overline{Y}}_{p_j}\right) \leq MSE\left(\widehat{\overline{Y}}_i\right)$ if $\overline{Y}\left(\frac{\theta_i C_x - \rho C_y}{S_x}\right) \leq R_{p_j} \leq \overline{Y}\left(\frac{\rho C_y - \theta_i C_x}{S_x}\right)$

or

$$\overline{Y}\left(\frac{\rho C_y - \theta_i C_x}{S_x}\right) \leq R_{p_j} \leq \overline{Y}\left(\frac{\theta_i C_x - \rho C_y}{S_x}\right)$$