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Methods for Estimating the Parameters of the Power Function Distribution

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Abstract

In this paper, we present some methods for estimating the parameters of the two parameter Power Function Distribution. We use the Least Squares Method (LSM), Relative Least Squares Method (RLSM) and Ridge Regression Method (RRM). Sampling behavior of the estimates is indicating by a Monte Carlo Simulation. The objective of identifying the best estimator amongst them we use the Total Deviation (TD) and Mean Square Error (MSE) as performance index. We determine the best method for estimation using different values for the parameters and different sample sizes.

Keywords

Power Function distributions, Parameter estimation, Least Squares method, Ridge Regression, Monte Carlo study

1. Introduction

Numerous Parametric Models are used in the analysis of lifetime data and in problems related to the modeling of failure processes. Among Univariate Models, a few particular Distributions occupy a central role because of their demonstrated usefulness in a wide range of situations. Foremost in this category are the Exponential, Weibull, Gamma and Lognormal Distributions.

The Power Function Distribution is also a flexible life time Distribution Model that may offer a Good Fit to some sets of failure data. Theoretically, Power Function Distribution is a special case of Pareto Distribution.

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Meniconi and Barry (1995) discussed the application of Power Function Distribution. They proved that the Power Function Distribution is the best Distribution to check the Reliability of any electrical component. They used Exponential Distribution, Lognormal Distribution and Weibull Distribution and showed from Reliability and Hazard Function that Power Function Distribution is the best Distribution.

The probability Distribution of Power Function Distribution is,

$$f(t) = \frac{\gamma t^{\gamma-1}}{\beta^{\gamma}}; \qquad 0 < t < \beta \tag{1.1}$$

With shape parameter γ and scale parameter β, the interval $(0,\beta)$.

Rider (1964) derived Distributions of the product and quotients of the Order Statistics from a Power Function Distribution. Moments of Order Statistics for a Power Function Distribution were calculated by Malik (1967). Lwin (1972) discussed Bayesian estimation for the scale parameter of the Pareto Distribution using a Power Function Prior. Ahsanullah and Kabir (1975) discussed the Estimation of the location and scale parameters of a Power Function Distribution.

Cohen and Whitten (1982) used the moment and Modified Moment Estimators for the Weibull Distribution. Samia and Mohammad (1993) used five modifications of moments to estimate the parameters of the Pareto Distribution. Lalitha and Anand (1996) used Modified Maximum Likelihood to estimate the scale parameter of the Rayleigh Distribution. Rafiq (1996) discussed the parameters of the Gamma Distribution. Rafiq (1999) discussed the method of Fractional Moments to estimate the parameters of Weibull Distribution. Kang and Young (1997) estimated the parameters of a Pareto Distribution by Jackknife and Bootstrap Methods. Neil (2005) estimated the parameters of Weibull Distribution with the help of percentiles. He called it Common Percentile Method. Zaka and Akhter (2013) discussed the different modifications of the parameter estimation methods and proved that the modified estimators appear better than the traditional maximum likelihood, moments and percentile estimators.

In this paper, we use the Least Squares Method, Relative Least Squares and Ridge Regression to estimate the two parameter of the Power Function Distribution. The present paper introduces the Ridge Regression estimators by taking different values of ‘k’. Where ‘k’ is the Ridge coefficient. Also, we compare between these methods using two parameter Power Function Distribution to find the most accurate method (the method which has least MSE).

2. Methodology

2.1 Least Squares Method (LSM): The Least Square Method (LSM) is extensively used in reliability engineering, mathematics problems and the estimation of Probability Distribution parameters.

The Cumulative Distribution Function of Power Function Distribution is given by

$$F(t_i) = \left(\frac{t_i}{\beta}\right)^{\gamma}$$

$$\left(\frac{t_i}{\beta}\right) = [F(t_i)]^{(1/\gamma)}$$

$$t_i = \beta[F(t_i)]^{(1/\gamma)} \qquad (2.1.1)$$

To get a linear relation between the two parameters taking the natural logarithm of above equation as follows:

$$\log t_i = \log(\beta) + \left(\frac{1}{\gamma}\right)\log\big(F(t_i)\big) \qquad (2.1.2)$$

Let

$Y_i = \log t_i$

$a = \log(\beta)$

$d = \frac{1}{\gamma}$

$x_i = \log\big(F(t_i)\big)$

wherei = 1, 2 ….n and n is the sample size.

Let $t_1, t_2, t_3, \dots, t_n$ be a random sample of t_i and $F(t_i)$ is estimated and replaced by the Median Rank method as follows:

$$F(t_i) = \frac{(i - 0.3)}{(n + 0.4)} t_i, i = 1,2, \dots, n \text{ and } (t_1 < t_2 < t_3 \dots < t_n)$$

Because $F(t_i)$ of the Mean Rank Method $F(t_i) = \frac{i}{(n+1)}$, may be a larger value for smaller ‘i’ and a smaller value for larger ‘i’. Therefore, we use Median Rank Method. Thus, equation (2.1.2) is a linear equation and is expressed as:

$Y_i = a + dX_i$

To compute ‘a’and ‘d’ by Simple Linear Regression we proceed as follows:

Let

$$S(a,d) = \sum_{i=1}^{n} (y_i - a - dx_i)^2$$

We obtain the Least Square Estimates (LSE) of ‘a’and ‘d’ as:

$$a = \bar{y} - d\,\bar{x}$$

$$d = \frac{\sum_{i=1}^{n} x_i\, y_i - \frac{(\sum_{i=1}^{n} x_i y_i)}{n}}{\sum_{i=1}^{n} {x_i}^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}}$$

$$a = \frac{\sum_{i=1}^{n} \log t_i}{n} - d\,\frac{\sum_{i=1}^{n} \log F(t_i)}{n}$$

$$d = \frac{\sum_{i=1}^{n} \log\big(F(t_i)\big)(\log t_i) - \frac{(\sum_{i=1}^{n} \log F(t_i))(\sum_{i=1}^{n} \log t_i)}{n}}{\sum_{i=1}^{n}\big(\log\big(F(t_i)\big)\big)^2 - \frac{(\sum_{i=1}^{n} \log F(t_i))^2}{n}}$$

where $a = \log(\beta)$ and $d = \frac{1}{\gamma}$

Therefore $\hat{\beta} = \text{Antilog}\left\{\frac{\sum_{i=1}^{n} \log t_i}{n} - d\,\frac{\sum_{i=1}^{n} \log F(t_i)}{n}\right\}$ (2.1.3)

and $\hat{\gamma} = \frac{\sum_{i=1}^{n}\big(\log\big(F(t_i)\big)\big)^2 - \frac{(\sum_{i=1}^{n} \log F(t_i))^2}{n}}{\sum_{i=1}^{n} \log\big(F(t_i)\big)(\log t_i) - \frac{(\sum_{i=1}^{n} \log F(t_i))(\sum_{i=1}^{n} \log t_i)}{n}}$ (2.1.4)

2.2. Relative Least Squares Method (RLSM): The Relative Least Squares estimators of ‘a’ and ‘d’ can be obtained by minimizing the sum of squares of the relative residuals, Pablo and Bruce (1992), w.r.t. ‘a’ and ‘d’ as follows:

$$S = \sum_{i=1}^{n} \left(\frac{y_i - a - dx_i}{y_i}\right)^2$$

$$S = \sum_{i=1}^{n} (1 - aw_i - dz_i)^2$$

where $w_i = \frac{1}{y_i}$, $z_i = \frac{x_i}{y_i}$

Differentiating w.r.t.‘a’and ‘d’ then equate to zero

$$\sum_{i=1}^{n} w_i = a\sum_{i=1}^{n} {w_i}^2 + d\sum_{i=1}^{n} w_i\, z_i$$

$$\sum_{i=1}^{n} z_i = a\sum_{i=1}^{n} w_i\, z_i + d\sum_{i=1}^{n} {z_i}^2$$

After simplification, we get

$$a = \frac{\sum_{i=1}^{n} w_i\, z_i \sum_{i=1}^{n} z_i - \sum_{i=1}^{n} w_i \sum_{i=1}^{n} {z_i}^2}{(\sum_{i=1}^{n} w_i\, z_i)^2 - \sum_{i=1}^{n} {z_i}^2 \sum_{i=1}^{n} {w_i}^2}$$

$$d = \frac{\sum_{i=1}^{n} w_i\, z_i \sum_{i=1}^{n} w_i - \sum_{i=1}^{n} z_i \sum_{i=1}^{n} {w_i}^2}{(\sum_{i=1}^{n} w_i\, z_i)^2 - \sum_{i=1}^{n} {z_i}^2 \sum_{i=1}^{n} {w_i}^2}$$

where $w_i = \frac{1}{y_i}$, $z_i = \frac{x_i}{y_i}$

Also $Y_i = \log t_i$ and $x_i = \log\big(F(t_i)\big)$

$$a = \frac{\sum_{i=1}^{n}\left(\frac{1}{\log t_i}\right)\left(\frac{\log(F(t_i))}{\log t_i}\right)\sum_{i=1}^{n}\left(\frac{\log(F(t_i))}{\log t_i}\right) - \sum_{i=1}^{n}\left(\frac{1}{\log t_i}\right)\sum_{i=1}^{n}\left(\frac{\log(F(t_i))}{\log t_i}\right)^2}{\left(\sum_{i=1}^{n}\left(\frac{1}{\log t_i}\right)\left(\frac{\log(F(t_i))}{\log t_i}\right)\right)^2 - \sum_{i=1}^{n}\left(\frac{\log(F(t_i))}{\log t_i}\right)^2\sum_{i=1}^{n}\left(\frac{1}{\log t_i}\right)^2}$$

$$d = \frac{\sum_{i=1}^{n}\left(\frac{1}{\log t_i}\right)\left(\frac{\log(F(t_i))}{\log t_i}\right)\sum_{i=1}^{n}\left(\frac{1}{\log t_i}\right) - \sum_{i=1}^{n}\left(\frac{1}{\log t_i}\right)^2\sum_{i=1}^{n}\left(\frac{\log(F(t_i))}{\log t_i}\right)}{\left(\sum_{i=1}^{n}\left(\frac{1}{\log t_i}\right)\left(\frac{\log(F(t_i))}{\log t_i}\right)\right)^2 - \sum_{i=1}^{n}\left(\frac{\log(F(t_i))}{\log t_i}\right)^2\sum_{i=1}^{n}\left(\frac{1}{\log t_i}\right)^2}$$

Also $a = \log(\beta)$ and $d = \frac{1}{\gamma}$

$$\therefore \hat{\beta} = \text{Antilog}\left\{\frac{\sum_{i=1}^{n}\left(\frac{1}{\log t_i}\right)\left(\frac{\log(F(t_i))}{\log t_i}\right)\sum_{i=1}^{n}\left(\frac{\log(F(t_i))}{\log t_i}\right) - \sum_{i=1}^{n}\left(\frac{1}{\log t_i}\right)\sum_{i=1}^{n}\left(\frac{\log(F(t_i))}{\log t_i}\right)^2}{\left(\sum_{i=1}^{n}\left(\frac{1}{\log t_i}\right)\left(\frac{\log(F(t_i))}{\log t_i}\right)\right)^2 - \sum_{i=1}^{n}\left(\frac{\log(F(t_i))}{\log t_i}\right)^2\sum_{i=1}^{n}\left(\frac{1}{\log t_i}\right)^2}\right\} \quad (2.2.1)$$

and

$$\hat{\gamma} = \frac{\left(\sum_{i=1}^{n}\left(\frac{1}{\log t_i}\right)\left(\frac{\log(F(t_i))}{\log t_i}\right)\right)^2 - \sum_{i=1}^{n}\left(\frac{\log(F(t_i))}{\log t_i}\right)^2\sum_{i=1}^{n}\left(\frac{1}{\log t_i}\right)^2}{\sum_{i=1}^{n}\left(\frac{1}{\log t_i}\right)\left(\frac{\log(F(t_i))}{\log t_i}\right)\sum_{i=1}^{n}\left(\frac{1}{\log t_i}\right) - \sum_{i=1}^{n}\left(\frac{1}{\log t_i}\right)^2\sum_{i=1}^{n}\left(\frac{\log(F(t_i))}{\log t_i}\right)} \quad (2.2.2)$$

2.3 Ridge Regression Method (RRM): The Ridge Regression estimators are given by

$$\beta_{rid} = (X'X + K\,I)^{-1}X'Y$$

where $0<k<1$ is the Ridge coefficient, 'I' is the p×p identity matrix and 'p' is the number of parameters.

$$X'X = \begin{bmatrix} n & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} {x_i}^2 \end{bmatrix}$$

$$X'Y = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \end{bmatrix}$$

$$\beta_{rid} = \begin{bmatrix} a \\ d \end{bmatrix}$$

$$(X'X + K\,I) = \begin{bmatrix} n & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} {x_i}^2 \end{bmatrix} + K \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$(X'X + K\,I) = \begin{bmatrix} n + k & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} {x_i}^2 + K \end{bmatrix}$$

$$(X'X + K\,I)^{-1} = \begin{bmatrix} n + k & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} {x_i}^2 + K \end{bmatrix}^{-1}$$

$$\beta_{rid} = (X'X + K\,I)^{-1} X'Y$$

$$\beta_{rid} = \begin{bmatrix} a \\ d \end{bmatrix} = \begin{bmatrix} n + k & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} {x_i}^2 + K \end{bmatrix}^{-1} \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \end{bmatrix}$$

After simplification we get,

$$\hat{a} = \frac{\left(\sum_{i=1}^{n} {x_i}^2 + K\right) \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i y_i}{(n + k)\left(\sum_{i=1}^{n} {x_i}^2 + K\right) - \left(\sum_{i=1}^{n} x_i\right)^2}$$

$$\hat{d} = \frac{-\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i + (n+k)\ \sum_{i=1}^{n} x_i y_i}{(n+k)(\sum_{i=1}^{n} x_i^2 + K) - (\sum_{i=1}^{n} x_i)^2}$$

where $a = \log(\beta)$ and $d = \frac{1}{\gamma}$

Also $Y_i = \log t_i$ and $x_i = \log\big(F(t_i)\big)$

$$\hat{\beta} = \text{Antilog}\left[\frac{\left(\sum_{i=1}^{n}\{\log(F(t_i))\}^2 + K\right)\sum_{i=1}^{n} \log t_i - \sum_{i=1}^{n} \log(F(t_i)) \sum_{i=1}^{n}\{\log(F(t_i))\} \log t_i}{(n+k)\left(\sum_{i=1}^{n}\{\log(F(t_i))\}^2 + K\right) - \left(\sum_{i=1}^{n} \log(F(t_i))\right)^2}\right] \quad (2.3.1)$$

and

$$\hat{\gamma} = \frac{(n+k)\left(\sum_{i=1}^{n}\{\log(F(t_i))\}^2 + K\right) - \left(\sum_{i=1}^{n}(F(t_i))\right)^2}{-\sum_{i=1}^{n}(F(t_i))\sum_{i=1}^{n} \log t_i + (n+k)\ \sum_{i=1}^{n}(F(t_i))\ \log t_i} \quad (2.3.2)$$

where 0<k<1 is the Ridge coefficient the readers may see Ronald and Raymond (1978) if k=0, we obtain the Least Square Estimates.

3. Performance Indices (Goodness of Fit Analysis)

Some methods of Goodness of Fit analysis are employed here. Mean Square Error (MSE) and Total Deviation (TD) are two measurements that give an indication of the accuracy of parameter estimation. Al-Fawzan (2000) referred to the use of the procedure of MSE and TD.

3.1 Mean Square Error (MSE): The MSE can be calculated as below:

$$\text{MSE} = \sum_{i=1}^{n}\{\hat{F}(t_i) - F(t_i)\}^2 \Big/ n$$

where $F(t_i) = \frac{i-.3}{n+.4}$ is the value of the Cumulative Distribution Function of the two parameter Power Function Distribution using the estimated parameters, and $\hat{F}(t_i) = \left(\frac{t_i}{\hat{\beta}}\right)^{\hat{\gamma}}$

Also,

Standard bias, Bias = E $(\hat{\beta})$ – β and MSE $(\hat{\beta})$ = $E\left[\left(\hat{\beta} - \beta\right)^2\right]$

Standard bias, Bias = E $(\hat{\gamma})$ – γ and MSE $(\hat{\gamma})$ = $E\left[\left(\hat{\gamma} - \gamma\right)^2\right]$

3.2. Total Derivation (TD): The Total Derivation (TD), calculated for each method is as follows:

$$\mathrm{TD} = \left|\frac{\hat{\gamma} - \gamma}{\gamma}\right| + \left|\frac{\hat{\beta} - \beta}{\beta}\right|$$

where γ and β are the known parameters, and $\hat{\gamma}$ and $\hat{\beta}$ are the estimated parameters by any method. These techniques are used to measure the variability of parameter estimates for each Simulation. These are used to determine the overall "best" parameter estimation method.

4. Application

A Simulation study is used in order to compare the performance of the proposed estimation methods. We carry out this comparison taking the samples of sizes as n = 20, 60 and 100 with pairs of $(\beta,\gamma) = \{(1, 2), (3, 2), (4, 3)\}$. We generated random samples of different sizes by observing that if R is uniform (0,1), then $t = \beta R^{1/\gamma}$ is the random number generation of Power Function Distribution with (γ,β) parameters. All results are based on 10,000 replications (Using R-Language).

5. Results and Conclusion

All the results are listed in Table 1, 2 and 3. From these tables, we see that the LSM estimates of parameters are too close to the true values, and the values of MSE and TD are very small. The parameter estimates from RLSM, RRM method are close to the true values but not as LSM estimates, because the values of MSE and TD are greater than the corresponding values from LSM.

It was observed that MSEs and TDs of all estimators of scale parameter as well as all the estimators of shape parameter are decreasing with the increase of sample size.

Consequently, we recommend using the LSM method for the parameters estimation of the Power Function Distribution. After LSM, the RRM (0.1) and RLSM method are best for estimation of scale and shape parameters of the Power Function Distribution.

Table 1: Estimates for the parameters β and γ of Power Function Distribution under the sample size 20

Methods	True Values		Estimated Values		MSE		TD
	β	γ	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	
LSM	1	2	1.001218	2.062543	0.006737	0.309703	0.032489
	4	3	3.999863	3.063222	0.051887	0.643331	0.021108
	3	2	2.996574	1.997344	0.098993	0.266866	0.00247
RLSM	1	2	0.997928	2.4512	0.000267	1.600062	0.227672
	4	3	3.913205	3.176421	0.239967	0.867286	0.080506
	3	2	2.636301	2.602738	0.490666	1.77547	0.422602
RRM(.1)	1	2	0.984061	2.135137	0.006174	0.344699	0.083507
	4	3	3.895059	3.265332	0.055787	0.837077	0.114679
	3	2	2.908783	2.098611	0.094425	0.310977	0.079711
RRM(.2)	1	2	0.968673	2.206826	0.006247	0.390202	0.13474
	4	3	3.80006	3.476352	0.079031	1.144886	0.208769
	3	2	2.829907	2.20134	0.104646	0.379475	0.157368
RRM(.3)	1	2	0.954808	2.277641	0.006778	0.445823	0.184012
	4	3	3.713498	3.696877	0.116492	1.583523	0.303918
	3	2	2.758647	2.305551	0.125486	0.473517	0.233227
RRM(.4)	1	2	0.942266	2.34761	0.007638	0.511195	0.231539
	4	3	3.634248	3.927562	0.164373	2.172441	0.400625
	3	2	2.693942	2.411268	0.153914	0.594307	0.307654
RRM(.5)	1	2	0.930878	2.416761	0.008729	0.585972	0.277503
	4	3	3.561378	4.169131	0.219839	2.934324	0.499366
	3	2	2.634917	2.518513	0.187708	0.743099	0.380951
RRM(.6)	1	2	0.920504	2.485121	0.009979	0.669827	0.322057
	4	3	3.494104	4.422391	0.280754	3.895759	0.600604
	3	2	2.580845	2.627308	0.225224	0.921199	0.453373
RRM(.7)	1	2	0.911024	2.552716	0.011334	0.762452	0.365334
	4	3	3.43177	4.688242	0.345506	5.088094	0.704805
	3	2	2.531117	2.737679	0.265245	1.129968	0.525134
RRM(.8)	1	2	0.902337	2.619569	0.012754	0.863556	0.407448
	4	3	3.373814	4.967692	0.412866	6.548536	0.812444
	3	2	2.485221	2.849651	0.30686	1.370824	0.596418
RRM(.9)	1	2	0.894356	2.685705	0.014207	0.972861	0.448496
	4	3	3.31976	5.261882	0.481898	8.321574	0.924021
	3	2	2.442719	2.963247	0.349391	1.645248	0.667384

Table 2: Estimation for the parameters β and γ of Power Function Distribution for sample size 60

Methods	True Values		Estimated Values		MSE		TD
	β	γ	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	
LSM	1	2	0.994648	2.019416	0.003304	0.111938	0.01506
	4	3	4.006399	2.992553	0.021194	0.240086	0.004082
	3	2	3.033886	1.96236	0.034403	0.103488	0.030115
RLSM	1	2	0.998754	2.136961	6.98E-05	0.186667	0.069727
	4	3	3.844167	3.278693	0.287488	1.42537	0.131856
	3	2	2.661028	2.466177	0.339682	1.405301	0.346079
RRM(.1)	1	2	0.989673	2.039686	0.003258	0.115163	0.03017
	4	3	3.9736	3.046982	0.020863	0.254506	0.022261
	3	2	3.006583	1.990433	0.031844	0.105801	0.006977
RRM(.2)	1	2	0.984856	2.059879	0.003264	0.119207	0.045084
	4	3	3.941755	3.102009	0.022636	0.275542	0.048564
	3	2	2.980138	2.018611	0.030834	0.109792	0.015926
RRM(.3)	1	2	0.980189	2.079997	0.003317	0.124061	0.059809
	4	3	3.910822	3.157644	0.026331	0.30342	0.074843
	3	2	2.954511	2.046892	0.031233	0.115478	0.038609
RRM(.4)	1	2	0.975667	2.10004	0.003413	0.129717	0.074353
	4	3	3.88076	3.213895	0.031786	0.338374	0.101108
	3	2	2.929664	2.075277	0.032914	0.12288	0.061084
RRM(.5)	1	2	0.971283	2.120009	0.003547	0.136165	0.088721
	4	3	3.851532	3.270771	0.03885	0.380645	0.127374
	3	2	2.905563	2.103766	0.035761	0.132017	0.083362
RRM(.6)	1	2	0.967032	2.139905	0.003716	0.143396	0.102921
	4	3	3.823102	3.328282	0.047385	0.430481	0.153652
	3	2	2.882174	2.132359	0.039669	0.142908	0.105455
RRM(.7)	1	2	0.962907	2.159728	0.003917	0.151402	0.116957
	4	3	3.795437	3.386436	0.057267	0.488139	0.179953
	3	2	2.859465	2.161057	0.044542	0.155575	0.127373
RRM(.8)	1	2	0.958904	2.179479	0.004146	0.160175	0.130836
	4	3	3.768505	3.445243	0.06838	0.553885	0.206288
	3	2	2.837408	2.18986	0.050292	0.170037	0.149127
RRM(.9)	1	2	0.955018	2.19916	0.004402	0.169705	0.144562
	4	3	3.742275	3.504713	0.080617	0.627991	0.232669
	3	2	2.815975	2.218769	0.05684	0.186315	0.170726

Table 3: Estimation for the parameters β and γ of Power Function Distribution for sample size 100

Methods	True Values		Estimated Values		M.S.E		T.D
	β	γ	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	
LSM	1	2	1.000677	2.023245	0.001921	0.071615	0.0123
	4	3	4.018766	2.973905	0.017037	0.196505	0.01339
	3	2	3.02700	1.98070	0.0174251	0.0590387	0.0186514
RLSM	1	2	0.999534	2.126007	1.8E-05	0.108957	0.063469
	4	3	3.779907	3.278657	0.325124	1.191472	0.147909
	3	2	2.78427	2.26152	0.168653	0.360695	0.202670
RRM(.1)	1	2	0.997791	2.034987	0.001883	0.07305	0.019702
	4	3	3.999364	3.004927	0.016253	0.201713	0.001801
	3	2	3.01122	1.99702	0.0163858	0.0598607	0.0052325
RRM(.2)	1	2	0.994958	2.046704	0.001864	0.07476	0.028394
	4	3	3.980293	3.036141	0.016225	0.209021	0.016974
	3	2	2.99573	2.01337	0.0158587	0.0612337	0.0081072
RRM(.3)	1	2	0.992177	2.058395	0.00186	0.076743	0.03702
	4	3	3.961544	3.06755	0.016915	0.218469	0.032131
	3	2	2.98052	2.02975	0.0158166	0.0631612	0.0213702
RRM(.4)	1	2	0.989447	2.070061	0.001873	0.078998	0.045584
	4	3	3.943109	3.099156	0.018287	0.230099	0.047275
	3	2	2.96558	2.04617	0.0162336	0.0656468	0.0345591
RRM(.5)	1	2	0.986766	2.081703	0.0019	0.081522	0.054085
	4	3	3.92498	3.130959	0.020306	0.243954	0.062408
	3	2	2.95090	2.06262	0.0170851	0.0686940	0.0476764
RRM(.6)	1	2	0.984133	2.093319	0.001942	0.084315	0.062526
	4	3	3.907149	3.162962	0.022941	0.260076	0.077533
	3	2	2.93649	2.07911	0.0183481	0.0723064	0.0607244
RRM(.7)	1	2	0.981547	2.104911	0.001996	0.087373	0.070908
	4	3	3.889608	3.195165	0.026161	0.278508	0.092653
	3	2	2.92233	2.09563	0.0200005	0.0764875	0.0737054
RRM(.8)	1	2	0.979007	2.116478	0.002063	0.090696	0.079232
	4	3	3.87235	3.227571	0.029935	0.299296	0.10777
	3	2	2.90841	2.11218	0.0220215	0.0812411	0.0866217
RRM(.9)	1	2	0.976512	2.128021	0.002142	0.094282	0.087499
	4	3	3.855367	3.260181	0.034237	0.322485	0.122885
	3	2	2.89473	2.12877	0.0243914	0.0865706	0.0994754

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