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The Non-Response in the Population Ratio of Mean for Current Occasion in Sampling on Two Occasions

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Abstract

In this article, we attempt the problem of estimation of the population Ratio of mean in mail surveys. This problem is conducted for current occasion in the context of sampling on two occasions when there is Non-response (i) on both occasions, (ii) only on the first occasion and (iii) only on the second occasion. We obtain the loss in precision of all the Estimators with respect to the Estimator of the population Ratio of mean when there is no Non-response. We derive the sample sizes and the saving in cost for all the Estimators, which have the same precision than the Estimator of the population Ratio of mean when there is no Non-response. An empirical study that allows us to investigate the performance of the proposed strategy is carried out.

Keywords

Successive sampling, Non-response, Estimator of the ratio

1. Introduction

Eckler (1955), Jessen (1942), Patterson (1950), Raj (1968), Tikkiwal (1951) and Yates (1949) contributed towards the development of the theory of Unbiased estimation of mean of characteristics in Successive sampling. In many practical situations the Estimate of the population Ratio and Product of two characters for the most recent occasion may be of considerable interest. The theory of estimation of the population Ratio of two characters over two occasions has been considered by Artes and Garcia (2001), Garcia and Artes (2002), Okafor (1992), Okafor and Arnab (1987), Rao (1957) and Rao and Pereira (1968) among others.

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Further, Garcia (2008) presented some sampling strategies for estimating, by a Linear Estimate, the population Product of two characters over two occasions.

Hansen and Hurwitz (1946) suggested a Technique for handling the Non-response in mail surveys. These surveys have the advantage that the data can be collected in a relatively inexpensive way. Okafor (2001) extended these surveys to the estimation of the population total in element sampling on two successive occasions. Later, Choudhary et al. (2004) used the Hansen and Hurwitz (HH) Technique to Estimate the population mean for current occasion in the context of sampling on two occasions when there is Non-response on both occasions. More recently, Singh and Kumar (2010) used the HH Technique to Estimate the population product for current occasion in the context of sampling on two occasions when there is Non-response on both occasions and Garcia and Ona (2011) used the HH Technique to Estimate the change of mean and the sum of mean for current occasion in the context of sampling on two occasions when there is Non-response on both occasions. However, Non-response is a common problem with mail surveys: Kumar (2012), Kumar et al. (2011), Singh et al. (2011), Singh, G. N. and Karna, J. P. (2012) and Singh and Kumar (2011).

Also, Cochran (1977) and Okafor and Lee (2000) extended the HH Technique to the case when the information on the characteristic under study is also available on auxiliary characteristic.

In this article, we develop the HH Technique to Estimate the population Ratio of mean for current occasion in the context of sampling on two occasions when there is Non-response (i) on both occasions, (ii) only on the first occasion and (iii) only on the second occasion. An empirical study that allows us to investigate the performance of the proposed strategy is carried out.

2. The Technique

Consider a finite population of N identifiable units. Let (xi; yi) be, for I = 1, 2, …, N, the values of the characteristic on the first and second occasions, respectively. We assume that the population can be divided into two classes, those who respond at the first attempt and those who not. Let, the sizes of these two classes be N_1 and N_2, respectively. Let on the first occasion, schedules through mail are sent to 'n' units selected by simple random sampling. On the second occasion, a simple random sample of $m = np$ units, for $0 < p < 1$, is retained while an independent sample of $u = nq = n-m$ units, for $q = 1-p$, is selected (unmatched with the first occasion). We assume that in the unmatched portion of the sample on two occasions, 'u_1' units respond and 'u_2' units do not. Similarly, in the matched portion 'm_1' units respond and 'm_2' units do not.

Let m_{h_2} denotes the size of the subsample drawn from the Non-response class from the matched portion of the sample on the two occasions for collecting information through personal interview. Similarly, denote by u_{h_2} the size of the sub-sample drawn from the Non-response class in the unmatched portion of the sample on the two occasions. Also, let $\sigma^2_{x_j}$, $\sigma^2_{y_j}$; j=1, 2 and $\sigma^2_{x_{j\,(2)}}, \sigma^2_{y_{j(2)}}$ j = 1, 2 denote the population variance and population variance pertaining to the Non-response class, respectively. In addition, let $\bar{x}^*_{1m}$, $\bar{y}^*_{1m}$, $\bar{x}^*_{1u}$ and $\bar{y}^*_{1u}$ denote the Estimator for matched and unmatched portions of the sample on the first occasion, respectively. Let the corresponding Estimator for the second occasion be denoted by $\bar{x}^*_{2m}, \bar{y}^*_{2m}, \bar{x}^*_{2u}$ and $\bar{y}^*_{2u}$. Thus, have the following setup:

x_i (y_i), the variable x (y) on ith occasion, i = 1, 2.

$R_1 = \frac{\bar{Y}_1}{X_1}$ ($R_2 = \frac{\bar{Y}_2}{X_2}$) the population Ratio on the first (second) occasion,

$\hat{R}_1 = \frac{\bar{y}_1}{x_1}$ ($\hat{R}_2 = \frac{\bar{y}_2}{x_2}$) the Estimator of the population Ratio on the first (second) occasion,

$\hat{R}^*_{1m} = \frac{\bar{y}^*_{1m}}{\bar{x}^*_{1m}}$ ($\hat{R}^*_{2m} = \frac{\bar{y}^*_{2m}}{\bar{x}^*_{2m}}$) the Estimator of the population Ratio on the first (second) occasion based on the matched sample of 'm' units,

$\hat{R}^*_{1u} = \frac{\bar{y}^*_{1u}}{\bar{x}^*_{1u}}$ ($\hat{R}^*_{2u} = \frac{\bar{y}^*_{2u}}{\bar{x}^*_{2u}}$) the Estimator of the population Ratio on the first (second) occasion based on the unmatched sample of 'u' units.

ρ_1 (ρ_2), the correlation coefficients between the variables y_1 and x_1 (y_2 and x_2),

ρ_3 (ρ_4), the correlation coefficients between the variables y_2 and x_1 (y_1 and x_2),

ρ_5 (ρ_6), the correlation coefficients between the variables x_1 and x_2 (y_1 and y_2),

$\rho_{1(2)}$ ($\rho_{2(2)}$), the correlation coefficients between the variables $y_{1(2)}$ and $x_{1(2)}$ ($y_{2(2)}$ and $x_{2(2)}$), $\rho_{3(2)}$ ($\rho_{4(2)}$), the correlation coefficients between the variables $y_{2(2)}$ and $x_{1(2)}$ ($y_{1(2)}$ and $x_{2(2)}$), $\rho_{5(2)}$ ($\rho_{6(2)}$), the correlation coefficients between the variables $x_{1(2)}$ and $x_{2(2)}$ ($y_{1(2)}$ and $y_{2(2)}$).

1st Occasion →	$\hat{R}^*_{1u}$	$\hat{R}^*_{1m}$	
2nd Occasion →		$\hat{R}^*_{2m}$	$\hat{R}^*_{2u}$

where

$$\bar{y}_{1m}^{*}=\frac{m_1\bar{y}_{1m_1}+m_2\bar{y}_{1m_{h2}}}{m}$$

Hansen and Hurwitz (1946) Estimator for the population mean $\bar{Y}_1$ on the first occasion for matched portion of the sample.

$$\bar{y}_{2m}^{*}=\frac{m_1\bar{y}_{2m_1}+m_2\bar{y}_{2m_{h2}}}{m}$$

Hansen and Hurwitz (1946) Estimator for the population mean $\bar{Y}_2$ on the second occasion for matched portion of the sample.

$$\bar{y}_{1u}^{*}=\frac{u_1\bar{y}_{1u_1}+u_2\bar{y}_{1u_{h2}}}{u}$$

Hansen and Hurwitz (1946) Estimator for the population mean $\bar{Y}_1$ on the first occasion for matched portion of the sample.

$$\bar{y}_{2u}^{*}=\frac{u_1\bar{y}_{2u_1}+u_2\bar{y}_{2u_{h2}}}{u}$$

Hansen and Hurwitz (1946) Estimator for the population mean $\bar{Y}_2$ on the second occasion for matched portion of the sample.

$$x_{1m}^{*}=\frac{m_1\bar{x}_{1m_1}+m_2\bar{x}_{1m_{h2}}}{m}$$

Hansen and Hurwitz (1946) Estimator for the population mean $\bar{X}_1$ on the first occasion for matched portion of the sample.

$$\bar{x}_{2m}^{*}=\frac{m_1\bar{x}_{2m_1}+m_2\bar{x}_{2m_{h2}}}{m}$$

Hansen and Hurwitz (1946) Estimator for the population mean $\bar{X}_2$ on the second occasion for matched portion of the sample.

$$\bar{x}_{1u}^{*}=\frac{u_1\bar{x}_{1u_1}+u_2\bar{x}_{1u_{h2}}}{u}$$

Hansen and Hurwitz (1946) Estimator for the population mean $\bar{X}_1$ on the first occasion for matched portion of the sample.

$$\bar{x}_{2u}^{*}=\frac{u_1\bar{x}_{2u_1}+u_2\bar{x}_{2u_{h2}}}{u}$$

Hansen and Hurwitz (1946) Estimator for the population mean $\bar{X}_2$ on the second occasion for matched portion of the sample.

$$i,j=1,2\begin{cases}\bar{y}_{jm_1}=\frac{1}{m_1}\sum_{i=1}^{m_1}y_{ji} & \bar{x}_{jm_1}=\frac{1}{m_1}\sum_{i=1}^{m_1}x_{ji} & \bar{y}_{jm_{h_2}}=\frac{1}{m_{h_2}}\sum_{l=1}^{m_{h_2}}y_{jl}\\ \bar{x}_{jm_{h_2}}=\frac{1}{m_{h_2}}\sum_{l=1}^{m_{h_2}}x_{jl} & \bar{y}_{ju_1}=\frac{1}{u_1}\sum_{\alpha=1}^{u_1}y_{j\alpha} & \bar{x}_{ju_1}=\frac{1}{u_1}\sum_{\alpha=1}^{u_1}x_{j\alpha}\\ \bar{y}_{ju_{h_2}}=\frac{1}{u_{h_2}}\sum_{\beta=1}^{u_{h_2}}y_{j\beta} & \bar{x}_{ju_{h_2}}=\frac{1}{u_{h_2}}\sum_{\beta=1}^{u_{h_2}}x_{j\beta} & \end{cases}$$

The procedure of obtaining Hansen-Hurwitz Estimators can be seen in Singh and Kumar (2010). Also, it can be easily seen that (see Singh and Kumar, 2010)

$$Cov\left(\hat{R}_{1u}^{*},\hat{R}_{1m}^{*}\right)=Cov\left(\hat{R}_{1u}^{*},\hat{R}_{2m}^{*}\right)=Cov\left(\hat{R}_{1u}^{*},\hat{R}_{2u}^{*}\right)=Cov\left(\hat{R}_{1m}^{*},\hat{R}_{2u}^{*}\right)$$
$$=Cov\left(\hat{R}_{2m}^{*},\hat{R}_{2u}^{*}\right)=0.$$

3. Estimation of the Population Ratio of Mean

3.1 Estimation of the Population Ratio of Mean for Current Occasion in the Presence of Non-response on both Occasions: We want to estimate, R_2, the population Ratio for the second period by a Linear Estimate (HH Technique) of the form

$$\hat{R}_2^{*}=a\hat{R}_{1u}^{*}+b\hat{R}_{1m}^{*}+c\hat{R}_{2m}^{*}+d\hat{R}_{2u}^{*}$$

We have

$$E(\hat{R}_{1u}^{*})=E(\hat{R}_{1m}^{*})=R_1 \quad and \quad E(\hat{R}_{2u}^{*})=E(\hat{R}_{2m}^{*})=R_2$$

we find that

$$E(\hat{R}_2^{*})=(a+b)R_1+(c+d)R_2$$

If we now require that $\hat{R}_2^{*}$ be an Unbiased Estimate of R_2, we must have

$$a+b=0 \qquad and \qquad c+d=1$$

so that

$$\hat{R}_2^{*}=a\left(\hat{R}_{1u}^{*}-\hat{R}_{1m}^{*}\right)+c\hat{R}_{2m}^{*}+(1-c)\hat{R}_{2u}^{*}$$

The variance of $\hat{R}_2^{*}$ is given by

$$V(\hat{R}_2^*) = a^2\left(\frac{1}{q}+\frac{1}{p}\right)\frac{1}{n\bar{X}_1^2}D^* + c^2\frac{1}{pn\bar{X}_2^2}E^* + (1-c)^2\frac{1}{qn\bar{X}_2^2}E^* - 2ac\frac{1}{pn\bar{X}_1\bar{X}_2}C^*$$

where

$D^* = \{A + W_2(k-1)A_{(2)}\}$; $E^* = \{B + W_2(k-1)B_{(2)}\}$

$W_2 = N_2/N$; $k = m_2/m_{h_2} = u_2/u_{h_2}$

$A = S_{y_1}^2 + R_1^2 S_{x_1}^2 - 2R_1\, Cov\,(y_1, x_1)$;

$A_{(2)} = S_{y_1(2)}^2 + R_1^2 S_{x_1(2)}^2 - 2R_1\, Cov\,(y_1, x_1)_{(2)}$

$B = S_{y_2}^2 + R_2^2 S_{x_2}^2 - 2R_2\, Cov\,(y_2, x_2)$;

$B_{(2)} = S_{y_2(2)}^2 + R_2^2 S_{x_2(2)}^2 - 2R_2\, Cov\,(y_2, x_2)_{(2)}$

$$C^* = [\,Cov\,(y_1,y_2) - R_1\,Cov\,(y_2,x_1) - R_2\,Cov\,(y_1,x_2) + R_1R_2\,Cov\,(x_1,x_2)]$$
$$+W_2(k-1)[\,Cov\,(y_1,y_2)_{(2)} - R_1\,Cov\,(y_2,x_1)_{(2)} - R_2\,Cov\,(y_1,x_2)_{(2)} + R_1R_2\,Cov\,(x_1,x_2)_{(2)}]$$

The values of ‘a’ and ‘c’ can be chosen to minimize $V(\hat{R}_2^*)$. Equating to zero the derivatives of $V(\hat{R}_2^*)$ with respect to ‘a’ and ‘c’, it follows that the optimum values are,

$$a_{opt} = \frac{pq\bar{X}_1E^*C^*}{\bar{X}_2(D^*E^* - q^2C^{*2})} \quad and \quad c_{opt} = \frac{pD^*E^*}{D^*E^* - q^2C^{*2}}$$

Thus, the Estimate with optimum values for ‘a’ and ‘c’ may be written:

$$\hat{R}_2^{**} = \frac{pq\bar{X}_1E^*C^*}{\bar{X}_2(D^*E^* - q^2C^{*2})}(\hat{R}_{1u}^* - \hat{R}_{1m}^*) + \frac{pD^*E^*}{D^*E^* - q^2C^{*2}}\hat{R}_{2m}^* + \left(1 - \frac{pD^*E^*}{D^*E^* - q^2C^{*2}}\right)\hat{R}_{2u}^* \qquad (3.1.1)$$

and its variance is,

$$V(\hat{R}_2^{**}) = \frac{E^*}{\bar{X}_2^2 n}\frac{D^*E^* - qC^{*2}}{D^*E^* - q^2C^{*2}} \qquad (3.1.2)$$

Note that if $q = 0$, $p = 1$, complete matching or $p = 0$, $q = 1$, no matching this variance equation (3.1.2) has the same value,

$$V(\hat{R}_2^{**}) = \frac{E^*}{\bar{X}_2^2 n}$$

Thus, for current Estimates, equal precision is obtained either by keeping the same sample or by changing it on every occasion. If $\bar{X}_1 = \bar{X}_2$, the Estimate give by equation (3.1.1) is somewhat simplified.

$$\hat{R}_2^{**} = \frac{pqE^*C^*}{(D^*E^* - q^2C^{*2})}(\hat{R}_{1u}^* - \hat{R}_{1m}^*) + \frac{pD^*E^*}{D^*E^* - q^2C^{*2}}\hat{R}_{2m}^* + \left(1 - \frac{pD^*E^*}{D^*E^* - q^2C^{*2}}\right)\hat{R}_{2u}^*$$

but its variance is unchanged, that is

$$V(\hat{R}_2^{**}) = \frac{E^*}{\bar{X}_2^2 n}\frac{D^*E^* - qC^{*2}}{D^*E^* - q^2C^{*2}}$$

while if $W_2 = 0$, i.e., there is Non-response, the $V\,(\hat{R}_2^{**})$ reduces to

$$V(\hat{R}_2) = \frac{B}{\bar{X}_2^2 n}\frac{AB - qk_1^2}{AB - q^2k_1^2}$$

where $\hat{R}_2$ is the usual Estimator of the Ratio of mean for the current occasion in the context of sampling on two occasions when there is complete response, that is $\hat{R}_2 = a\,\hat{R}_{1u} + b\,\hat{R}_{1m} + c\,\hat{R}_{2m} + d\,\hat{R}_{2u}$.

Similarly an Estimate of the first occasion is,

$$\hat{R}_1^{**} = \frac{pq\bar{X}_2E^*C^*}{\bar{X}_1(D^*E^* - q^2C^{*2})}(\hat{R}_{2u}^* - \hat{R}_{2m}^*) + \frac{pD^*E^*}{D^*E^* - q^2C^{*2}}\hat{R}_{1m}^* + \left(1 - \frac{pD^*E^*}{D^*E^* - q^2C^{*2}}\right)\hat{R}_{1u}^*$$

Its variance is,

$$V(\hat{R}_1^{**}) = \frac{D^*}{\bar{X}_1^2 n}\frac{D^*E^* - qC^{*2}}{D^*E^* - q^2C^{*2}}$$

Equating to zero the derivative of $V(\hat{R}_2^{**})$ with respect to q, we find that the variance $V(\hat{R}_2^{**})$ will have its minimum value if we choose:

$$q_{opt}^{(0)} = \frac{D^*E^* - \sqrt{D^{*2}E^{*2} - C^{*2}D^*E^*}}{C^{*2}} \qquad (3.1.3)$$

and

$$V_{min}(\hat{R}_2^{**}) = \frac{E^*}{\bar{X}_2^2 n}\frac{D^*E^* + \sqrt{D^{*2}E^{*2} - C^{*2}D^*E^*}}{2D^*E^*}$$

However, if only the Estimate using information gathered on the second occasion is considered, the Estimator of the population Ratio is,

$$\hat{R}^* = p\hat{R}^*_{2m} + q\hat{R}^*_{2u}$$

and its variance is,

$$V(\hat{R}^*) = \frac{E^*}{\bar{X}_2^2 n}$$

and we find

$$\frac{E^*}{\bar{X}_2^2 n}\frac{D^*E^* + \sqrt{D^{*2}E^{*2} - C^{*2}D^*E^*}}{2D^*E^*} \leq \frac{E^*}{\bar{X}_2^2 n}$$

3.2 Estimation of the Population Ratio of Mean for the Current Occasion in the Presence of Non-response on the First Occasion: When there is Non-response only on the first occasion, the Minimum Variance Linear Unbiased Estimator for the population Ratio on current occasion can be obtained as follows:

$$\hat{R}^*_{21} = a\left(\hat{R}^*_{1u} - \hat{R}^*_{1m}\right) + c\hat{R}_{2m} + (1-c)\hat{R}_{2u}$$

where

$$\hat{R}_{2m} = \frac{\bar{y}_{2m}}{\bar{x}_{2m}} \quad and \quad \hat{R}_{2u} = \frac{\bar{y}_{2u}}{\bar{x}_{2u}}$$

The variance of $\hat{R}^*_{21}$ is given by

$$V(\hat{R}^*_{21}) = a^2\left(\frac{1}{q} + \frac{1}{p}\right)\frac{1}{n\bar{X}_1^2}D^* + c^2\frac{1}{pn\bar{X}_2^2}B + (1-c)^2\frac{1}{qn\bar{X}_2^2}B - 2ac\frac{1}{pn\bar{X}_1\bar{X}_2}k_1$$

which is minimum, when

$$a_{opt} = \frac{pq\bar{X}_1 Bk_1}{\bar{X}_2(D^*B - q^2k_1^2)}$$

and

$$c_{opt} = \frac{pD^*B}{D^*B - q^2k_1^2}$$

where

$$k_1 = Cov\,(y_1, y_2) - R_1\,Cov\,(y_2, x_1) - R_2\,Cov\,(y_1, x_2) + R_1R_2\,Cov\,(x_1, x_2)$$

$$V(\hat{R}_{2m}) = \frac{1}{pn\bar{X}_2^2}B; \quad V(\hat{R}_{2u}) = \frac{1}{qn\bar{X}_2^2}B$$

Thus the Estimator $\hat{R}^*_{21}$ turns out to be

$$\hat{R}_{21}^{**} = \frac{pq\bar{X}_1 B k_1}{\bar{X}_2(D^*B - q^2 k_1^{*2})}\left(\hat{R}_{1u}^{*} - \hat{R}_{1m}^{*}\right) + \frac{pD^*B}{D^*B - q^2 k_1^{*2}}\hat{R}_{2m} + \left(1 - \frac{pD^*B}{D^*B - q^2 k_1^{*2}}\right)\hat{R}_{2u}$$

with the variance

$$V(\hat{R}_{21}^{**}) = \frac{B}{\bar{X}_2^2 n}\frac{D^*B - qk_1^2}{D^*B - q^2k_1^2}$$

while if $W_2 = 0$, i.e., there is Non-response, the $V(\hat{R}_{21}^{**})$ reduces to

$$V(\hat{R}_2) = \frac{B}{\bar{X}_2^2 n}\frac{AB - qk_1^2}{AB - q^2k_1^2}$$

where $\hat{R}_2$ is the usual Estimator of the Ratio of mean for the current occasion in the context of sampling on two occasions when there is complete Response, that is,

$$\hat{R}_2 = a\,\hat{R}_{1u} + b\,\hat{R}_{1m} + c\,\hat{R}_{2m} + d\,\hat{R}_{2u}.$$

The optimum fraction to be unmatched is given by

$$q_{opt}^{(1)} = \frac{D^*B - \sqrt{D^{*2}B^2 - k_1^2 D^* B}}{k_1^2} \tag{3.2.1}$$

and thus the minimum variance of $\hat{R}_{21}^{**}$ is,

$$V_{min}(\hat{R}_{21}^{**}) = \frac{B}{\bar{X}_2^2 n}\frac{D^*B + \sqrt{D^{*2}B^2 - k_1^2 D^* B}}{2D^*B}$$

3.3 Estimation of the Population Ratio of Mean for the Current Occasion in the Presence of Non-response on the Second Occasion: When there is Non-response only on the second occasion, the Minimum Variance Linear Unbiased Estimator for the population Ratio on current occasion can be obtained as follows:

$$\hat{R}_{22}^{*} = a\left(\hat{R}_{1u} - \hat{R}_{1m}\right) + c\hat{R}_{2m}^{*} + (1-c)\hat{R}_{2u}^{*}$$

where

$$\hat{R}_{1m} = \frac{\bar{y}_{1m}}{\bar{x}_{1m}} \quad and \quad \hat{R}_{1u} = \frac{\bar{y}_{1u}}{\bar{x}_{1u}}$$

The variance of $\hat{R}_{22}^{*}$ is given by

$$V(\hat{R}_{22}^{*}) = a^2\left(\frac{1}{q} + \frac{1}{p}\right)\frac{1}{n\bar{X}_1^2}A + c^2\frac{1}{pn\bar{X}_2^2}E^* + (1-c)^2\frac{1}{qn\bar{X}_2^2}E^* - 2ac\frac{1}{pn\bar{X}_1\bar{X}_2}k_1$$

which is minimum when

$$a_{opt} = \frac{pq\bar{X}_1 E^* k_1}{\bar{X}_2(AE^* - q^2 k_1^2)} \quad and \quad c_{opt} = \frac{pAE^*}{AE^* - q^2 k_1^2}$$

where

$$k_1 = Cov\,(y_1, y_2) - R_1\, Cov\,(y_2, x_1) - R_2\, Cov\,(y_1, x_2) + R_1 R_2\, Cov\,(x_1, x_2)$$

$$V(\hat{R}_{1m}) = \frac{1}{pn\bar{X}_1^2} A; \quad V(\hat{R}_{1u}) = \frac{1}{qn\bar{X}_1^2} A$$

Thus the Estimator $\hat{R}_{22}^{*}$ turns out to be

$$\hat{R}_{22}^{**} = \frac{pq\bar{X}_1 E^* k_1}{\bar{X}_2(AE^* - q^2 k_1^2)}\left(\hat{R}_{1u} - \hat{R}_{1m}\right) + \frac{pAE^*}{AE^* - q^2 k_1^2}\hat{R}_{2m}^{*} + \left(1 - \frac{pAE^*}{AE^* - q^2 k_1^2}\right)\hat{R}_{2u}^{*}$$

with the variance

$$V(\hat{R}_{22}^{**}) = \frac{E^*}{\bar{X}_2^2 n}\frac{E^* A - qk_1^2}{E^* A - q^2 k_1^2}$$

while if $W_2 = 0$, i.e., there is Non-response, the $V\,(\hat{R}_{22}^{**})$ reduces to

$$V(\hat{R}_2) = \frac{B}{\bar{X}_2^2 n}\frac{AB - qk_1^2}{AB - q^2 k_1^2}$$

where $\hat{R}_2$ is the usual Estimator of the Ratio of mean for the current occasion in the context of sampling on two occasions when there is complete Response, that is,

$\hat{R}_2 = a\,\hat{R}_{1u} + b\,\hat{R}_{1m} + c\,\hat{R}_{2m} + d\,\hat{R}_{2u}$.

The optimum fraction to be unmatched is given by

$$q_{opt}^{(2)} = \frac{AE^* - \sqrt{E^{*2}A^2 - k_1^2 E^* A}}{k_1^2} \tag{3.3.1}$$

and thus the minimum variance of $\hat{R}_{22}^{**}$ is,

$$V_{min}(\hat{R}_{22}^{**}) = \frac{E^*}{\bar{X}_2^2 n}\frac{E^* A + \sqrt{E^{*2}A^2 - k_1^2 E^* A}}{2E^* A}$$

3.4 Comparison between Variances of the Estimators, $\hat{R}_2$, $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$: In this subsection, we carry out an analysis based on the loss in precision of $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$ with respect to $\hat{R}_2$. This loss is expressed in percentage and given by

$$L_{12} = \left[\frac{V(\hat{R}_2^{**})}{V(\hat{R}_2)} - 1\right] \times 100,$$

$$L_1 = \left[\frac{V\left(\hat{R}_{21}^{**}\right)}{V\left(\hat{R}_2\right)} - 1\right] \times 100,$$

$$and \quad L_2 = \left[\frac{V(\hat{R}_{22}^{**})}{V(\hat{R}_2)} - 1\right] \times 100$$

respectively. Now, we assume that

$C_{y_1} = C_{y_2} = C_{x_1} = C_{x_2} = C_0,$
$C_{y_1(2)} = C_{y_2(2)} = C_{x_1(2)} = C_{x_2(2)} = C_{0(2)}$
$\rho_1 = \rho_2 = \rho_3 = \rho_4 = \rho$
$\rho_{1(2)} = \rho_{2(2)} = \rho_{3(2)} = \rho_{4(2)} = \rho_{(2)}$
$\rho_5 = \rho_6 = \rho_0, \quad \rho_{5(2)} = \rho_{6(2)} = \rho_{0(2)}$

The expressions of D^*, E^* and C^* becomes

$$D^* = 2\bar{Y}_1^2 d, \quad E^* = 2\bar{Y}_2^2 d, \quad and \quad C^* = 2\bar{Y}_1\bar{Y}_2 t$$

where

$$d = (1-\rho)C_0^2 + W_2(k-1)\left(1-\rho_{(2)}\right)C_{0(2)}^2, \quad t = (\rho_0 - \rho)C_0^2 + W_2(k-1)\left(\rho_{0(2)} - \rho_{(2)}\right)C_{0(2)}^2$$

$$L_{12} = \left[\frac{d^2 - qt^2}{d^2 - q^2t^2}\frac{d}{(1-\rho)C_0^2}\frac{(1-\rho)^2 - (\rho_0-\rho)^2q^2}{(1-\rho)^2 - (\rho_0-\rho)^2q} - 1\right] \times 100$$

$$L_1 = \left[\frac{d(1-\rho) - (\rho_0-\rho)^2C_0^2q}{d(1-\rho) - (\rho_0-\rho)^2C_0^2q^2}\frac{(1-\rho)^2 - (\rho_0-\rho)^2q^2}{(1-\rho)^2 - (\rho_0-\rho)^2q} - 1\right] \times 100$$

$$L_2 = \left[\frac{d(1-\rho) - (\rho_0-\rho)^2C_0^2q}{d(1-\rho) - (\rho_0-\rho)^2C_0^2q^2}\frac{(1-\rho)^2 - (\rho_0-\rho)^2q^2}{(1-\rho)^2 - (\rho_0-\rho)^2q}\frac{d}{(1-\rho)C_0^2} - 1\right] \times 100$$

The losses in precision of $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$ with respect to $\hat{R}_2$ for different values of C_0, $C_{0(2)}$, ρ, ρ_0, $\rho_{(2)}$ and $\rho_{0(2)}$. are presented in Tables 1-2. It is assumed that $N = 300$ and $n = 50$. From these tables, we obtain the following conclusions:

- For the case $C_0 < C_{0(2)}$, the loss in precision of $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$ with respect to $\hat{R}_2$ increases as the values of $C_{0(2)}$ increase.
- For the case $C_0 > C_{0(2)}$, the loss in precision of $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$ with respect to $\hat{R}_2$ decreases as the values of C_0 increase.
- For the case $C_0 = C_{0(2)}$, the loss in precision of all the Estimators with respect to $\hat{R}_2$ remain constant as the values of C_0 and $C_{0(2)}$ increase.
- For the case $\rho > \rho_0$, the loss in precision of all the Estimators with respect to $\hat{R}_2$ decreases as the values of ρ_0 increase.
- For the case $\rho < \rho_0$, the loss in precision of $\hat{R}_{21}^{**}$ with respect to $\hat{R}_2$ decreases as the values of ρ increase, whereas the loss in precision of $\hat{R}_2^{**}$ and $\hat{R}_{22}^{**}$ with respect to $\hat{R}_2$ increases as the values of ρ increase.

- For the case $\rho = \rho_0$, the loss in precision of $\hat{R}_2^{**}$ and $\hat{R}_{22}^{**}$ with respect to $\hat{R}_2$ increases as the values of ρ and ρ_0 increase, whereas the loss in precision of $\hat{R}_{21}^{**}$ remain constant as the values of ρ and ρ_0 increase.
- For the case $\rho_{(2)} > \rho_{0(2)}$, the loss in precision of $\hat{R}_2^{**}$ with respect to $\hat{R}_2$ increases as the values of $\rho_{0(2)}$ increase, whereas the loss in precision of $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$ with respect to $\hat{R}_2$ remains constant as the values of $\rho_{0(2)}$ increase.
- For the case $\rho_{(2)} < \rho_{0(2)}$, the loss in precision $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$ with respect to $\hat{R}_2$ decreases as the values of $\rho_{(2)}$ increase.
- For the case $\rho_{(2)} = \rho_{0(2)}$, the loss in precision of $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$ with respect to $\hat{R}_2$ decreases as the values of $\rho_{(2)}$ and $\rho_{0(2)}$ increase.
- The loss in precision of $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$ with respect to $\hat{R}_2$ increases as the values of W_2 increase.
- The loss in precision of $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$ with respect to $\hat{R}_2$ increases as the values of $k-1$ increase.
- The loss in precision of $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$, $\hat{R}_{22}^{**}$ with respect to $\hat{R}_2$ increases as the values of q increase.

4. Comparing Estimators in terms of Survey Cost

We give some ideas about how saving in cost through mail surveys in the context of Successive sampling on two occasions for different assumed values of C_0, $C_{0(2)}$, ρ, ρ_0, $\rho_{(2)}$, $\rho_{0(2)}$, W_2, $(k-1)$ and q. Let $N = 300$, $n = 50$, $c_0 = 1$, $c_1 = 4$, and $c_2 = 45$ (see Choudhary et al., 2004) where c_0, c_1, and c_2 denote the cost per unit for mailing a questionnaire, processing the results from the first attempt respondents, and collecting data through personal interview, respectively. In addition, C_{00} is the total cost incurred for collecting the data by personal interview from the whole sample, i.e., when there is no Non-Response. The Cost Function in this case is given by (assuming the cost incurred on data collection for the matched and unmatched portion of the sample are same and cost incurred on the data collection on both occasions is same).

$$C_{00}^{*} = 2nc_2. \tag{4.1}$$

Substituting the values of n and c_2 in equation (4.1), the total cost work out to be 4500.

Let n_1 denotes the number of units which respond at the first attempt and n_2 denotes the number of units which do not respond. Thus,

1. The Cost Function for the case when there is Non-Response on both occasions is,

$$C_0^* = 2\left[c_0 n + c_1 n_1 + \frac{c_2 n_2}{k-1}\right].$$

The expected cost is given by

$$\mathrm{E}(C_0^*) = 2n_0^*\left[c_0 + c_1 W_1 + \frac{c_2 W_2}{k-1}\right],$$

where $W_1 = N_1/N$ and $W_2 = N_2/N$, such that $W_1 + W_2 = 1$ and

$$n_0^* = \left[\frac{d^2 - qt^2}{d^2 - q^2t^2}\frac{d}{(1-\rho)C_0^2}\frac{(1-\rho)^2 - (\rho_0-\rho)^2 q^2}{(1-\rho)^2 - (\rho_0-\rho)^2 q}\right]$$

2. The Cost Function for the case when there is only Non-Response on the second occasion is,

$$C_1^* = 2c_0 n + c_1 n + \left[c_1 n_1 + \frac{c_2 n_2}{k-1}\right]$$

and the expected cost is given by

$$\mathrm{E}(C_1^*) = n_1^*\left[2c_0 + c_1(W_1+1) + \frac{c_2 W_2}{k-1}\right].$$

where

$$n_1^* = \left[\frac{d(1-\rho) - (\rho_0-\rho)^2 C_0^2 q}{d(1-\rho) - (\rho_0-\rho)^2 C_0^2 q^2}\frac{(1-\rho)^2 - (\rho_0-\rho)^2 q^2}{(1-\rho)^2 - (\rho_0-\rho)^2 q}\right]$$

3. The Cost Function for the case when there is Non-Response on first occasion only is,

$$C_2^* = \left[c_1 n_1 + \frac{c_2 n_2}{k-1}\right] + 2c_0 n + c_1 n,$$

which expected cost is expressed as:

$$\mathrm{E}(C_2^*) = n_2^*\left[2c_0 + c_1(W_1+1) + \frac{c_2 W_2}{k-1}\right].$$

where

$$n_2^* = \left[\frac{d(1-\rho) - (\rho_0-\rho)^2 C_0^2 q}{d(1-\rho) - (\rho_0-\rho)^2 C_0^2 q^2}\frac{(1-\rho)^2 - (\rho_0-\rho)^2 q^2}{(1-\rho)^2 - (\rho_0-\rho)^2 q}\frac{d}{(1-\rho)C_0^2}\right]$$

By equating the variances $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$, and $\hat{R}_{22}^{**}$, respectively, to $V(\hat{R}_2)$ and using the assumed values of different parameters, the values of the sample size for the three cases and the corresponding expected cost of survey were determined with respect of $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$, and $\hat{R}_{22}^{**}$. The sample sizes associated with the three Estimators which provide equal precision to the Estimator $\hat{R}_2$ are denoted by n_0^*, n_1^* and n_2^*. The results of this exercise are presented in Tables 3-4. From these tables, we obtain the following conclusions:

- For the case $C_0 < C_{0(2)}$, the saving in cost for $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$ decreases as the values of $C_{0(2)}$ increase. The sample sizes for $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$, which have the same precision than $\hat{R}_2$, increase as the values of $C_{0(2)}$ increase.
- For the case $C_0 > C_{0(2)}$, the saving in cost for $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$ increases as the values of C_0 increase. The sample sizes for $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$ which have the same precision than $\hat{R}_2$, decrease as the values of C_0 increase.
- For the case $C_0 = C_{0(2)}$ the saving in cost for all the Estimators remains constant as the values of C_0 and $C_{0(2)}$ increase. The sample sizes for all the Estimators, which have the same precision than $\hat{R}_2$, remain constant as the values of C_0 and $C_{0(2)}$ increase.
- For the case $\rho > \rho_0$, the saving in cost for all the Estimators increases as the values of ρ_0 increase. The sample sizes for the three Estimators, which have the same precision than $\hat{R}_2$, decreases as the values of ρ_0 increase.
- For the case $\rho < \rho_0$, the saving in cost for $\hat{R}_2^{**}$ and $\hat{R}_{22}^{**}$ the saving in cost decreases as the values of ρ increase, whereas for $\hat{R}_{21}^{**}$ the saving in cost increases as the values of ρ increase. The sample sizes for $\hat{R}_2^{**}$ and $\hat{R}_{22}^{**}$, which give equal precision to $\hat{R}_2$ increase as the values of ρ increase, whereas the sample size for $\hat{R}_{21}^{**}$, which has the same precision than $\hat{R}_2$, remains constant as the values of ρ increase.
- For the case $\rho = \rho_0$, the saving in cost for all the Estimators decreases as the values of ρ and ρ_0 increase. The sample sizes for the three Estimators, which have the same precision than $\hat{R}_2$, increases as the values of ρ and ρ_0 increase.
- For the case $\rho_{(2)} > \rho_{0(2)}$, the saving in cost for $\hat{R}_2^{**}$ decreases as the values of $\rho_{0(2)}$ increase, whereas for $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$ the saving in cost

remains constant as the values of $\rho_{0(2)}$ increase. The sample size for $\hat{R}_2^{**}$, which have the same precision than $\hat{R}_2$, increases as the values of $\rho_{0(2)}$ increase, whereas for $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$ which give equal precision to $\hat{R}_2$ remains constant as the values of $\rho_{0(2)}$ increase.

- For the case $\rho_{(2)} < \rho_{0(2)}$, the saving in cost for all the Estimators increases as the values of $\rho_{(2)}$ increase. The sample sizes for $\hat{R}_2^{**}$ and $\hat{R}_{22}^{**}$, which have the same precision than $\hat{R}_2$, decreases as the values of $\rho_{(2)}$ increase, whereas the sample size for $\hat{R}_{21}^{**}$, which have the same precision than $\hat{R}_2$ remains constant as the values of $\rho_{(2)}$.
- For the case $\rho_{(2)} = \rho_{0(2)}$, the saving in cost for all the Estimators increases as the values of of $\rho_{(2)}$ and $\rho_{0(2)}$ increase. The sample sizes for $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$, which have the same precision than $\hat{R}_2$, decrease as the values of $\rho_{(2)}$ and $\rho_{0(2)}$ increase.
- The saving in cost for $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$ decreases as the values of W_2 increase. The sample sizes for $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$, which have the same precision than $\hat{R}_2$, increases as the values of W_2 increase.
- The saving in cost for $\hat{R}_2^{**}$ and $\hat{R}_{21}^{**}$ increases as the values of $k-1$ increase whereas for $\hat{R}_{22}^{**}$ the saving in cost decreases as the values of $k-1$ increase. The sample sizes for $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$, which have the same precision than $\hat{R}_2$, increases as the values of $k-1$ increase.
- The saving in cost for all the Estimators decreases as the values of q increase. The sample sizes for $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$, which have the same precision than $\hat{R}_2$, increases as the values of q increase.

5. Conclusions

In this paper, we have used the HH Technique for estimating the population Ratio of mean in mail surveys. This problem is conducted for current occasion in the context of sampling on two occasions when there is Non-Response (i) on both occasions, (ii) only on the first occasion and (iii) only on the second occasion. The results obtained reveals that the loss in precision is maximum for the estimation of the Ratio of mean when there is Non-Response only on the second occasion, whereas it is least for the estimation of the Ratio of mean when there is Non-Response on both occasions and when there is Non-Response only on the first occasion. Also, we derive the sample sizes and the saving in cost for all the Estimators, which have the same precision than the Estimator of the population

Ratio of mean when there is no non-Response. In the majority of the cases the sample sizes and the saving in cost is maximum for the estimation of the Ratio of mean when there is Non-Response on both occasions, whereas it is least for the estimation of the Ratio of mean when there is Non-Response only on the first occasion and when there is Non-Response only on the second occasion.

Table 1: Loss in precision, expressed in percentage, of $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$ with respect to $\hat{R}_2$ for different values of C_0, $C_{0(2)}$, ρ, ρ_0, $\rho_{(2)}$ and $\rho_{0(2)}$

ρ	ρ_0	$\rho_{(2)}$	$\rho_{0(2)}$	$(k-1)$	W_2	C_0	$C_{0(2)}$	q	$L_{(12)}$	$L_{(1)}$	$L_{(2)}$
				$C_0 < C_{0(2)}$							
0.6	0.2	0.5	0.3	1.5	0.8	0.4	1	0.7	1577.23	66.39	1626.28
0.6	0.2	0.5	0.3	1.5	0.8	0.4	1.5	0.7	3497.89	68.35	3619.43
0.6	0.2	0.5	0.3	1.5	0.8	0.4	2	0.7	6185.67	69.06	6408.84
				$C_0 > C_{0(2)}$							
0.6	0.4	0.4	0.2	0.5	0.6	0.3	0.2	0.5	20.98	1.26	21.51
0.6	0.4	0.4	0.2	0.5	0.6	0.5	0.2	0.5	7.56	0.51	7.75
0.6	0.4	0.4	0.2	0.5	0.6	0.7	0.2	0.5	3.86	0.27	3.95
				$C_0 = C_{0(2)}$							
0.6	0.3	0.5	0.3	1	0.7	0.1	0.1	0.3	98.17	6.82	100.28
0.6	0.3	0.5	0.3	1	0.7	0.3	0.3	0.3	98.17	6.82	100.28
0.6	0.3	0.5	0.3	1	0.7	0.8	0.8	0.3	98.17	6.82	100.28
				$\rho > \rho_0$							
0.7	0.3	0.6	0.7	0.5	0.8	0.5	0.6	0.4	357.08	84.62	261.85
0.7	0.4	0.6	0.7	0.5	0.8	0.5	0.6	0.4	167.79	21.33	137.81
0.7	0.5	0.6	0.7	0.5	0.8	0.5	0.6	0.4	120.32	6.59	108.92
				$\rho < \rho_0$							
0.2	0.7	0.6	0.7	0.5	0.4	0.7	0.3	0.3	2.04	0.17	2.01
0.4	0.7	0.6	0.7	0.5	0.4	0.7	0.3	0.3	2.59	0.14	2.59
0.6	0.7	0.6	0.7	0.5	0.4	0.7	0.3	0.3	3.67	0.05	3.72
				$\rho = \rho_0$							
0.2	0.2	0.5	0.1	1.5	0.5	0.4	0.6	0.6	124.65	0	126.56
0.5	0.5	0.5	0.1	1.5	0.5	0.4	0.6	0.6	150.85	0	168.75
0.8	0.8	0.5	0.1	1.5	0.5	0.4	0.6	0.6	360.21	0	421.88
				$\rho_{(2)} > \rho_{0(2)}$							
0.7	0.6	0.5	0.2	1.5	0.8	0.5	0.6	0.4	270.52	2.08	296.07
0.7	0.6	0.5	0.3	1.5	0.8	0.5	0.6	0.4	284.46	2.08	296.07
0.7	0.6	0.5	0.4	1.5	0.8	0.5	0.6	0.4	293.52	2.08	296.07
				$\rho_{(2)} < \rho_{0(2)}$							
0.5	0.6	0.1	0.4	2.5	0.6	0.5	0.6	0.8	384.19	0.53	391.38

ρ	ρ_0	$\rho_{(2)}$	$\rho_{0(2)}$	$(k-1)$	W_2	C_0	$C_{0(2)}$	q	$L_{(12)}$	$L_{(1)}$	$L_{(2)}$
0.5	0.6	0.2	0.4	2.5	0.6	0.5	0.6	0.8	344.30	0.52	347.90
0.5	0.6	0.3	0.4	2.5	0.6	0.5	0.6	0.8	303.44	0.50	304.41
				$\rho_{(2)} = \rho_{0(2)}$							
0.6	0.2	0.5	0.5	1.5	0.6	0.2	0.6	0.5	1565.37	46.55	1530.39
0.6	0.2	0.7	0.7	1.5	0.6	0.2	0.6	0.5	955.92	44.51	922.38
0.6	0.2	0.8	0.8	1.5	0.6	0.2	0.6	0.5	650.00	42.19	618.05

Table 2: Loss in precision, expressed in percentage, of $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$ with respect to $\hat{R}_2$ for different values of W_2, $(k-1)$ and q

ρ	ρ_0	$\rho_{(2)}$	$\rho_{0(2)}$	$(k-1)$	W_2	C_0	$C_{0(2)}$	q	$L_{(12)}$	$L_{(1)}$	$L_{(2)}$
				W_2							
.6	0.2	0.5	0.5	1.5	0.2	0.2	0.6	0.8	680.69	72.29	653.77
0.6	0.2	0.5	0.5	1.5	0.4	0.2	0.6	0.8	1291.24	75.95	1263.61
0.6	0.2	0.5	0.5	1.5	0.8	0.2	0.6	0.8	2508.01	77.92	2479.87
				$(k-1)$							
.7	0.3	0.4	0.6	0.5	0.8	0.2	0.6	0.4	1922.81	134.34	1821.60
0.7	0.3	0.4	0.6	1.0	0.8	0.2	0.6	0.4	3667.69	140.70	3606.79
0.7	0.3	0.4	0.6	1.5	0.8	0.2	0.6	0.4	5406.33	142.96	5390.82
				q							
.5	0.3	0.4	0.6	1.5	0.8	0.2	0.6	0.2	1314.78	2.46	1330.29
0.5	0.3	0.4	0.6	1.5	0.8	0.2	0.6	0.4	1325.42	3.82	1349.27
0.5	0.3	0.4	0.6	1.5	0.8	0.2	0.6	0.6	1326.95	3.96	1351.28

Table 3: Sample sizes and corresponding expected cost of survey, which have the same precision than $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$, with respect to $\hat{R}_2$ for different values of C_0, $C_{0(2)}$, ρ, ρ_0, $\rho_{(2)}$ and $\rho_{0(2)}$.

ρ	ρ_0	$\rho_{(2)}$	$\rho_{0(2)}$	$(k-1)$	W_2	C_0	$C_{0(2)}$	q	n_0^*	n_1^*	n_2^*	$E(C_0^*)$	$E(C_1^*)$	$E(C_2^*)$
				$C_0 < C_{0(2)}$										
0.6	0.2	0.5	0.3	1.5	0.8	0.4	1.0	0.7	839	83	863	43272.44	2562.38	26584.72
0.6	0.2	0.5	0.3	1.5	0.8	0.4	1.5	0.7	1799	84	1860	92825.56	2592.56	57279.19
0.6	0.2	0.5	0.3	1.5	0.8	0.4	2.0	0.7	3143	85	3254	162170.33	2603.54	100236.13
				$C_0 > C_{0(2)}$										
0.6	0.4	0.4	0.2	0.5	0.6	0.3	0.2	0.5	60	51	61	6847.15	3118.68	3742.42
0.6	0.4	0.4	0.2	0.5	0.6	0.5	0.2	0.5	54	50	54	6087.93	3095.69	3318.58
0.6	0.4	0.4	0.2	0.5	0.6	0.7	0.2	0.5	52	50	52	5878.40	3088.30	3201.74

				$C_0 = C_{0(2)}$										
0.6	0.3	0.5	0.3	1	0.7	0.1	0.1	0.3	99	53	100	6678.40	2066.88	3875.40
0.6	0.3	0.5	0.3	1	0.7	0.3	0.3	0.3	99	53	100	6678.40	2066.88	3875.40
0.6	0.3	0.5	0.3	1	0.7	0.8	0.8	0.3	99	53	100	6678.40	2066.88	3875.40
				$\rho > \rho_0$										
0.7	0.3	0.6	0.7	0.5	0.8	0.5	0.6	0.4	229	92	181	33732.27	7274.01	14257.05
0.7	0.4	0.6	0.7	0.5	0.8	0.5	0.6	0.4	134	61	119	19762.95	4780.53	9369.85
0.7	0.5	0.6	0.7	0.5	0.8	0.5	0.6	0.4	110	53	104	16259.47	4199.78	8231.56
				$\rho < \rho_0$										
.2	0.7	0.6	0.7	0.5	0.4	0.7	0.3	0.3	51	50	51	4020.68	2223.85	2264.70
0.4	0.7	0.6	0.7	0.5	0.4	0.7	0.3	0.3	51	50	51	4042.06	2223.08	2277.52
0.6	0.7	0.6	0.7	0.5	0.4	0.7	0.3	0.3	52	50	52	4084.73	2221.06	2302.65
				$\rho = \rho_0$										
.2	0.2	0.5	0.1	1.5	0.5	0.4	0.6	0.6	94	50	97	3402.91	1151.66	2231.35
0.5	0.5	0.5	0.1	1.5	0.5	0.4	0.6	0.6	116	50	121	4184.56	1154.58	2778.20
0.8	0.8	0.5	0.1	1.5	0.5	0.4	0.6	0.6	183	51	195	6587.56	1174.50	4477.78
				$\rho_{(2)} > \rho_{0(2)}$										
.7	0.6	0.5	0.2	1.5	0.8	0.5	0.6	0.4	185	51	198	9559.77	1572.05	6099.54
0.7	0.6	0.5	0.3	1.5	0.8	0.5	0.6	0.4	192	51	198	9919.16	1572.05	6099.54
0.7	0.6	0.5	0.4	1.5	0.8	0.5	0.6	0.4	197	51	198	10152.91	1572.05	6099.54
				$\rho_{(2)} < \rho_{0(2)}$										
.5	0.6	0.1	0.4	2.5	0.6	0.5	0.6	0.8	242	50	246	6488.11	924.86	4520.73
0.5	0.6	0.2	0.4	2.5	0.6	0.5	0.6	0.8	222	50	224	5953.61	924.74	4120.66
0.5	0.6	0.3	0.4	2.5	0.6	0.5	0.6	0.8	202	50	202	5406.04	924.60	3720.59
				$\rho_{(2)} = \rho_{0(2)}$										
.6	0.2	0.5	0.5	1.5	0.6	0.2	0.6	0.5	833	73	815	34306.67	1875.86	20868.97
0.6	0.2	0.7	0.7	1.5	0.6	0.2	0.6	0.5	528	72	511	21752.01	1849.67	13086.42
0.6	0.2	0.8	0.8	1.5	0.6	0.2	0.6	0.5	375	71	359	15450.02	1820	9191

Table 4: Sample sizes and corresponding expected cost of survey, which have the same precision than $\hat{R}_2^{**}$, $\hat{R}_{21}^{**}$ and $\hat{R}_{22}^{**}$, with respect to $\hat{R}_2$ for different values of W_2, $(k-1)$ and q.

ρ	ρ_0	$\rho_{(2)}$	$\rho_{0(2)}$	$(k-1)$	W_2	C_0	$C_{0(2)}$	q	n_0^*	n_1^*	n_2^*	$E(C_0^*)$	$E(C_1^*)$	$E(C_2^*)$
				W_2										
0.6	0.2	0.5	0.5	1.5	0.2	0.2	0.6	0.8	390	86	377	7963.03	1309.40	5728.61
0.6	0.2	0.5	0.5	1.5	0.4	0.2	0.6	0.8	696	88	682	21425.16	1794.68	13908.80
0.6	0.2	0.5	0.5	1.5	0.8	0.2	0.6	0.8	1304	89	1290	67286.60	2740	39730
				$(k-1)$										
0.7	0.3	0.4	0.6	0.5	0.8	0.2	0.6	0.4	1011	117	961	149283.09	9233.04	75710.93

0.7	0.3	0.4	0.6	1.0	0.8	0.2	0.6	0.4	1884	120	1853	142418.76	5150.99	79325.32
0.7	0.3	0.4	0.6	1.5	0.8	0.2	0.6	0.4	2753	121	2745	142063.27	3741.53	84558.58
				q										
0.5	0.3	0.4	0.6	1.5	0.8	0.2	0.6	0.2	707	51	715	36501.43	1577.83	22026.47
0.5	0.3	0.4	0.6	1.5	0.8	0.2	0.6	0.4	713	52	725	36775.80	1598.76	22318.71
0.5	0.3	0.4	0.6	1.5	0.8	0.2	0.6	0.6	713	52	726	36815.43	1600.98	22349.70

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