ISSN 1684-8403
Journal of Statistics
Volume 21, 2014. pp. 128-156

Posterior Analysis of Burr Type V Distribution under Different Types of Censoring

Navid Feroze[1] and Muhammad Aslam[2]

Abstract

This paper aims to discuss the Bayesian Analysis of Burr Type V Distribution based on Left, Singly Type II and Doubly Type II censored samples. The Bayes Point and Interval Estimators of the parameter for the said Distribution have been derived under four Priors (Informative and Non-informative) using a couple of Loss Functions. The Posterior Predictive Intervals have also been constructed. The performance of the Bayes Estimators has been compared under a Simulation study. A real life data set has also been analyzed for illustration. The results indicate that the performance of the estimates under Inverse Levy Prior based on Generalized Entropy Loss Function using Singly Type II censored samples is superior to their counter-parts.

Keywords

Bayes Estimators, Posterior Risks, Loss Functions, Censoring, Prior distributions.

1. Introduction

Burr family of Distributions consists of twelve Distributions. These Distributions can be used to fit almost any given set of Uni-modal data. Burr (1942) proposed these Distributions. Among these twelve Distributions, Burr Type X and XII have received the sizeable attention of the analysts. Surles and Padgett (2001) introduced two-parameter Burr Type X Distribution and showed that this particular skewed Distribution can be used quite effectively in analyzing time to failure data. Soliman (2002) derived the Bayes Estimates of the parameter of Burr Type XII Distribution three Loss Functions.

[1] Department of Mathematics and Statistics, Allama Iqbal Open University, Islamabad, Pakistan.
Email: navidferoz@gmail.com
[2] Department of Statistics, Quaid-i-Azam University, Islamabad, Pakistan.
Email: aslamsdqu@yahoo.com

The Bayes and Maximum Likelihood Estimates have been compared by using Lindley's approximation. Shao (2004) addressed the problem of Maximum Likelihood Estimation for the three-parameter Burr Type XII Distribution. Shao et al. (2004) applied the Burr Type XII Distribution for flood Frequency Analysis. Soliman (2005) argued that the versatility and flexibility of the Burr Type XII Distribution turns it quite attractive as a tentative model for data whose underlying Distribution is unknown. Wu et al. (2007) considered the estimation of Burr Distribution based on progressive Type II censoring with random removals. Silva et al. (2008) proposed a Location-Scale Regression Model based on Burr XII Distribution and referred it as the Log-Burr XII Regression Model. Dasgupta (2011) discussed that under certain conditions, the Distribution of Burr can be shown to follow an Extreme-value Distribution. Makhdoom and Jafari (2011) obtained Bayesian Estimators for the shape parameter of the Burr Type XII Distribution using grouped and un-grouped data. Panahi and Asadi (2011) considered the statistical inferences based on a Type II Hybrid censored sample from a Burr Type XII Distribution. Feroze and Aslam (2012a) dealt with Posterior Analysis of Burr Type X Distribution using complete and censored data. The other authors dealing with Bayesian and Classical Inference of Burr Type X and XII Distribution include: Aludaat et al. (2008), Amjad and Ayman (2006), Mousa and Jaheen (2002), Wahed (2006), Wu and Yu (2005), Yarmohammadi and Pazira (2010). The remaining types of the Burr family of Distributions haven't received a considerable interest of the analysts; same is the case with Burr Type V Distributions. The Burr Type V Distribution can also be used to model the lifetime data. We have considered the Bayesian Estimation of the Distribution. The application of the Distribution in Reliability Analysis further gave the motivation to carry out the analysis under censored samples.

Censoring is very important technique which has wide range of applications in reliability studies. It has many types each of whom can be used in analysis of different kinds of data representing various real life circumstances. We have used Left, Singly Type II and Doubly Type II censoring for the Bayes Estimation of the parameter of Burr Type V Distribution. The introduction of all types of censoring has been presented in the following.

The left censored data is very likely to occur in Survivor Analysis. It can happen where an event of interest has already occurred at the observation time, but it is not known exactly when. For example, the situations including: the infection with a sexually-transmitted disease such as HIV/AIDS, onset of a pre-symptomatic illness such as cancer and time at which teenagers begin to drink alcohol can lead to left censored data. In such cases, we can only observe those individuals whose event time is greater than some truncation point. This truncation point may or may not be the same for all individuals. For example, in case of actuarial life studies, the individuals those died in the womb are often ignored. Another example: suppose we wish to study how long patients who have been hospitalized for a heart attack survive taking some treatment at home. In such situations, the starting time is often considered to be the time of the heart attack. Only those patients who survive their stay in hospital are able to be included in the study. The more illustrations on left censoring can be seen from Antweller and Taylor (2008), Asselineau et al. (2007), Jerald and Lawless (2003), Sinha et al. (2006), Thompson et al. (2011).

On the other hand, suppose it is desired to estimate the average life of electric bulb produced in a certain factory. The simple method would be to take a certain number of bulbs at random and burn them out to get the required number of bulbs for analysis. Instead of wasting the bulbs it might be decided to stop the experiment when a fixed number have burnt out. The random sample hence obtained would be a Type II censored sample. Further, the biologists are often required to perform experiments on animals (say, rabbits or mice) to determine the effect of certain drugs on them. A fixed number of animals are exposed to the drug for this purpose and their reaction times are observed. Experience shows that some animals take an extremely long time to react. If instead of waiting until all animals have reacted, the experiment is stopped when a fixed number have reacted, it will be called Type II censoring, which may result in economical experimentation. The authors considering analysis under Singly and Doubly Type II censored samples are: Akhter and Hirai (2009), Al-Hussaini and Hussein (2011), Fauzy (2004), Feroze and Aslam (2012b) and Yarmohammadi and Pazira (2010).

The organization of the paper is as follows: Section 2 contains the Model and expressions for Likelihood Functions under Left, Singly Type II and Doubly Type II censored samples. The Posterior Distributions under different Priors based on above mentioned censoring methods have been derived and presented in section 3. Section 4 contains the introduction of the Loss Function and expressions for Bayes Estimates and Posterior Risks under these Loss Functions. Section 5 deals with construction of Bayesian Credible Intervals. On the other hand, the Posterior Predictive Intervals have been derived in section 6. The elicitation of the hyper-parameters in Informative Priors has been discussed in section 7. Section 8 includes the numerical results and discussions based on a Simulation study. The applicability of the results has been discussed by analyzing a real life data in section 9. Finally the conclusion is presented in section 10.

2. Model and Likelihood Functions

The probability density function of the Distribution is,

$$f(y) = \theta e^{-\tan y} \sec^2 y \left(1+e^{-\tan y}\right)^{-\theta-1}, \; -\frac{\pi}{2} < y < \frac{\pi}{2}, \; \theta > 0 \quad (2.1)$$

The cumulative distribution function of this Distribution is,

$$F(y) = \left(1+e^{-\tan y}\right)^{-\theta} \quad (2.2)$$

where θ is the location parameter of the Distribution.

2.1 Likelihood Function under Left Censored Sample: Let $Y_{r+1},...,Y_n$ be last n-r Order Statistics from a sample of size n from Burr Type V Distribution. Then the Likelihood Function for the $Y_{r+1},...,Y_n$ left censored observations is,

$$L(\theta|\mathrm{y}) \propto \left\{F(y_{r+1})\right\}^r \prod_{i=r+1}^{n} f(y_i)$$

$$L(\theta|\mathrm{y}) \propto \theta^{n-r} \exp\left[-\theta\left\{-r\ln\left(1+e^{-\tan y_{r+1}}\right) - \sum_{i=r+1}^{n} \ln\left(1+e^{-\tan y_i}\right)\right\}\right] \quad (2.1.1)$$

2.3 Likelihood Function under Singly Type II Censored Sample: Suppose 'n' items are put on a life-testing experiment and only first 'm' failure times have been observed, that is, $y_1 < y_2 ... < y_m$ and remaining n-m items are still working. Under the assumptions that the lifetimes of the items are independently and identically distributed Burr Type V random variable, then the Likelihood Function for the Singly Type II censored sample is,

$$L(\theta|\mathrm{y}) \propto \left[\prod_{i=1}^{m} f(y_i)\right]\left[1-F(y_m)\right]^{n-m}$$

$$L(\theta|\mathrm{y}) \propto \sum_{j=0}^{n-m} (-1)^j \binom{n-m}{j} \theta^m \exp\left[-\theta\left\{-\sum_{i=1}^{m} \ln\left(1+e^{-\tan y_i}\right) - j\ln\left(1+e^{-\tan y_m}\right)\right\}\right] \quad (2.3.1)$$

2.4 Likelihood Function under Doubly Type II Censored Sample: Consider a random sample of size 'n' from an Burr Type V Distribution, and let $y_r,..., y_s$ be the ordered observations remaining when the 'r – 1' smallest observations and the 'n – s' largest observations have been censored, the Likelihood Function for θ given the Type II Doubly censored sample $\underline{y} = (y_r,..., y_s)$, is,

$$L(\theta|\mathrm{y}) \propto \left[F\left(y_r|\theta\right)\right]^{r-1}\left[1-F\left(y_s|\theta\right)\right]^{n-s}\prod_{i=r}^{s} f\left(y_i|\theta\right)$$

$$L(\theta|\mathrm{y}) \propto \sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\theta^k \exp\left[-\theta\left\{-\sum_{i=r}^{s}\ln\left(1+e^{-\tan y_i}\right)-(r-1)\ln\left(1+e^{-\tan y_r}\right)-j\ln\left(1+e^{-\tan y_s}\right)\right\}\right] \quad (2.4.1)$$

3. Prior and Posterior Distributions

Bayesian methods can provide more precision of estimate than the classical methods of inference. This happens due to additional information that is present in terms of Prior information and the estimates are obtained from the combined sources of information. Thus Bayesian inference is one that modifies one's initial probability statements about the parameters before observing the data to updated or Posterior knowledge that combines both Prior information and the data at hand. Therefore, Prior subject-matter knowledge about a parameter is a vital aspect of the inference process. We have considered Uniform, Jeffreys, Gamma and Inverse Levy Priors to derive the Posterior Distributions and to complete the corresponding analysis. The expressions for each Prior are given below:

Uniform Prior: $g(\theta) \propto 1$ (3.1)

Jeffreys Prior: $g(\theta) \propto \theta^{-1}$ $\quad \theta > 0$ (3.2)

Gamma Prior: $g(\theta) = \dfrac{b^a}{\Gamma(a)}\theta^{a-1}e^{-b\theta}$ $\quad \theta > 0,\ a,b > 0$ (3.3)

Inverse Levy Prior: $g(\theta) = \sqrt{\dfrac{c}{2\pi}}(\theta)^{-0.5}e^{-0.5c\theta}$ $\quad \theta > 0,\ c > 0$ (3.4)

where a,b and c are the hyper-parameters.

The Posterior Distribution summarizes the current state of knowledge about all the uncertain qualities in a Bayesian Analysis. Analytically, the Posterior Distribution is the product of the Prior Density and the Likelihood. Consider the Prior Distribution $p(\theta)$ which reflects the Prior information before collecting the data y and the Likelihood Function $L(\theta|\mathrm{y})$ that represents the observed data then the Posterior Density $p(\theta|\mathrm{y})$ is calculated as proportional to the multiplication of Prior Distribution and the Likelihood Function i.e.

Posterior Density $\propto$ (Prior Density)×(Likelihood Function)

$$p(\theta|\mathrm{y}) \propto p(\theta)L(\theta|\mathrm{y}) \qquad \text{Or} \qquad p(\theta|\mathrm{y}) = \frac{p(\theta)L(\theta|\mathrm{y})}{\int_{-\infty}^{\infty} p(\theta)L(\theta|\mathrm{y})d\theta}$$

The Posterior Distribution updates the information. All inferences and interpretations about parameters are made through the Posterior Distribution.

The Posterior Distribution under Uniform Prior based on left censored samples is,

$$p(\theta|\mathrm{y}) = \frac{\left\{-r\ln\left(1+e^{-\tan y_{r+1}}\right)-\sum_{i=r+1}^{n}\ln\left(1+e^{-\tan y_i}\right)\right\}^{n-r+1}}{\Gamma(n-r+1)}\theta^{n-r}\exp\left[-\theta\left\{-r\ln\left(1+e^{-\tan y_{r+1}}\right)-\sum_{i=r+1}^{n}\ln\left(1+e^{-\tan y_i}\right)\right\}\right] \quad (3.5)$$

The Posterior Distribution under Uniform Prior based on Singly Type II censored samples is,

$$p(\theta|\mathrm{y}) = \frac{1}{A_{1j}}\sum_{j=0}^{n-m}(-1)^j\binom{n-m}{j}\theta^m\exp\left[-\theta\left\{-\sum_{i=1}^{m}\ln\left(1+e^{-\tan y_i}\right)-j\ln\left(1+e^{-\tan y_m}\right)\right\}\right] \quad (3.6)$$

$$A_{1j} = \sum_{j=0}^{n-m}(-1)^j\binom{n-m}{j}\Gamma(m+1)\left\{-\sum_{i=1}^{m}\ln\left(1+e^{-\tan y_i}\right)-j\ln\left(1+e^{-\tan y_m}\right)\right\}^{-m-1}$$

The Posterior Distribution under Uniform Prior based on Doubly Type II censored samples is,

$$p(\theta|\underline{x}) = \frac{1}{A_{2j}}\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\theta^k\exp\left[-\theta\left\{-\sum_{i=r}^{s}\ln\left(1+e^{-\tan y_i}\right)-(r-1)\ln\left(1+e^{-\tan y_r}\right)-j\ln\left(1+e^{-\tan y_s}\right)\right\}\right] \quad (3.7)$$

$$A_{2j} = \sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\Gamma(k+1)\left\{-\sum_{i=r}^{s}\ln\left(1+e^{-\tan y_i}\right)-(r-1)\ln\left(1+e^{-\tan y_r}\right)-j\ln\left(1+e^{-\tan y_s}\right)\right\}^{-k-1}$$

The Posterior Distributions under the assumption of remaining Priors have been presented in the appendix.

4. Loss Functions and Bayes Point Estimation

The performance of the Bayes Point Estimates can be compared in terms of magnitudes of Posterior Risks. The Posterior Risk is defined to be the expected value of the Loss Function. So, the employment of Loss Function is fundamental in order to draw conclusion about the performance of the Bayes Estimators. Here, Generalized Entropy and Linear Exponential Loss Function have been used for Bayes estimation of the parameter of Burr Type V Distribution. Their description is as under

4.1 Generalized Entropy Loss Function (GELF): The generalized Entropy Loss Function suggested by Calabria and Pulcini (1996) can be given as $L\left(\theta,\theta_{GELF}\right)=\left(\theta^{-1}\theta_{GELF}\right)^{p}-p\ln\left(\theta^{-1}\theta_{GELF}\right)-1$ where $\theta_{GELF}=\left[\mathrm{E}\left(\theta^{-p}\right)\right]^{-\frac{1}{p}}$ is the Bayes Estimator under this Loss Functions and p is the shape parameter of the Loss Function.

4.2 LINEX Loss Function (LLF): LINEX (Linear Exponential) Loss Function has been defined by Klebanov (1972). The expression of the Loss Function is $L\left(\theta_{LLF},\theta\right)=\phi\left\{e^{\omega\left(\theta_{LLF}-\theta\right)}-\omega\left(\theta_{LLF}-\theta\right)-1\right\}$. Without loss of generality we can assume $\phi=1$, then $L\left(\theta_{LLF},\theta\right)=\left\{e^{\omega\left(\theta_{LLF}-\theta\right)}-\omega\left(\theta_{LLF}-\theta\right)-1\right\}$. And the Bayes Estimator based on this Loss Function is,

$$\theta_{LLF}=-\omega^{-1}\ln\left[\mathrm{E}\left(e^{-\omega\theta}\right)\right]$$

Now, the GELF and LLF have been used to derive the Bayes Estimators and Posterior Risks under each Prior for all three censoring schemes. The expressions for the derived results have been presented in the following. The comparisons among the performance of these Estimators have been made in sections 8-9.

The Bayes Estimator and Posterior Risk under Uniform Prior for left censored samples using GELF are,

$$\theta_{GELF}=\left[\frac{\Gamma\left(n-r-p+1\right)\left\{-r\ln\left(1+e^{-\tan y_{r+1}}\right)-\sum_{i=r+1}^{n}\ln\left(1+e^{-\tan y_i}\right)\right\}^{p}}{\Gamma\left(n-r+1\right)}\right]^{-\frac{1}{p}}$$

$$\rho\left(\theta_{GELF}\right)=\ln\left[\frac{\Gamma\left(n-r-p+1\right)\left\{-r\ln\left(1+e^{-\tan y_{r+1}}\right)-\sum_{i=r+1}^{n}\ln\left(1+e^{-\tan y_i}\right)\right\}^{p}}{\Gamma\left(n-r+1\right)}\right]+p\left[\psi\left(n-r+1\right)-\ln\left\{-r\ln\left(1+e^{-\tan y_{r+1}}\right)-\sum_{i=r+1}^{n}\ln\left(1+e^{-\tan y_i}\right)\right\}\right]$$

where $\psi\left(z\right)$ is a Di-gamma Function.

The Bayes Estimator and Posterior Risk under Uniform Prior for left censored samples using LLF are,

$$\theta_{LLF}=-\frac{n-r+1}{\omega}\ln\left\{\frac{-r\ln\left(1+e^{-\tan y_{r+1}}\right)-\sum_{i=r+1}^{n}\ln\left(1+e^{-\tan y_i}\right)}{-r\ln\left(1+e^{-\tan y_{r+1}}\right)-\sum_{i=r+1}^{n}\ln\left(1+e^{-\tan y_i}\right)+\omega}\right\}$$

$$\rho\left(\theta_{LLF}\right)=\left(n-r+1\right)\ln\left\{\frac{-r\ln\left(1+e^{-\tan y_{r+1}}\right)-\sum_{i=r+1}^{n}\ln\left(1+e^{-\tan y_i}\right)}{-r\ln\left(1+e^{-\tan y_{r+1}}\right)-\sum_{i=r+1}^{n}\ln\left(1+e^{-\tan y_i}\right)+\omega}\right\}+\frac{\omega\left(n-r+1\right)}{-r\ln\left(1+e^{-\tan y_{r+1}}\right)-\sum_{i=r+1}^{n}\ln\left(1+e^{-\tan y_i}\right)}$$

The Bayes Estimator and Posterior Risk under Uniform Prior for Singly Type II censored samples using GELF are,

$$\theta_{GELF}=\left[\frac{\Gamma\left(m-p+1\right)}{A_{1j}}\sum_{j=0}^{n-m}\left(-1\right)^{j}\binom{n-m}{j}\left\{-\sum_{i=1}^{m}\ln\left(1+e^{-\tan y_i}\right)-j\ln\left(1+e^{-\tan y_m}\right)\right\}^{p-m-1}\right]^{-\frac{1}{p}}$$

$$\rho\left(\theta_{GELF}\right)=\ln\left[\frac{\Gamma\left(m-p+1\right)}{A_{1j}}\sum_{j=0}^{n-m}\left(-1\right)^{j}\binom{n-m}{j}\left\{-\sum_{i=1}^{m}\ln\left(1+e^{-\tan y_i}\right)-j\ln\left(1+e^{-\tan y_m}\right)\right\}^{p-m-1}\right]$$
$$+p\left[\frac{1}{A_{1j}}\sum_{j=0}^{n-m}\left(-1\right)^{j}\binom{n-m}{j}\left\{\psi\left(m+1\right)-\ln\left\{-\sum_{i=1}^{m}\ln\left(1+e^{-\tan y_i}\right)-j\ln\left(1+e^{-\tan y_m}\right)\right\}\right\}\right]$$

The Bayes Estimator and Posterior Risk under Uniform Prior for Singly Type II censored samples using LLF are,

$$\theta_{LLF}=-\frac{1}{\omega}\ln\left[\frac{\Gamma\left(m+1\right)}{A_{1j}}\sum_{j=0}^{n-m}\left(-1\right)^{j}\binom{n-m}{j}\left\{\omega-\sum_{i=1}^{m}\ln\left(1+e^{-\tan y_i}\right)-j\ln\left(1+e^{-\tan y_m}\right)\right\}^{-m-1}\right]$$

$$\rho\left(\theta_{LLF}\right)=\ln\left[\frac{\Gamma\left(m+1\right)}{A_{1j}}\sum_{j=0}^{n-m}\left(-1\right)^{j}\binom{n-m}{j}\left\{\omega-\sum_{i=1}^{m}\ln\left(1+e^{-\tan y_i}\right)-j\ln\left(1+e^{-\tan y_m}\right)\right\}^{-m-1}\right]$$
$$+\frac{\omega\Gamma\left(m+2\right)}{A_{1j}}\sum_{j=0}^{n-m}\left(-1\right)^{j}\binom{n-m}{j}\left\{-\sum_{i=1}^{m}\ln\left(1+e^{-\tan y_i}\right)-j\ln\left(1+e^{-\tan y_m}\right)\right\}^{-m-2}$$

The Bayes Estimator and Posterior Risk under Uniform Prior for Doubly Type II censored samples using GELF are,

$$\theta_{GELF}=\left[\frac{\Gamma\left(k-p+1\right)}{A_{2j}}\sum_{j=0}^{n-s}\left(-1\right)^{j}\binom{n-s}{j}\left\{-\sum_{i=r}^{s}\ln\left(1+e^{-\tan y_i}\right)-\left(r-1\right)\ln\left(1+e^{-\tan y_r}\right)-j\ln\left(1+e^{-\tan y_s}\right)\right\}^{p-k-1}\right]^{-\frac{1}{p}}$$

$$\rho\left(\theta_{GELF}\right)=\ln\left[\frac{\Gamma\left(k-p+1\right)}{A_{2j}}\sum_{j=0}^{n-s}\left(-1\right)^{j}\binom{n-s}{j}\left\{-\sum_{i=r}^{s}\ln\left(1+e^{-\tan y_i}\right)-\left(r-1\right)\ln\left(1+e^{-\tan y_r}\right)-j\ln\left(1+e^{-\tan y_s}\right)\right\}^{p-k-1}\right]$$
$$+p\left[\frac{1}{A_{2j}}\sum_{j=0}^{n-s}\left(-1\right)^{j}\binom{n-s}{j}\left\{\psi\left(k+1\right)-\ln\left\{-\sum_{i=r}^{s}\ln\left(1+e^{-\tan y_i}\right)-\left(r-1\right)\ln\left(1+e^{-\tan y_r}\right)-j\ln\left(1+e^{-\tan y_s}\right)\right\}\right\}\right]$$

The Bayes Estimator and Posterior Risk under Uniform Prior for Doubly Type II censored samples using LLF are,

$$\theta_{LLF}=-\frac{1}{\omega}\ln\left[\frac{\Gamma\left(k+1\right)}{A_{2j}}\sum_{j=0}^{n-s}\left(-1\right)^{j}\binom{n-s}{j}\left\{\omega-\sum_{i=r}^{s}\ln\left(1+e^{-\tan y_i}\right)-\left(r-1\right)\ln\left(1+e^{-\tan y_r}\right)-j\ln\left(1+e^{-\tan y_s}\right)\right\}^{-k-1}\right]$$

$$\rho\left(\theta_{LLF}\right)=\ln\left[\frac{\Gamma\left(k+1\right)}{A_{2j}}\sum_{j=0}^{n-s}\left(-1\right)^{j}\binom{n-s}{j}\left\{\omega-\sum_{i=r}^{s}\ln\left(1+e^{-\tan y_i}\right)-\left(r-1\right)\ln\left(1+e^{-\tan y_r}\right)-j\ln\left(1+e^{-\tan y_s}\right)\right\}^{-k-1}\right]$$
$$+\frac{\omega\Gamma\left(k+2\right)}{A_{2j}}\sum_{j=0}^{n-s}\left(-1\right)^{j}\binom{n-s}{j}\left\{-\sum_{i=r}^{s}\ln\left(1+e^{-\tan y_i}\right)-\left(r-1\right)\ln\left(1+e^{-\tan y_r}\right)-j\ln\left(1+e^{-\tan y_s}\right)\right\}^{-k-2}$$

The expressions for Bayes Estimators and associated Posterior Risks under Jeffreys, Gamma and Inverse Levy Priors have been reported in the appendix.

5. Credible Intervals

The Credible Interval for θ based on Uniform Prior using Left, Singly Type II and Doubly Type II censored samples, as discussed by Feroze and Aslam (2012b), have respectively been presented in the following.

$$\frac{\chi^2_{\left(1-\alpha/2\right)\{2(n-r+1)\}}}{2\{T_{1j}\}}<\theta<\frac{\chi^2_{\left(\alpha/2\right)\{2(n-r+1)\}}}{2\{T_{1j}\}}$$

where $T_{1j}=-r\ln\left(1+e^{-\tan y_{r+1}}\right)-\sum_{i=r+1}^{n}\ln\left(1+e^{-\tan y_i}\right)$

$$\frac{\chi^2_{\left(1-\alpha/2\right)\{2(m+1)\}}\sum_{j=0}^{n-m}(-1)^j\binom{n-m}{j}\{T_{2j}\}^{-m-1}}{2\sum_{j=0}^{n-m}(-1)^j\binom{n-m}{j}\{T_{2j}\}^{-m-2}}<\theta<\frac{\chi^2_{\left(\alpha/2\right)\{2(m+1)\}}\sum_{j=0}^{n-m}(-1)^j\binom{n-m}{j}\{T_{2j}\}^{-m-1}}{2\sum_{j=0}^{n-m}(-1)^j\binom{n-m}{j}\{T_{2j}\}^{-m-2}}$$

where $T_{2j}=-\sum_{i=1}^{m}\ln\left(1+e^{-\tan y_i}\right)-j\ln\left(1+e^{-\tan y_m}\right)$

$$\frac{\chi^2_{\left(1-\alpha/2\right)\{2(k+1)\}}\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\{T_{3j}\}^{-k-1}}{2\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\{T_{3j}\}^{-k-2}}<\theta<\frac{\chi^2_{\left(\alpha/2\right)\{2(k+1)\}}\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\{T_{3j}\}^{-k-1}}{2\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\{T_{3j}\}^{-k-2}}$$

where $T_{3j}=-\sum_{i=r}^{s}\ln\left(1+e^{-\tan y_i}\right)-(r-1)\ln\left(1+e^{-\tan y_r}\right)-j\ln\left(1+e^{-\tan y_s}\right)$

The Credible Intervals on the basis of rest of the Priors have been presented in the appendix.

6. Posterior Predictive Distributions and Intervals

The Posterior Predictive Distribution is used to make predictions of future observations, based on the inferences drawn from the data at hand. It can be defined as:

$$p\left(x|\mathrm{y}\right)=\int_0^\infty p(\theta|\mathrm{y})f\left(x;\theta\right)d\theta \qquad (6.1)$$

where x = y_{n+1} be the future observation given the sample information $y = y_1$, y_2, ..., y_n, from the model (3.5), (3.6) and (3.7). The Posterior Predictive Distribution under Uniform Prior based on left censored sample is,

$$p(x|\mathrm{y}) = \frac{(n-r+1)e^{-\tan x}\sec^2(x)\left\{T_{1j}\right\}^{n-r+1}}{\left(1+e^{-\tan x}\right)\left\{T_{1j}-\ln\left(1+e^{-\tan x}\right)\right\}^{n-r+1}} \qquad (6.2)$$

The Posterior Predictive Distribution under Uniform Prior based on Singly Type II censored sample is,

$$p(x|\mathrm{y}) = \frac{\Gamma(m+2)e^{-\tan x}\sec^2(x)}{A_{1j}\left(1+e^{-\tan x}\right)}\sum_{j=0}^{n-m}(-1)^j\binom{n-m}{j}\left\{T_{2j}-\ln\left(1+e^{-\tan x}\right)\right\}^{-m-2} \qquad (6.3)$$

The Posterior Predictive Distribution under Uniform Prior based on Doubly Type II censored sample is,

$$p(x|\mathrm{y}) = \frac{\Gamma(k+2)e^{-\tan x}\sec^2(x)}{A_{2j}\left(1+e^{-\tan x}\right)}\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\left\{T_{3j}-\ln\left(1+e^{-\tan x}\right)\right\}^{-k-2} \qquad (6.4)$$

The Posterior Predictive interval for prediction of the future observation of the variable following Burr Type V Distribution can be defined as:

$$\int_{-\pi/2}^{L} p(x|\mathrm{y})dx = \frac{\alpha}{2}, \qquad \int_{U}^{\pi/2} p(x|\mathrm{y})dx = \frac{\alpha}{2}$$

The Posterior Predictive Interval for the future observation from the Burr Type V Distribution under Uniform Prior on the basis of left censored samples can be obtained by solving the following two equations.

$$\left\{\frac{T_{1j}}{T_{1j}-\ln\left(1+e^{-\tan L}\right)}\right\}^{n-r+1} = 1-\frac{\alpha}{2} \qquad \text{and} \qquad \left\{\frac{T_{1j}}{T_{1j}-\ln\left(1+e^{-\tan U}\right)}\right\}^{n-r+1} = \frac{\alpha}{2}$$

The Posterior Predictive Interval under Uniform Prior using Singly Type II censored sample can be derived as:

$$\frac{\Gamma(m+2)}{A_{1j}(m+1)}\sum_{j=0}^{n-m}(-1)^j\binom{n-m}{j}\left[\left\{T_{2j}\right\}^{-m-1}-\left\{T_{2j}-\ln\left(1+e^{-\tan L}\right)\right\}^{-m-1}\right] = \frac{\alpha}{2}$$

$$\frac{\Gamma(m+2)}{A_{1j}(m+1)}\sum_{j=0}^{n-m}(-1)^j\binom{n-m}{j}\left[\left\{T_{2j}\right\}^{-m-1}-\left\{T_{2j}-\ln\left(1+e^{-\tan U}\right)\right\}^{-m-1}\right] = 1-\frac{\alpha}{2}$$

The Posterior Predictive Interval under Uniform Prior using Doubly Type II censored sample can be given as:

$$\frac{\Gamma(k+2)}{A_{2j}(k+1)}\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\left[\{T_{3j}\}^{-k-1}-\left\{T_{3j}-\ln\left(1+e^{-\tan L}\right)\right\}^{-k-1}\right]=\frac{\alpha}{2}$$

$$\frac{\Gamma(k+2)}{A_{2j}(k+1)}\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\left[\{T_{3j}\}^{-k-1}-\left\{T_{3j}-\ln\left(1+e^{-\tan U}\right)\right\}^{-k-1}\right]=1-\frac{\alpha}{2}$$

As in some cases, the closed form expressions for L and U cannot be obtained, we have used the numerical solutions of these limits by Iterative Methods. The Posterior Predictive Intervals based on remaining Priors have been reported in the appendix.

7. Prior Elicitation

Elicitation is a process that can be used to formulate an expert's knowledge or belief about a certain quantity in probabilistic form. In the context of Bayesian inference, it can be regarded as a method for specifying the Prior Distribution for one or more unknown parameters of a statistical model. In literature, there are various methods for Elicitation of a Prior Distribution. We have used the method of Elicitation proposed by Aslam (2003) which is based on the Prior Predictive approach. In order to Elicit the Prior by the mentioned method, we have to derive the Prior Predictive Distribution for each Informative Prior. The Prior Predictive Distribution can be defined as:

$$g(y) = \int_0^\infty g(\theta)f(y|\theta)d(\theta) \tag{7.1}$$

According to (7.1), the Prior Predictive Distribution under Gamma Prior is,

$$g(y)=\frac{ab^a e^{-\tan y}\sec^2 y}{\left(1+e^{-\tan y}\right)\left\{b-\ln\left(1+e^{-\tan y}\right)\right\}^{a+1}} \tag{7.2}$$

As we have to Elicit two Hyper-parameters so we have to consider two integrals. The set of Hyper-parameters with minimum values has been chosen to be the Elicited values of the Hyper-parameters. By considering the Prior Predictive Distribution in (7.2), we have assumed the expert's probabilities to be 0.30 for each integral. We considered the following integrals:

$$\int_{-1.1}^{0.0} g(y)=0.30,\ \int_{0.1}^{1.0} g(y)=0.30\,.$$

Now, these integrals have been simultaneously solved through a program written in SAS package using the "PROC SYSLIN" command and the Elicited values of the Hyper-parameters have been found to be $(a,b)=(0.687645, 0.478683)$.

Again, by (7.1), the Prior Predictive Distribution under Inverse Levy Prior is,

$$g(y)=\frac{\Gamma(3/2)c^{1/2}e^{-\tan y}\sec^2 y}{(2\pi)^{1/2}\left(1+e^{-\tan y}\right)\left\{\frac{c}{2}-\ln\left(1+e^{-\tan y}\right)\right\}^{3/2}} \tag{7.3}$$

Following the similar process as mentioned above, the Elicited value of the Hyper-parameter is $c=1.026478$.

8. Numerical Results and Discussions

As the analytical comparisons among the Estimators derived above are not possible, we have carried out a Simulation study to make the comparisons numerically. The performance of the Point Estimates has been compared in terms of magnitudes of Posterior Risks associated with each estimate. While, the Interval Estimators have been judged against their widths. The samples of size n = 20, 30, 50, 100 and 150 have been generated by inverse transformation method from the Burr Type V Distribution by using the function $X=Tan^{-1}\left\{-\ln\left(U^{-1/\theta}-1\right)\right\}$ where U is a uniformly distributed random variable over $(0,1)$. All the samples have been assumed to be 15% censored. The parametric space includes: $\theta\in(0.50, 1.00, 1.50)$.

By assuming different values of the parameters in GELF and LLF it has been found that the results for p = -0.50 and ω = 1 are the most precise. So, the Bayes Estimates and Posterior Risks have been presented for these parametric values only. All the results have been replicated 10,000 times and their average has been presented in the following Tables. The amounts of Posterior Risks have been given in parenthesis in Tables.

It is immediate from Tables 1-9 that the estimated value of the parameter becomes close to the true value as sample size increases. The parameter has been under estimated in almost all the cases. The degree of under estimation is more serious for larger values of the parameter. It can also be assessed that the amounts of Posterior Risks tend to decrease by increasing the sample size. This simply indicates that the Estimators are consistent. The estimates under Inverse Levy Prior are having the least amounts of Posterior Risks. The larger choice of true parametric values inflates the magnitudes of Risks associated with estimates under LLF. How, the amounts of Risks under GELF are independent of the choice of true parametric values. It is interesting to note that the performance of the estimates under Informative Priors is better than those under Non-informative Priors. Similarly, the estimates under GELF are significantly superior to those under LLF. The level of efficiency achieved under n = 20 for GELF can be obtained for n = 100 to 150 in case of LLF. So, implementation of the GELF can save 80% of the time and cost on experimentation. It can also be observed that the performance of the Estimates using Singly Type II censored samples is better than those based on rest of censoring schemes. Therefore, the Simulation study suggests the use of GELF under Inverse Levy Prior for estimation.

Here, LL: Lower Limit; UL: Upper Limit; LC: Left Censoring; STTC: Singly Type II Censoring; DTTC: Doubly Type II Censoring.

Tables 10-11 contain the 95% Bayesian Credible and Posterior Predictive Intervals under different Priors. The widths of intervals are declining with increasing the sample size. Both of the Interval Estimates tend to be more specific under Informative Priors especially in case of Inverse Levy Prior. Further, the estimates based on the Singly Type II censored samples are more precise than their counterparts. So, the Interval Estimation replicated the findings reached in case of the Point Estimation. The most efficient Interval Estimation (Credible and Prediction) has been observed under the assumption of Inverse Levy Prior using Singly Type II censoring.

8.1 Concluding Remarks: The results from the Tables 1-11 suggest that the efficiency of the Estimates (Point and Interval) increases by increasing the sample size. The performance of the Informative Priors is found to be superior to Non-informative Priors. Moreover, the most efficient Bayes Estimation of the parameter of the Burr Type V Distribution has been observed under the assumption of Inverse Levy Prior along with Generalized Entropy Loss Function.

9. Real Life Example

In order to discuss the practical applicability of the results obtained under above sections, the real life data presented by Bekker et al. (2000) regarding cancer Survival times have been used for analysis.

From Tables 12-14, it can be assessed that the findings from the analysis of the real life data are completely in accordance with those of the Simulation study. Here, the Point Estimation is more accurate under Inverse Levy Prior based GELF using Singly Type II censored samples. The shortest widths of the Bayesian Credible Intervals have been reported under Inverse Levy Prior using Singly Type II censored samples. The Posterior Predictions also tend to be more precise under the said case/combination.

10. Conclusion

The paper discusses the Bayesian Estimation and Prediction of the Burr Type V Distribution under different censoring schemes using GELF and LLF based on a class of Priors. It has been assessed that the Point Estimation of the parameter of the said Distribution is more precise under the assumption of Inverse Levy Prior using GELF on the basis of Singly Type II censored samples. The use of GELF significantly reduces the cost and time of experimentation. The Bayesian Credible Intervals and Posterior Predictive Intervals tend to be more specific again under Levy Prior. The real life data analysis further strengthened these findings. The proposed Estimators are consistent and can work efficiently even in the small samples. The findings of the article are useful for analysts from various fields dealing with analysis of the censored data under a Life-time Model.

Acknowledgement

Special thanks to referees for their precious comments/suggestions for the improvement of the quality of the paper.

Appendix

Derivation of Posterior Distributions, Bayes Estimators, Risks, Credible Intervals and Posterior Predictive Intervals under Jeffreys, Gamma and Inverse Levy Priors

As the re-written of the different formulas with little modifications is not economical, we have derived various results in a generalized version. The results for Jeffreys, Gamma and Inverse Levy Priors can be obtained as special cases by putting $t_1 = 1, t_2 = 0, t_3 = 0$; $t_1 = 0, t_2 = 1, t_3 = 0$ and $t_1 = 0, t_2 = 0, t_3 = 1$, respectively, in the following expressions.

The Generalized Posterior Distribution based on left censored samples is,

$$p(\theta|\text{y}) = \frac{\left\{\xi(x_{1i}) + t_2 b + 0.5t_3 c\right\}^{n-r-t_1+t_2(a-1)-0.5t_3+1}}{\Gamma\left(n-r-t_1+t_2(a-1)-0.5t_3+1\right)} \theta^{n-r-t_1+t_2(a-1)-0.5t_3} \exp\left[-\theta\left\{\xi(x_{1i}) + t_2 b + 0.5t_3 c\right\}\right] \quad \text{(A1)}$$

where $\xi(x_{1i}) = -r\ln\left(1+e^{-\tan y_{r+1}}\right) - \sum_{i=r+1}^{n} \ln\left(1+e^{-\tan y_i}\right)$

The Generalized Posterior Distribution based on Singly Type II censored samples is,

$$p(\theta|\text{y}) = \frac{1}{A_{1j}} \sum_{j=0}^{n-m} (-1)^j \binom{n-m}{j} \theta^{m-t_1+t_2(a-1)-0.5t_3} \exp\left[-\theta\left\{\xi(x_{2i}) + t_2 b + 0.5t_3 c\right\}\right] \quad \text{(A2)}$$

where

$$A_{1j} = \sum_{j=0}^{n-m} (-1)^j \binom{n-m}{j} \Gamma\left(m-t_1+t_2(a-1)-0.5t_3+1\right) \left\{\xi(x_{2i}) + t_2 b + 0.5t_3 c\right\}^{-m-\left\{-t_1+t_2(a-1)-0.5t_3\right\}-1}$$

and $\xi(x_{2i}) = -\sum_{i=1}^{m} \ln\left(1+e^{-\tan y_i}\right) - j\ln\left(1+e^{-\tan y_m}\right)$

The Generalized Posterior Distribution based on Doubly Type II censored samples is,

$$p(\theta|\text{y}) = \frac{1}{A_{2j}} \sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \theta^{k-t_1+t_2(a-1)-0.5t_3} \exp\left[-\theta\left\{\xi(x_{3i}) + t_2 b + 0.5t_3 c\right\}\right] \quad \text{(A3)}$$

where

$$A_{2j} = \sum_{j=0}^{n-s} (-1)^j \binom{n-s}{j} \Gamma\left(k-t_1+t_2(a-1)-0.5t_3+1\right) \left\{\xi(x_{3i}) + t_2 b + 0.5t_3 c\right\}^{-k-\left\{-t_1+t_2(a-1)-0.5t_3\right\}-1}$$

and $\xi\left(x_{3i}\right)=-\sum_{i=r}^{s}\ln\left(1+e^{-\tan y_i}\right)-\left(r-1\right)\ln\left(1+e^{-\tan y_r}\right)-j\ln\left(1+e^{-\tan y_s}\right)$

The Generalized Bayes Estimator and Posterior Risk for Left censored samples using GELF are,

$$\theta_{GELF}=\left[\frac{\Gamma\left(n-r-t_1+t_2\left(a-1\right)-0.5t_3-p+1\right)\left\{\xi\left(x_{1i}\right)+t_2b+0.5t_3c\right\}^{p}}{\Gamma\left(n-r-t_1+t_2\left(a-1\right)-0.5t_3+1\right)}\right]^{-\frac{1}{p}}$$

$$\rho\left(\theta_{GELF}\right)=\ln\left[\frac{\Gamma\left(n-r-t_1+t_2\left(a-1\right)-0.5t_3-p+1\right)\left\{\xi\left(x_{1i}\right)+t_2b+0.5t_3c\right\}^{p}}{\Gamma\left(n-r-t_1+t_2\left(a-1\right)-0.5t_3+1\right)}\right]+p\left[\psi\left(n-r-t_1+t_2\left(a-1\right)-0.5t_3+1\right)-\ln\left\{\xi\left(x_{1i}\right)+t_2b+0.5t_3c\right\}\right]$$

where $\psi\left(z\right)$ is a Di-gamma Function.

The Generalized Bayes Estimator and Posterior Risk for Left censored samples using LLF are,

$$\theta_{LLF}=-\frac{n-r-t_1+t_2\left(a-1\right)-0.5t_3+1}{\omega}\ln\left\{\frac{\xi\left(x_{1i}\right)+t_2b+0.5t_3c}{\xi\left(x_{1i}\right)+t_2b+0.5t_3c+\omega}\right\}$$

$$\rho\left(\theta_{LLF}\right)=\left(n-r-t_1+t_2\left(a-1\right)-0.5t_3+1\right)\ln\left\{\frac{\xi\left(x_{1i}\right)+t_2b+0.5t_3c}{\xi\left(x_{1i}\right)+t_2b+0.5t_3c+\omega}\right\}+\frac{\omega\left(n-r-t_1+t_2\left(a-1\right)-0.5t_3+1\right)}{\xi\left(x_{1i}\right)+t_2b+0.5t_3c}$$

The Generalized Bayes Estimator and Posterior Risk for Singly Type II censored samples using GELF are,

$$\theta_{GELF}=\left[\frac{\Gamma\left(m-t_1+t_2\left(a-1\right)-0.5t_3-p+1\right)}{A_{1j}}\sum_{j=0}^{n-m}\left(-1\right)^{j}\binom{n-m}{j}\left\{\xi\left(x_{2i}\right)+t_2b+0.5t_3c+\omega\right\}^{p-m-\left\{-t_1+t_2\left(a-1\right)-0.5t_3\right\}-1}\right]$$

$$\rho\left(\theta_{GELF}\right)=\ln\left[\frac{\Gamma\left(m-t_1+t_2\left(a-1\right)-0.5t_3-p+1\right)}{A_{1j}}\sum_{j=0}^{n-m}\left(-1\right)^{j}\binom{n-m}{j}\left\{\xi\left(x_{2i}\right)+t_2b+0.5t_3c+\omega\right\}^{p-m-\left\{-t_1+t_2\left(a-1\right)-0.5t_3\right\}-1}\right]$$
$$+p\left[\frac{1}{A_{1j}}\sum_{j=0}^{n-m}\left(-1\right)^{j}\binom{n-m}{j}\left\{\psi\left(m-t_1+t_2\left(a-1\right)-0.5t_3+1\right)-\ln\left\{\xi\left(x_{2i}\right)+t_2b+0.5t_3c+\omega\right\}\right\}\right]$$

The Generalized Bayes Estimator and Posterior Risk for Singly Type II censored samples using LLF are,

$$\theta_{LLF}=-\frac{1}{\omega}\ln\left[\frac{\Gamma\left(m-t_1+t_2\left(a-1\right)-0.5t_3+1\right)}{A_{1j}}\sum_{j=0}^{n-m}\left(-1\right)^j\binom{n-m}{j}\left\{\omega+\xi\left(x_{2i}\right)+t_2b+0.5t_3c\right\}^{-m-\left\{-t_1+t_2\left(a-1\right)-0.5t_3\right\}-1}\right]$$

$$\rho\left(\theta_{LLF}\right)=\ln\left[\frac{\Gamma\left(m-t_1+t_2\left(a-1\right)-0.5t_3+1\right)}{A_{1j}}\sum_{j=0}^{n-m}\left(-1\right)^j\binom{n-m}{j}\left\{\omega+\xi\left(x_{2i}\right)+t_2b+0.5t_3c\right\}^{-m-\left\{-t_1+t_2\left(a-1\right)-0.5t_3\right\}-1}\right]$$
$$+\frac{\omega\Gamma\left(m-t_1+t_2\left(a-1\right)-0.5t_3+2\right)}{A_{1j}}\sum_{j=0}^{n-m}\left(-1\right)^j\binom{n-m}{j}\left\{\xi\left(x_{2i}\right)+t_2b+0.5t_3c\right\}^{-m-\left\{-t_1+t_2\left(a-1\right)-0.5t_3\right\}-2}$$

The Generalized Bayes Estimator and Posterior Risk for Doubly Type II censored samples using GELF are,

$$\theta_{GELF}=\left[\frac{\Gamma\left(k-t_1+t_2\left(a-1\right)-0.5t_3-p+1\right)}{A_{2j}}\sum_{j=0}^{n-s}\left(-1\right)^j\binom{n-s}{j}\left\{\xi\left(x_{3i}\right)+t_2b+0.5t_3c\right\}^{p-k-\left\{-t_1+t_2\left(a-1\right)-0.5t_3\right\}-1}\right]$$

$$\rho\left(\theta_{GELF}\right)=\ln\left[\frac{\Gamma\left(k-t_1+t_2\left(a-1\right)-0.5t_3-p+1\right)}{A_{2j}}\sum_{j=0}^{n-s}\left(-1\right)^j\binom{n-s}{j}\left\{\xi\left(x_{3i}\right)+t_2b+0.5t_3c\right\}^{p-k-\left\{-t_1+t_2\left(a-1\right)-0.5t_3\right\}-1}\right]$$
$$+p\left[\frac{1}{A_{2j}}\sum_{j=0}^{n-s}\left(-1\right)^j\binom{n-s}{j}\left\{\psi\left(k-t_1+t_2\left(a-1\right)-0.5t_3+1\right)-\ln\left\{\xi\left(x_{3i}\right)+t_2b+0.5t_3c\right\}\right\}\right]$$

The Generalized Bayes Estimator and Posterior Risk for Doubly Type II censored samples using LLF are,

$$\theta_{LLF}=-\frac{1}{\omega}\ln\left[\frac{\Gamma\left(k-t_1+t_2\left(a-1\right)-0.5t_3+1\right)}{A_{2j}}\sum_{j=0}^{n-s}\left(-1\right)^j\binom{n-s}{j}\left\{\omega+\xi\left(x_{3i}\right)+t_2b+0.5t_3c\right\}^{-k-\left\{-t_1+t_2\left(a-1\right)-0.5t_3\right\}-1}\right]$$

$$\rho\left(\theta_{LLF}\right)=\ln\left[\frac{\Gamma\left(k-t_1+t_2\left(a-1\right)-0.5t_3+1\right)}{A_{2j}}\sum_{j=0}^{n-s}\left(-1\right)^j\binom{n-s}{j}\left\{\omega+\xi\left(x_{3i}\right)+t_2b+0.5t_3c\right\}^{-k-\left\{-t_1+t_2\left(a-1\right)-0.5t_3\right\}-1}\right]$$
$$+\frac{\omega\Gamma\left(k-t_1+t_2\left(a-1\right)-0.5t_3+2\right)}{A_{2j}}\sum_{j=0}^{n-s}\left(-1\right)^j\binom{n-s}{j}\left\{\xi\left(x_{3i}\right)+t_2b+0.5t_3c\right\}^{-k-\left\{-t_1+t_2\left(a-1\right)-0.5t_3\right\}-2}$$

The Generalized Credible Interval for θ based on Left, Singly Type II and Doubly Type II censored samples have respectively been presented in the following.

$$\frac{\chi^2_{\left(1-\alpha/2\right)\left\{2\left(n-r-t_1+t_2(a-1)-0.5t_3+1\right)\right\}}}{2\left\{\xi\left(x_{1i}\right)+t_2b+0.5t_3c\right\}}<\theta<\frac{\chi^2_{\left(\alpha/2\right)\left\{2\left(n-r-t_1+t_2(a-1)-0.5t_3+1\right)\right\}}}{2\left\{\xi\left(x_{1i}\right)+t_2b+0.5t_3c\right\}}$$

$$\frac{\chi^2_{\left(1-\alpha/2\right)\left\{2\left(m-t_1+t_2(a-1)-0.5t_3+1\right)\right\}}\sum_{j=0}^{n-m}(-1)^j\binom{n-m}{j}\left\{\xi\left(x_{2i}\right)+t_2b+0.5t_3c\right\}^{-m-\left\{-t_1+t_2(a-1)-0.5t_3\right\}-1}}{2\sum_{j=0}^{n-m}(-1)^j\binom{n-m}{j}\left\{\xi\left(x_{2i}\right)+t_2b+0.5t_3c\right\}^{-m-\left\{-t_1+t_2(a-1)-0.5t_3\right\}-2}}<\theta<\frac{\chi^2_{\left(\alpha/2\right)\left\{2\left(m-t_1+t_2(a-1)-0.5t_3+1\right)\right\}}\sum_{j=0}^{n-m}(-1)^j\binom{n-m}{j}\left\{\xi\left(x_{2i}\right)+t_2b+0.5t_3c\right\}^{-m-\left\{-t_1+t_2(a-1)-0.5t_3\right\}-1}}{2\sum_{j=0}^{n-m}(-1)^j\binom{n-m}{j}\left\{\xi\left(x_{2i}\right)+t_2b+0.5t_3c\right\}^{-m-\left\{-t_1+t_2(a-1)-0.5t_3\right\}-2}}$$

$$\frac{\chi^2_{\left(1-\alpha/2\right)\left\{2\left(k-t_1+t_2(a-1)-0.5t_3+1\right)\right\}}\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\left\{\xi\left(x_{3i}\right)+t_2b+0.5t_3c\right\}^{-k-\left\{-t_1+t_2(a-1)-0.5t_3\right\}-1}}{2\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\left\{\xi\left(x_{3i}\right)+t_2b+0.5t_3c\right\}^{-k-\left\{-t_1+t_2(a-1)-0.5t_3\right\}-2}}<\theta<\frac{\chi^2_{\left(\alpha/2\right)\left\{2\left(k-t_1+t_2(a-1)-0.5t_3+1\right)\right\}}\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\left\{\xi\left(x_{3i}\right)+t_2b+0.5t_3c\right\}^{-k-\left\{-t_1+t_2(a-1)-0.5t_3\right\}-1}}{2\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\left\{\xi\left(x_{3i}\right)+t_2b+0.5t_3c\right\}^{-k-\left\{-t_1+t_2(a-1)-0.5t_3\right\}-2}}$$

The Generalized Posterior Predictive Distribution based on Left censored sample is,

$$p\left(x\middle|\mathrm{y}\right)=\frac{\left(n-r-t_1+t_2\left(a-1\right)-0.5t_3+1\right)e^{-\tan x}\sec^2\left(x\right)\left\{\xi\left(x_{1i}\right)+t_2b+0.5t_3c\right\}^{n-r-t_1+t_2(a-1)-0.5t_3+1}}{\left(1+e^{-\tan x}\right)\left\{\xi\left(x_{1i}\right)+t_2b+0.5t_3c-\ln\left(1+e^{-\tan x}\right)\right\}^{n-r-t_1+t_2(a-1)-0.5t_3+1}}$$

The Generalized Posterior Predictive Distribution based on Singly Type II censored sample is,

$$p\left(x\middle|\mathrm{y}\right)=\frac{\Gamma\left(m-t_1+t_2\left(a-1\right)-0.5t_3+2\right)e^{-\tan x}\sec^2\left(x\right)}{A_{1j}\left(1+e^{-\tan x}\right)}\sum_{j=0}^{n-m}(-1)^j\binom{n-m}{j}\left\{\xi\left(x_{2i}\right)+t_2b+0.5t_3c-\ln\left(1+e^{-\tan x}\right)\right\}^{-m-\left\{-t_1+t_2(a-1)-0.5t_3\right\}-2}$$

The Generalized Posterior Predictive Distribution based on Doubly Type II censored sample is,

$$p\left(x\middle|\mathrm{y}\right)=\frac{\Gamma\left(k-t_1+t_2\left(a-1\right)-0.5t_3+2\right)e^{-\tan x}\sec^2\left(x\right)}{A_{2j}\left(1+e^{-\tan x}\right)}\sum_{j=0}^{n-s}(-1)^j\binom{n-s}{j}\left\{\xi\left(x_{3i}\right)+t_2b+0.5t_3c-\ln\left(1+e^{-\tan x}\right)\right\}^{-k-\left\{-t_1+t_2(a-1)-0.5t_3\right\}-2}$$

The Generalized Posterior Predictive Interval for the future observation from the Burr Type V Distribution on the basis of Left censored samples can be obtained by solving the following two equations.

$$\left\{\frac{\xi\left(x_{1i}\right)+t_2b+0.5t_3c}{\xi\left(x_{1i}\right)+t_2b+0.5t_3c-\ln\left(1+e^{-\tan L}\right)}\right\}^{n-r-t_1+t_2(a-1)-0.5t_3+1}=1-\frac{\alpha}{2}$$

$$\left\{\frac{\xi\left(x_{1i}\right)+t_2b+0.5t_3c}{\xi\left(x_{1i}\right)+t_2b+0.5t_3c-\ln\left(1+e^{-\tan U}\right)}\right\}^{n-r-t_1+t_2(a-1)-0.5t_3+1}=\frac{\alpha}{2}$$

The Generalized Posterior Predictive Interval using Singly Type II censored sample can be derived as:

$$\frac{\Gamma\left(m-t_1+t_2\left(a-1\right)-0.5t_3+2\right)}{A_{1j}\left(m-t_1+t_2\left(a-1\right)-0.5t_3+1\right)}\sum_{j=0}^{n-m}\left(-1\right)^j\binom{n-m}{j}\left[\left\{\xi\left(x_{2i}\right)+t_2b+0.5t_3c\right\}^{-m-\left\{-t_1+t_2\left(a-1\right)-0.5t_3\right\}-1}-\left\{\xi\left(x_{2i}\right)+t_2b+0.5t_3c-\ln\left(1+e^{-\tan L}\right)\right\}^{-m-\left\{-t_1+t_2\left(a-1\right)-0.5t_3\right\}-1}\right]$$

$$\frac{\Gamma\left(m-t_1+t_2\left(a-1\right)-0.5t_3+2\right)}{A_{1j}\left(m-t_1+t_2\left(a-1\right)-0.5t_3+1\right)}\sum_{j=0}^{n-m}\left(-1\right)^j\binom{n-m}{j}\left[\left\{\xi\left(x_{2i}\right)+t_2b+0.5t_3c\right\}^{-m-\left\{-t_1+t_2\left(a-1\right)-0.5t_3\right\}-1}-\left\{\xi\left(x_{2i}\right)+t_2b+0.5t_3c-\ln\left(1+e^{-\tan U}\right)\right\}^{-m-\left\{-t_1+t_2\left(a-1\right)-0.5t_3\right\}-1}\right]$$

The Generalized Posterior Predictive Interval using Doubly Type II censored sample can be given as:

$$\frac{\Gamma\left(k-t_1+t_2\left(a-1\right)-0.5t_3+2\right)}{A_{2j}\left(k-t_1+t_2\left(a-1\right)-0.5t_3+1\right)}\sum_{j=0}^{n-s}\left(-1\right)^j\binom{n-s}{j}\left[\left\{\xi\left(x_{3i}\right)+t_2b+0.5t_3c\right\}^{-k-\left\{-t_1+t_2\left(a-1\right)-0.5t_3\right\}-1}-\left\{\xi\left(x_{3i}\right)+t_2b+0.5t_3c-\ln\left(1+e^{-\tan L}\right)\right\}^{-k-\left\{-t_1+t_2\left(a-1\right)-0.5t_3\right\}-1}\right]$$

$$\frac{\Gamma\left(k-t_1+t_2\left(a-1\right)-0.5t_3+2\right)}{A_{2j}\left(k-t_1+t_2\left(a-1\right)-0.5t_3+1\right)}\sum_{j=0}^{n-s}\left(-1\right)^j\binom{n-s}{j}\left[\left\{\xi\left(x_{3i}\right)+t_2b+0.5t_3c\right\}^{-k-\left\{-t_1+t_2\left(a-1\right)-0.5t_3\right\}-1}-\left\{\xi\left(x_{3i}\right)+t_2b+0.5t_3c-\ln\left(1+e^{-\tan U}\right)\right\}^{-k-\left\{-t_1+t_2\left(a-1\right)-0.5t_3\right\}-1}\right]$$

Table 1: Bayes Estimates and Posterior Risks under Left censored samples for θ = 0.50

n	Uniform		Jeffreys		Gamma		Inverse Levy	
	GELF	LLF	GELF	LLF	GELF	LLF	GELF	LLF
20	0.44783 (0.01628)	0.42856 (0.05808)	0.43887 (0.01596)	0.41999 (0.05692)	0.44140 (0.01449)	0.42675 (0.04789)	0.44582 (0.01228)	0.43102 (0.04587)
30	0.47014 (0.00817)	0.45840 (0.03303)	0.46074 (0.00801)	0.44923 (0.03237)	0.46340 (0.00727)	0.45647 (0.02723)	0.46803 (0.00616)	0.46103 (0.02608)
50	0.48717 (0.00409)	0.48047 (0.01806)	0.47742 (0.00401)	0.47086 (0.01770)	0.48018 (0.00364)	0.47845 (0.01489)	0.48498 (0.00309)	0.48323 (0.01427)
100	0.49435 (0.00273)	0.48966 (0.01248)	0.48446 (0.00268)	0.47986 (0.01224)	0.48726 (0.00243)	0.48759 (0.01029)	0.49213 (0.00206)	0.49247 (0.00986)
150	0.49531 (0.00205)	0.49178 (0.00932)	0.48540 (0.00201)	0.48194 (0.00914)	0.48820 (0.00183)	0.48971 (0.00769)	0.49308 (0.00155)	0.49460 (0.00736)

Table 2: Bayes Estimates and Posterior Risks under Left censored samples for θ = 1.00

n	Uniform		Jeffreys		Gamma		Inverse Levy	
	GELF	LLF	GELF	LLF	GELF	LLF	GELF	LLF
20	0.88771 (0.01628)	0.84952 (0.07585)	0.86995 (0.01596)	0.83253 (0.07433)	0.87497 (0.01449)	0.84594 (0.06254)	0.88372 (0.01228)	0.85440 (0.05990)
30	0.93194 (0.00817)	0.90867 (0.04313)	0.91330 (0.00801)	0.89049 (0.04227)	0.91857 (0.00727)	0.90484 (0.03556)	0.92776 (0.00616)	0.91389 (0.03406)
50	0.96569 (0.00409)	0.95242 (0.02359)	0.94637 (0.00401)	0.93337 (0.02312)	0.95183 (0.00364)	0.94841 (0.01945)	0.96135 (0.00309)	0.95789 (0.01863)
100	0.97993 (0.00273)	0.97062 (0.01630)	0.96033 (0.00268)	0.95121 (0.01598)	0.96588 (0.00243)	0.96653 (0.01344)	0.97553 (0.00206)	0.97620 (0.01288)
150	0.98182 (0.00205)	0.97483 (0.01217)	0.96219 (0.00201)	0.95533 (0.01193)	0.96774 (0.00183)	0.97072 (0.01004)	0.97742 (0.00155)	0.98043 (0.00961)

Table 3: Bayes Estimates and Posterior Risks under Left censored samples for θ = 1.50

n	Uniform		Jeffreys		Gamma		Inverse Levy	
	GELF	**LLF**	**GELF**	**LLF**	**GELF**	**LLF**	**GELF**	**LLF**
20	1.34288 (0.01628)	1.28511 (0.12511)	1.31603 (0.01596)	1.25941 (0.12260)	1.32362 (0.01449)	1.27969 (0.10316)	1.33686 (0.01228)	1.29249 (0.09880)
30	1.40979 (0.00817)	1.37459 (0.07114)	1.38159 (0.00801)	1.34710 (0.06972)	1.38957 (0.00727)	1.36880 (0.05866)	1.40347 (0.00616)	1.38248 (0.05618)
50	1.46084 (0.00409)	1.44078 (0.03891)	1.43163 (0.00401)	1.41196 (0.03813)	1.43989 (0.00364)	1.43471 (0.03208)	1.45429 (0.00309)	1.44906 (0.03073)
100	1.48239 (0.00273)	1.46831 (0.02689)	1.45275 (0.00268)	1.43894 (0.02635)	1.46113 (0.00243)	1.46212 (0.02217)	1.47574 (0.00206)	1.47674 (0.02124)
150	1.48526 (0.00205)	1.47468 (0.02008)	1.45555 (0.00201)	1.44519 (0.01968)	1.46395 (0.00183)	1.46847 (0.01656)	1.47859 (0.00155)	1.48315 (0.01586)

Table 4: Bayes Estimates and Posterior Risks under Singly Type II censored samples for θ = 0.50

n	Uniform		Jeffreys		Gamma		Inverse Levy	
	GELF	**LLF**	**GELF**	**LLF**	**GELF**	**LLF**	**GELF**	**LLF**
20	0.45086 (0.01590)	0.43147 (0.05671)	0.44185 (0.01558)	0.42284 (0.05557)	0.44440 (0.01415)	0.42965 (0.04676)	0.44884 (0.01199)	0.43394 (0.04478)
30	0.47333 (0.00798)	0.46151 (0.03224)	0.46386 (0.00782)	0.45228 (0.03160)	0.46654 (0.00710)	0.45956 (0.02659)	0.47120 (0.00602)	0.46416 (0.02547)
50	0.49047 (0.00400)	0.48373 (0.01764)	0.48066 (0.00392)	0.47406 (0.01728)	0.48343 (0.00356)	0.48169 (0.01454)	0.48827 (0.00302)	0.48651 (0.01393)
100	0.49770 (0.00267)	0.49298 (0.01219)	0.48775 (0.00261)	0.48312 (0.01195)	0.49056 (0.00237)	0.49090 (0.01005)	0.49547 (0.00201)	0.49581 (0.00963)
150	0.49866 (0.00200)	0.49511 (0.00910)	0.48869 (0.00196)	0.48521 (0.00892)	0.49151 (0.00178)	0.49303 (0.00750)	0.49643 (0.00151)	0.49796 (0.00719)

Table 5: Bayes Estimates and Posterior Risks under Singly Type II censored samples for $\theta = 1.00$

n	Uniform		Jeffreys		Gamma		Inverse Levy	
	GELF	**LLF**	**GELF**	**LLF**	**GELF**	**LLF**	**GELF**	**LLF**
20	0.89373 (0.01590)	0.85528 (0.07405)	0.87585 (0.01558)	0.83817 (0.07257)	0.88091 (0.01415)	0.85167 (0.06106)	0.88972 (0.01199)	0.86019 (0.05848)
30	0.93826 (0.00798)	0.91483 (0.04211)	0.91949 (0.00782)	0.89653 (0.04127)	0.92480 (0.00710)	0.91097 (0.03472)	0.93405 (0.00602)	0.92008 (0.03326)
50	0.97223 (0.00400)	0.95888 (0.02303)	0.95279 (0.00392)	0.93970 (0.02257)	0.95829 (0.00356)	0.95484 (0.01899)	0.96787 (0.00302)	0.96439 (0.01819)
100	0.98658 (0.00267)	0.97720 (0.01592)	0.96684 (0.00261)	0.95766 (0.01560)	0.97242 (0.00237)	0.97308 (0.01313)	0.98215 (0.00201)	0.98282 (0.01257)
150	0.98848 (0.00200)	0.98144 (0.01189)	0.96871 (0.00196)	0.96181 (0.01165)	0.97430 (0.00178)	0.97731 (0.00980)	0.98404 (0.00151)	0.98708 (0.00939)

Table 6: Bayes Estimates and Posterior Risks under Singly Type II censored samples for $\theta = 1.50$

n	Uniform		Jeffreys		Gamma		Inverse Levy	
	GELF	**LLF**	**GELF**	**LLF**	**GELF**	**LLF**	**GELF**	**LLF**
20	1.35199 (0.01590)	1.29382 (0.12214)	1.32495 (0.01558)	1.26795 (0.11970)	1.33260 (0.01415)	1.28837 (0.10071)	1.34592 (0.01199)	1.30125 (0.09646)
30	1.41935 (0.00798)	1.38391 (0.06945)	1.39096 (0.00782)	1.35623 (0.06807)	1.39899 (0.00710)	1.37808 (0.05727)	1.41298 (0.00602)	1.39186 (0.05485)
50	1.47075 (0.00400)	1.45055 (0.03799)	1.44134 (0.00392)	1.42154 (0.03723)	1.44966 (0.00356)	1.44444 (0.03132)	1.46415 (0.00302)	1.45888 (0.03000)
100	1.49245 (0.00267)	1.47827 (0.02626)	1.46260 (0.00261)	1.44870 (0.02573)	1.47104 (0.00237)	1.47204 (0.02165)	1.48575 (0.00201)	1.48676 (0.02074)
150	1.49533 (0.00200)	1.48468 (0.01960)	1.46542 (0.00196)	1.45499 (0.01921)	1.47388 (0.00178)	1.47842 (0.01616)	1.48862 (0.00151)	1.49321 (0.01548)

Table 7: Bayes Estimates and Posterior Risks under Doubly Type II censored samples for θ = 0.50

n	Uniform		Jeffreys		Gamma		Inverse Levy	
	GELF	**LLF**	**GELF**	**LLF**	**GELF**	**LLF**	**GELF**	**LLF**
20	0.44436 (0.01657)	0.42524 (0.05911)	0.43547 (0.01624)	0.41674 (0.05792)	0.43798 (0.01474)	0.42345 (0.04874)	0.44236 (0.01250)	0.42768 (0.04668)
30	0.46650 (0.00832)	0.45485 (0.03361)	0.45717 (0.00815)	0.44575 (0.03294)	0.45981 (0.00740)	0.45293 (0.02771)	0.46440 (0.00627)	0.45746 (0.02654)
50	0.48339 (0.00417)	0.47675 (0.01838)	0.47372 (0.00408)	0.46722 (0.01801)	0.47646 (0.00371)	0.47474 (0.01516)	0.48122 (0.00314)	0.47949 (0.01452)
100	0.49052 (0.00278)	0.48586 (0.01271)	0.48071 (0.00273)	0.47614 (0.01245)	0.48349 (0.00247)	0.48381 (0.01048)	0.48832 (0.00210)	0.48865 (0.01003)
150	0.49147 (0.00209)	0.48797 (0.00949)	0.48164 (0.00205)	0.47821 (0.00930)	0.48442 (0.00186)	0.48591 (0.00782)	0.48926 (0.00157)	0.49077 (0.00749)

Table 8: Bayes Estimates and Posterior Risks under Doubly Type II censored samples for θ = 1.00

n	Uniform		Jeffreys		Gamma		Inverse Levy	
	GELF	**LLF**	**GELF**	**LLF**	**GELF**	**LLF**	**GELF**	**LLF**
20	0.88083 (0.01657)	0.84293 (0.07719)	0.86321 (0.01624)	0.82608 (0.07564)	0.86820 (0.01474)	0.83938 (0.06364)	0.87688 (0.01250)	0.84778 (0.06096)
30	0.92472 (0.00832)	0.90163 (0.04389)	0.90622 (0.00815)	0.88359 (0.04301)	0.91145 (0.00740)	0.89783 (0.03619)	0.92057 (0.00627)	0.90681 (0.03466)
50	0.95820 (0.00417)	0.94504 (0.02401)	0.93904 (0.00408)	0.92614 (0.02353)	0.94446 (0.00371)	0.94106 (0.01979)	0.95391 (0.00314)	0.95047 (0.01896)
100	0.97234 (0.00278)	0.96310 (0.01659)	0.95289 (0.00273)	0.94384 (0.01626)	0.95839 (0.00247)	0.95904 (0.01368)	0.96798 (0.00210)	0.96863 (0.01310)
150	0.97422 (0.00209)	0.96728 (0.01239)	0.95473 (0.00205)	0.94793 (0.01214)	0.96024 (0.00186)	0.96320 (0.01022)	0.96985 (0.00157)	0.97283 (0.00978)

Table 9: Bayes Estimates and Posterior Risks under Doubly Type II censored samples for θ = 1.50

n	Uniform		Jeffreys		Gamma		Inverse Levy	
	GELF	LLF	GELF	LLF	GELF	LLF	GELF	LLF
20	1.33248 (0.01657)	1.27515 (0.12731)	1.30583 (0.01624)	1.24965 (0.12477)	1.31337 (0.01474)	1.26978 (0.10498)	1.32650 (0.01250)	1.28248 (0.10055)
30	1.39887 (0.00832)	1.36394 (0.07240)	1.37089 (0.00815)	1.33666 (0.07095)	1.37880 (0.00740)	1.35819 (0.05969)	1.39259 (0.00627)	1.37177 (0.05718)
50	1.44953 (0.00417)	1.42962 (0.03960)	1.42054 (0.00408)	1.40103 (0.03880)	1.42874 (0.00371)	1.42359 (0.03265)	1.44302 (0.00314)	1.43783 (0.03127)
100	1.47091 (0.00278)	1.45694 (0.02737)	1.44149 (0.00273)	1.42780 (0.02682)	1.44981 (0.00247)	1.45080 (0.02257)	1.46431 (0.00210)	1.46530 (0.02161)
150	1.47375 (0.00209)	1.46325 (0.02043)	1.44427 (0.00205)	1.43399 (0.02003)	1.45261 (0.00186)	1.45709 (0.01685)	1.46714 (0.00157)	1.47166 (0.01614)

Table 10: 95% Credible Intervals under different Priors and censoring techniques for θ = 0.50

n	Censoring	Uniform		Jeffreys		Gamma		Inverse Levy	
		LL	UL	LL	UL	LL	UL	LL	UL
20	LC	0.30267	0.59207	0.31355	0.58596	0.31629	0.58282	0.32766	0.57680
	STTC	0.30650	0.58873	0.31752	0.58265	0.32029	0.57953	0.33180	0.57355
	DTTC	0.30109	0.59410	0.31191	0.58797	0.31463	0.58482	0.32594	0.57878
30	LC	0.31775	0.58871	0.32917	0.58264	0.33205	0.57951	0.34398	0.57353
	STTC	0.32177	0.58539	0.33334	0.57935	0.33625	0.57624	0.34834	0.57030
	DTTC	0.31609	0.59073	0.32745	0.58464	0.33031	0.58150	0.34218	0.57550
50	LC	0.32926	0.58075	0.34109	0.57475	0.34408	0.57168	0.35644	0.56577
	STTC	0.33343	0.57747	0.34541	0.57151	0.34843	0.56845	0.36095	0.56258
	DTTC	0.32753	0.58274	0.33930	0.57673	0.34227	0.57364	0.35457	0.56772
100	LC	0.33412	0.56850	0.34612	0.56263	0.34915	0.55962	0.36170	0.55384
	STTC	0.33834	0.56529	0.35050	0.55946	0.35357	0.55646	0.36628	0.55071
	DTTC	0.33237	0.57045	0.34431	0.56456	0.34732	0.56154	0.35980	0.55574
150	LC	0.37577	0.55474	0.38562	0.54901	0.39268	0.54607	0.40298	0.54043
	STTC	0.38053	0.55161	0.39050	0.54591	0.39765	0.54299	0.40808	0.53738
	DTTC	0.37380	0.55664	0.38360	0.55090	0.39063	0.54794	0.40086	0.54229

Table 11: 95% Posterior Predictive Intervals under different censoring techniques for θ = 0.50

n	Censoring	Uniform		Jeffreys		Gamma		Inverse Levy	
		LL	**UL**	**LL**	**UL**	**LL**	**UL**	**LL**	**UL**
20	**LC**	0.10435	1.91514	0.10810	1.89537	0.10905	1.88522	0.11296	1.86576
	STTC	0.10567	1.90433	0.10947	1.88468	0.11043	1.87458	0.11439	1.85523
	DTTC	0.10380	1.92171	0.10753	1.90188	0.10847	1.89169	0.11237	1.87216
30	**LC**	0.10955	1.90428	0.11349	1.88462	0.11448	1.87453	0.11859	1.85517
	STTC	0.11094	1.89353	0.11492	1.87398	0.11593	1.86395	0.12009	1.84470
	DTTC	0.10898	1.91081	0.11289	1.89109	0.11388	1.88096	0.11797	1.8615 4
50	**LC**	0.11352	1.87852	0.11760	1.85913	0.11862	1.84917	0.12289	1.83008
	STTC	0.11495	1.86792	0.11908	1.84863	0.12013	1.83873	0.12444	1.81975
	DTTC	0.11292	1.88497	0.11698	1.86551	0.11800	1.85551	0.12224	1.83636
100	**LC**	0.11519	1.83889	0.11933	1.81991	0.12037	1.81016	0.12470	1.79148
	STTC	0.11665	1.82852	0.12084	1.80964	0.12190	1.79995	0.12628	1.78137
	DTTC	0.11459	1.84520	0.11871	1.82616	0.11974	1.81637	0.12405	1.79762
150	**LC**	0.12955	1.79438	0.13295	1.77586	0.13538	1.76634	0.13893	1.74811
	STTC	0.13119	1.78425	0.13463	1.76583	0.13710	1.75638	0.14069	1.73824
	DTTC	0.12887	1.80054	0.13225	1.78195	0.13467	1.77241	0.13820	1.75411

Table 12: Bayes Estimates and Posterior Risks under real life data

Censoring	Uniform		Jeffreys		Gamma		Inverse Levy	
	GELF	**LLF**	**GELF**	**LLF**	**GELF**	**LLF**	**GELF**	**LLF**
LC	1.13982 (0.02223)	1.09079 (0.17083)	1.11703 (0.02179)	1.06897 (0.16741)	1.12347 (0.01978)	1.08619 (0.14086)	1.13471 (0.01677)	1.09705 (0.13492)
STTC	1.14755 (0.02171)	1.09818 (0.16678)	1.12460 (0.02127)	1.07622 (0.16345)	1.13109 (0.01932)	1.09355 (0.13752)	1.14240 (0.01637)	1.10449 (0.13172)
DTTC	1.13099 (0.02263)	1.08233 (0.17385)	1.10837 (0.02217)	1.06069 (0.17037)	1.11477 (0.02013)	1.07777 (0.14335)	1.12592 (0.01707)	1.08855 (0.13730)

Table 13: 95% Credible Intervals for real life data

Censoring	Uniform		Jeffreys		Gamma		Inverse Levy	
	LL	**UL**	**LL**	**UL**	**LL**	**UL**	**LL**	**UL**
LC	0.77354	1.42973	0.75807	1.40114	0.76245	1.40922	0.77007	1.42332
STTC	0.78682	1.36759	0.77108	1.34024	0.77553	1.36183	0.78329	1.37545
DTTC	0.73128	1.38406	0.71666	1.35637	0.72079	1.37822	0.72800	1.39201

Table 14: 95% Posterior Predictive Intervals based on real life data

Censoring	Uniform		Jeffreys		Gamma		Inverse Levy	
	LL	**UL**	**LL**	**UL**	**LL**	**UL**	**LL**	**UL**
LC	0.08018	1.61050	0.07858	1.57829	0.07903	1.58740	0.07982	1.60327
STTC	0.08156	1.54050	0.07992	1.50969	0.08039	1.53401	0.08119	1.54935
DTTC	0.07580	1.55905	0.07428	1.52786	0.07471	1.55248	0.07546	1.56800

References

1. Akhter, A. S. and Hirai. A. S. (2009). Estimation of the scale parameter from the Rayleigh Distribution from type II singly and doubly censored data. *Pakistan Journal of Statistics and Operational Research*, **5**, 31-45.
2. Al-Hussaini, E. and Hussein. J. M. (2011). Estimation using censored data from Exponentiated Burr Type XII population. *American Open Journal of Statistics*, **1**, 33-45.
3. Aludaat, K. M., Alodat, M. T. and Alodat, T. T. (2008). Parameter estimation of Burr Type X Distribution for grouped data. *Journal of Applied Mathematical Sciences*, **2(9)**, 415-423.
4. Amjad, A. and Ayman, B. (2006). Interval estimation for the scale parameter of Burr Type X Distribution based on grouped data. *Journal of Modern Applied Statistical Methods*, **3**, 386-398.
5. Antweller, R. C. and Taylor, H. E. (2008). Evaluation of statistical treatments of Left-censored environmental data using coincident uncensored data sets: A Summary statistics. *Environmental Science and Technology*, **42**, 3732-3738.
6. Aslam, M. (2003). An application of Prior Predictive Distribution to Elicit the Prior Density. *Journal of Statistical - Theory and Applications*, **2(1)**, 183-197.
7. Asselineau, J., Thiebaut, R., Perez, P., Pinganaud, G. and Chene, G. (2007). Analysis of Left-censored quantitative outcome: Example of pro-calcitonin level. *Revue d'Epidemiologie et de Sante Publique*, **55(3)**, 213-20.
8. Bekker, A., Roux, J. and Mostert, P. (2000). A generalization of the compound Rayleigh Distribution: Using a Bayesian methods on cancer survival times. *Communication in Statistics - Theory and Methods*, **29(7)**, 1419-1433.
9. Burr, W. I. (1942). Cumulative Frequency Distribution. *Annals of the Mathematical Statistics*, **13**, 215-232.
10. Calabria, G. and Pulcini, G. (1996). Point estimation under asymmetric Loss Functions for Left-truncated Exponential samples. *Communication in Statistics - Theory and Methods*, 25(3), 585-600.

11. Dasgupta, R. (2011). On the Distribution of burr with applications. *Sankhya B*, **73**, 1-19.
12. Fauzy, A. (2004). Interval Estimation for parameters of Exponential Distribution under Doubly Type II censoring. *Journal of Applies Mathematics*, **10**, 71-79.
13. Feroze, N. and Aslam, M. (2012a). Bayesian Analysis of Burr Type X Distribution under complete and censored samples. *International Journal of Pure and Applied Sciences and Technology*, **11(2)**, 16-28.
14. Feroze, N. and Aslam. M. (2012b). Bayesian Analysis of Gumbel Type II Distribution under Doubly censored samples using different Loss Functions. *Caspian Journal of Applied Sciences Research*, **1(10)**, 1-10.
15. Jerald, F. and Lawless, J. F. (2003). *Statistical models and methods for life-time data (2nd Edition).*Wiley Series in Probability and Statistics, New Jersey.
16. Klebanov, L. B. (1972). Universal Loss Function and Unbiased estimation. *Doklady Akademii Nauk SSSR Soviet Mathematics*. *Doklady*, **203**, 1249-1251.
17. Makhdoom, I. and Jafari, A. (2011). Bayesian estimations on the Burr Type XII Distribution using grouped and un-grouped data. *Australian Journal of Basic and Applied Sciences*, **5(6)**, 1525-1531.
18. Mousa, M. A. M. and Jaheen, Z. F. (2002). Statistical inference for the burr model based on progressively censored data. *Computers and Mathematics with Applications*, **43(10-11)**, 1441-1449.
19. Panahi, H. and Asadi, S. (2011). Analysis of the type II Hybrid censored Burr Type XII Distribution under Linex Loss Function. *Applied Mathematical Sciences*, **5(79)**, 3929-3942.
20. Shao, Q. (2004). Notes on Maximum Likelihood Estimation for the three-parameter Burr XII Distribution. *Computational Statistics and Data Analysis*, **45**, 675-687.
21. Shao, Q., Wong, H. and Xia, J. (2004). Models for extremes using the extended three parameter Burr XII system with application to Flood Frequency Analysis. *Hydrological Sciences Journal des Sciences Hydrologiques*, **49**, 685-702.
22. Silva, G. O., Ortega, E. M. M., Garibay, V. C and Barreto, M. L. (2008). Log-Burr XII Regression Models with censored data. *Computational Statistics and Data Analysis*, **52**, 3820-3842.
23. Sinha, P., Lambert, M. B. and Trumbull, V. L. (2006). Evaluation of statistical methods for Left-censored environmental data with Non-uniform detection limits. *Environmental Toxicology and Chemistry*, **25(9)**, 2533-2540.

24. Soliman, A. A. (2002). Reliability estimation in a Generalized Life Model with application to the Burr XII. IEEE Tranactions on Reliability, **51**, 337-343.
25. Soliman, A. A. (2005). Estimation of parameters of life from progressively censored data using Burr XII Model. IEEE Transactions on Reliability, **54**, 34-42.
26. Surles, J. G. and Padgett, W.J. (2001). Inference for reliability and stress-length for a scaled Burr Type X Distribution. *Life-time Data Analysis*. **7**, 187-202.
27. Thompson, E. M., Hewlett, J. B. and Baise, L. G. (2011). The Gumbel hypothesis test for Left-censored observations using regional earth-quake records as an example. *Natural Hazards and Earth System Science*, **11**, 115-126.
28. Wahed, A. S. (2006). Bayesian inference using Burr Model under Asymmetric Loss Function: An application to carcinoma survival data. *Journal of Statistical Research*, **40(1)**, 45-57.
29. Wu, J. W. and Yu, H. Y. (2005). Statistical inference about the shape parameter of the Burr Type XII Distribution under the failure-censored sampling plan. *Applied Mathematics and Computation*, **163(1)**, 443-482.
30. Wu, S. J., Chen, Y. J. and Chang. C. T. (2007). Statistical inference based on progressively censored samples with random removals from the Burr Type XII Distribution. *Journal of Statistical Computation and Simulation*, **77**, 19-27.
31. Yarmohammadi, M. and Pazira, H. (2010). Minimax estimation of the parameter of the Burr Type XII Distribution. *Australian Journal of Basic and Applied Sciences*, **4(12)**, 6611-6622.