ISSN 1684-8403
Journal of Statistics
Volume 21, 2014. pp. 157-174

Using Transformation and Model-Based Approach to Enhance the Ratio and Product Type Estimators

Ayesha Nazuk[1], Sadia Nadir[2] and Javid Shabbir[3]

Abstract

In this paper, we have worked on Estimators by Prasad (1986) and Lui (1991). Some Ratio and Ratio Type Estimators have been introduced under weighting, transformation and Model based approach. We have introduced Estimators efficient than Estimators proposed by Chakrabarty (1979), Singh and Singh (1997), Singh (2002) and Singh et al. (2006).

Keywords

Model based approach, Percent Relative Efficiency, Product Estimator, Ratio Estimator, Regression Estimator.

1. Introduction

Motivation behind this study is to offer better Estimates of population mean when the information on the actual study variable is not available. In other words, we are talking about the family of Ratio, Product and Regression Estimators. The objective of this study is to provide more reliable Estimates of population mean so that better prediction is possible of different phenomena that rely on information of auxiliary variables.

Many researchers have worked on introducing efficient Estimators of population mean of the study variable (Y) by incorporating an observable auxiliary variable (X).

[1] NUST Business School, National University of Sciences and Technology (NUST), Islamabad, Pakistan.
Email: ayesha.nazuk@nbs.edu.pk,
[2] Department of Basic Sciences, Rippah International University, Islamabad, Pakistan.
[3] Department of Statistics, Quaid-i-Azam University, Islamabad, Pakistan.

A necessary condition for auxiliary variable is that it is highly correlated with the study variable but the nature of relationship may or may not be positive. The researchers did not stop on just formulating the Ratio and Product Estimators but introduced several variants of these in order to further improve the efficiency of Estimators. Some of the researchers who introduced several variants of the Ratio and Product Estimators include Bandyopadhyay (1994), Singh and Singh (1997), and Singh (2000, 2002).

We propose new Estimators by the procedure of (i) weighting of existing Estimators, (ii) transformation of the variables involved in Ratio and Regression Type Estimators (iii) using a Model based approach.

We have employed Durbin (1959) Model to estimate the population mean $\left(\bar{Y}\right)$. The Durbin (1959) Model is given as below:

$$\left.\begin{array}{l} y_i = \alpha + \beta x_i + e_i\,;\ \beta < 0, \\ \text{with the assumptions} \\ (i)\ \ E(e_i \mid x_i) = 0, \\ (ii)\ \ E(e_i\, e_j \mid x_i\, x_j) = 0 \text{ for } i \neq j, \\ (iii)\ \ V(e_i \mid x_i) = n\delta,\ \delta \text{ is a constant of order } n^{-1}, \\ (iv)\ \text{The variate } x_i / n \text{ have a Gamma Distribution with the parameter } m = n\,h. \\ (v)\ \ i=1,2,3,...n \end{array}\right] \quad (1.1)$$

where

$$\left.\begin{array}{l} e_i \text{ is } a \text{ White Noise process,} \\ \alpha = \bar{Y}\left[\left(K-\rho\right)/K\right], \\ \beta = \bar{Y}\left[\left(\rho\right)/Km\right], \\ \delta = \bar{Y}^2\left[\left(1-\rho^2\right)/K^2 m\right], \\ \rho = \text{Coefficient of correlation between } X \text{ and } Y, \\ K = C_X / C_Y, C_j = \text{Coeffecient of variation of } j,\ j = X, Y \end{array}\right] \quad (1.2)$$

In this paper, we propose a new weighted Estimator of the population mean. The proposed Estimator has been compared with the Estimators proposed by

Chakrabarty (1979), Singh and Singh (1997), Singh (2002) and Singh et al. (2006). We have also presented transformed Estimators of $\bar{Y}$.

2. The Proposed Estimator

In this section, we present the proposed Estimator for different conditions on the transforming weights.

2.1 The Proposed Estimator for Weights Summing to Unity: We propose a new weighted Estimator whose weights sum up to one. The proposed Estimator is compared with the Estimators proposed by Chakrabarty (1979), Singh and Singh (1997), Singh (2002) and Singh et al. (2006). Durbin's (1959) Model has been utilized to proceed further with the estimation process.

Prasad (1986) proposed a new Estimator which is given below:

$$t_{pra} = \bar{y} - \frac{\bar{x}}{\bar{X}} + 1, \tag{2.1.1}$$

where $\bar{y}$ and $\bar{x}$ are respectively the usual sample means.

Later on Lui (1991) applied design based approach to the Estimator given above. The contribution of Lui (1991) resulted in a new Estimator which is given as follows:

$$t_L = \alpha\,\bar{y} + (1-\alpha)\left(\bar{y} - \frac{\bar{x}}{\bar{X}} + 1\right), \text{ where } \alpha \text{ is a constant.} \tag{2.1.2}$$

Now, we use the Estimator of Prasad (1986) under Model based approach.

$$\bar{y}_{prop} = d_1 t_{pra} + d_2 \bar{y}, \tag{2.1.3}$$

subject to $d_1 + d_2 = 1,$ where d_1 and d_2 are weights.

The proposed Estimator is Unbiased Estimator of population mean and its minimum variance is given as below:

$$V(\bar{y}_{prop})_{\min} = \frac{\left(\beta^2 m+\delta\right)\left(\dfrac{\beta m(\beta m-2)+m\delta+1}{m}\right) - \left\{\beta(\beta m-1)+\delta\right\}^2}{\left(\left(\beta^2 m+\delta\right)+\left(\dfrac{\beta m(\beta m-2)+m\delta+1}{m}\right)-2\left\{\beta(\beta m-1)+\delta\right\}\right)}. \tag{2.1.4}$$

2.2 The Proposed Estimator when Weights are not Summing to Unity: We propose a new weighted Estimator whose weights do not necessarily sum up to one. The target has been to improve the efficiency of estimation of the population mean, in Model based approach.

The proposed Estimator is given by

$\bar{y}_{new} = h_1 t_{pra} + h_2 \bar{y},\ h_1 + h_2 \neq 1,$ where h_1 and h_2 are weights. (2.2.1)

The Bias and minimum Mean Square Error (MSE) are respectively, given as follows:

$$B(\bar{y}_{new}) = E(\bar{y}_{new} - \bar{Y}) = \left(h_1 + h_2 - 1\right)\left(\alpha + \beta m\right), \tag{2.2.2}$$

$$MSE(\bar{y}_{new})_{\min} = h_{1(opt)}^2 A^* + h_{2(opt)}^2 B^* + h_{3(opt)}^2 C^* + 2h_{1(opt)} h_{2(opt)} D^*, \tag{2.2.3}$$

where, $h_3 = \left(h_1 + h_2\right) - 1$ and

$$\left.\begin{aligned}
&A^* = E\left(\bar{y} - \bar{Y}\right)^2,\ B^* = E\left(t_{pra} - \bar{Y}\right)^2\\
&C^* = E\left(\bar{Y}^2\right),\ D^* = E\left(\bar{y} - \bar{Y}\right)\left(t_{pra} - \bar{Y}\right),\\
&E^* = E\left(t_{pra} - \bar{Y}\right)\bar{Y},\ h_3 = h_1 + h_2 - 1,\\
&h_{1(opt)} = \frac{C^*\left(B^* - D^*\right)}{\left(A^* + C^*\right)\left(B^* + C^*\right) - \left(C^* + D^*\right)^2},\\
&h_{2(opt)} = \frac{C^*\left(A^* - D^*\right)}{\left(A^* + C^*\right)\left(B^* + C^*\right) - \left(C^* + D^*\right)^2}.
\end{aligned}\right\} \tag{2.2.4}$$

Now, minimizing $MSE(\bar{y}_{new})$ with respect to $h_i,\ i = 1, 2,$ one may easily calculate the minimum MSE of $\bar{y}_{new}$.

3. Efficiency Comparison

3.1 Efficiency Comparison when $h_1 + h_2 \neq 1$***:*** Let us define the following expression for obtaining the Percentage Relative Efficiency (PRE).

$$PRE_j = \frac{MSE\left(\bar{y}\right)_{\min}}{MSE\left(i\right)_{\min}} \times 100, \text{ where } i = T_r,\ T_p,\ \bar{y}_{new}. \tag{3.1.1}$$

Note that;

$T_p = W_{1p}\bar{y} + W_{2p}\bar{y}_p$, proposed by Singh (2002),

$\bar{y} =$ Sample Mean Estimator,

$\bar{y}_p =$ usual Product Estimator,

$\bar{y}_r =$ usual Ratio Estimator,

(W_{1r}, W_{2r}) and (W_{1p}, W_{2p}) are suitably chosen scalars whose sums may not be unity,

$T_r = W_1\,\bar{y} + W_2\,\bar{y}_r$, W_1 *and* W_2 are unknown weights, whose sum may not be unity.

In Table 1, we have compared the proposed Estimator with $\bar{y}$, T_r and T_p. The proposed Estimator has been found to be more efficient than all of these Estimators. We have considered many data sets for comparison of the competitive Estimators. However, in Table 1, we have discussed thirty six different data sets which are given in appendix. We can easily see that that the proposed Estimators are efficient even for small sample sizes. We can see that, for a fixed n, h, α and δ, the proposed Estimator becomes more efficient as β increases (For instance, see the increase in efficiency upon moving from data set 1 to data set 3). We can also note that, for a fixed h, β and δ, efficiency increases as α and n increase (compare efficiency of data set 1 with that of data set 10). Similarly, for other parameters remaining constant we can see that δ is negatively related to efficiency (Compare efficiency of data set 1 with that of data set 4).

3.2 Efficiency Comparison when $d_1 + d_2 = 1$***:*** The following expression is used to obtain the Percent Relative Efficiency (PRE).

$$PRE(l) = \frac{V(\bar{y})_{\min}}{MSE(l)_{\min}} \times 100, \text{where } l = \bar{y}_{prop}, \bar{y}_a, \bar{y}_{rc1} \qquad (3.2.1)$$

where $\bar{y}_a = (1-w)\bar{y} + w\,\bar{y}_{pi}$; $0 < w < 1$, $\bar{y}_{pi} = (p^* / \bar{X})$,

$p^* = \left[2p - 0.5(p_1 + p_2)\right]$,

$p = \bar{y}\,\bar{x}$ and $p_i = \bar{y}_i\,\bar{x}_i$, $r_{C1} = (1-W)\,\bar{y} + W\,\bar{y}_r$; $W \geq 0$, W is a constant weight and

$$r^* = 2r - 0.5(r_1 + r_2),\ r_j = \frac{\bar{y}'_j}{\bar{x}'_j} = \frac{(n\bar{y} - p\bar{y}_j)/(n-p)}{(n\bar{x} - p\bar{x}_j)/(n-p)},\ j = 1, 2.$$

In Table 2, we have compared the proposed Estimator with $\bar{y}$ and the Estimator proposed by Singh et al. (2006). Note that the proposed Estimator is more efficient than $\bar{y}$ for different values of m, k and ρ. Scrutinizing Table 3, one can easily observe the numerical supremacy of the proposed Estimator. We can easily see that the efficiency is independent of all other parameters except the absolute value of population correlation coefficient with which it is positively correlated. We can see that a stronger relationship, demonstrated by a higher absolute value of the population correlation coefficient, yields a higher efficiency of the proposed Estimator.

3.3 Comparison of the Proposed Estimator $\bar{y}_{new}$ with Chakrabarty (1979), under varying Weights: In Table 4 and 5, all the comparisons are done under the varying weights situation. We have chosen different values of m, K and ρ under the varying weights of the proposed Estimator and r_{C1}. Following the convention by many researchers like Rao and Webster (1966) we have taken $\bar{Y} = 6$ across all the numerical computations.

Analyzing the numerical results we can easily conclude that under non-optimum weights, the proposed Estimator $\bar{y}_{prop}$, in which sum of the weights in assumed to be equal to one, is efficient than r_{C1} and $\bar{y}$. In Table 4 and Table 5, we can see that as $|\rho|$ increases the efficiency increases (See Data set 100 and 103), as m increases so does the efficiency and same is the case with k. (Data set 100, 101 and 118 may be seen).

4. Transformed Estimator

In this section, we introduce some variants of the proposed Estimator. Many researchers have used the transformation technique to further reduce the Bias and enhance the efficiency of estimation. Some of such researchers are Chakrabarty (1979), Srivenkataramana and Tracy (1986) and Mohanty and Sahoo (1995). We have also used the transformation technique of Mohanty and Sahoo (1995) to enhance the efficiency of estimation.

4.1 Transformed Estimator: Here, we present the setup for the transformation and apply it in our scenario. Let us have a finite population of N units, represented by $\Omega = \{(X_1, Y_1), (X_2, Y_2), (X_3, Y_3), ...(Y_N, Y_N)\}$. Let X, Y be two positively correlated random variables and let (x_i, y_i), $i = 1, 2, ...N$ be a simple random sample of size n. Using the transformation presented by Mohanty and Sahoo (1995), we have

$$\left.\begin{aligned} & u_i=\frac{x_i+x_m}{x_M+x_m},\ z_i=\frac{x_i+x_M}{x_M+x_m}, \\ & \text{then we have } \bar{u}=\frac{\bar{x}+x_m}{x_M+x_m},\ \bar{Z}=\frac{\bar{X}+x_M}{x_M+x_m},\ \bar{z}=\frac{\bar{x}+x_M}{x_M+x_m}, \text{ and } \bar{U}=\frac{\bar{X}+x_m}{x_M+x_m}. \end{aligned}\right\} \tag{4.1.1}$$

where x_M and x_m are respectively the minimum and maximum values of x. Also $(\bar{z},\bar{u})$ and $(\bar{Z},\bar{U})$ are respectively the sample and population means of transformed variables.

Now, we present the two transformed Estimators of $\bar{Y}$.

$$\left.\bar{y}_{tran(1)}=\bar{y}-\frac{\bar{u}}{\bar{U}}+1 \text{ and } \bar{y}_{tran(2)}=\bar{y}-\frac{\bar{z}}{\bar{Z}}+1\right\}. \tag{4.1.2}$$

Incorporating the transformation given in equation (4.1.1) we present a modified version of the proposed Estimator.

$$\bar{y}_{tran(1)}=\bar{y}-\frac{\left(\bar{x}+\bar{X}-\bar{X}+x_m\right)/\bar{X}}{\frac{\bar{X}}{\bar{X}}+\frac{x_m}{\bar{X}}}+1, \tag{4.1.3}$$

$$\bar{y}_{tran(1)}=\bar{y}-\frac{e_1}{c_1},\ \text{where } e_1=\frac{\bar{x}-\bar{X}}{\bar{X}}, c_1=1+\frac{x_m}{X}.$$

The Bias and variance of this Estimator are evaluated by using the difference equation given as below:

$$\bar{y}_{tran(1)}-\bar{Y}=\bar{y}-\frac{e_1}{c_1}-\bar{Y}, \tag{4.1.4}$$

Upon substitution of the values of $\bar{Y}$ and $\bar{y}$ from Durbin (1959) Model in equation (4.1.4), the difference equation takes on the following form:

$$\bar{y}_{tran(1)}-\bar{Y}=\beta\left(\bar{x}-m\right)+\bar{u}-\frac{e_1}{c_1}. \tag{4.1.5}$$

The Bias and variance of $\bar{y}_{tran(1)}$ are respectively given as follows:

$$B\left(\bar{y}_{tran(1)}\right)=0. \tag{4.1.6}$$

$$V\left(\bar{y}_{tran(1)}\right)=\beta^2 m+\delta+\frac{1}{c_1^2 m}-\frac{2\beta}{c_1}. \tag{4.1.7}$$

Similarly one may develop the expressions for Bias of $\bar{y}_{tran(2)}$. The Bias and variance of $\bar{y}_{tran(2)}$ are, respectively, given as follows:

$$B\left(\bar{y}_{tran(2)}\right)=0. \quad (4.1.8)$$

$$V\left(\bar{y}_{tran(2)}\right)=\beta^2 m+\delta+\frac{1}{c_2^2 m}-\frac{2\beta}{c_2}, \text{ where } c_2=1+\frac{x_M}{\bar{X}}. \quad (4.1.9)$$

Now, we propose a weighted Estimator where sum of weights is equal to one.

$\bar{y}_{tran(f)} = f_2\,\bar{y} + f_1\,\bar{y}_{tran(1)}$, f_1 and f_2 are weights such that $f_1+f_2=1$. (4.1.10)

The Bias of $\bar{y}_{tran(f)}$ is zero and the its variance is given below:

$$V\left(\bar{y}_{tran(f)}\right)=\left(1-f_1\right)^2 A^* + f_1^2 B^* + 2\,f_1\left(1-f_1\right)D^*. \quad (4.1.11)$$

From equation (2.2.3), we have $A^*=E\left(\bar{y}-\bar{Y}\right)^2=\beta^2 m+\delta$. Similarly from equation (4.1.7), one may substitute the values of $B^*=V\left(\bar{y}_{tran(1)}-\bar{Y}\right)^2$.

The value of D^* is given by

$$D^* = \beta^2 m+\delta-\frac{\beta}{c_1}. \quad (4.1.12)$$

The proposed weighted Estimator (under condition that $h_1+h_2=1$) is given as follows:

$$\bar{y}_{tran(h)}=h_2\bar{y}+h_1\,\bar{y}_{tran(2)},\; h_1+h_2=1. \quad (4.1.13)$$

The Bias and MSE of $\bar{y}_{tran(h)}$ are as follows:

$$B\left(\bar{y}_{tran(h)}\right)=0. \quad (4.1.14)$$

$$MSE\left(\bar{y}_{tran(h)}\right)=h_2^2\left(\beta m+\delta\right)+h_1^2\left\{\beta^2 m+\delta+\frac{1}{mc_2^2}-\frac{2\beta}{c_2}\right\}+2h_1 h_2\left\{\beta^2 m+\delta-\frac{\beta}{c_2}\right\}. \quad (4.1.15)$$

4.2 Comparison of the Proposed Estimator, $\bar{y}_{tran(f)}$, with $\bar{y}_{prop}$: In this section, we have checked whether or not the transformation has improved the efficiency of estimation.

In Table 6, we present the numerical comparison of $\bar{y}_{tran(f)}$ with $\bar{y}_{prop}$. Numerical computations show that $\bar{y}_{tran(f)}$ performs better than $\bar{y}_{prop}$. We can see that as m

increases the efficiency remains unchanged (for instance see Data set 190 and 198). A simultaneous increase in w and ρ causes an improvement in the efficiency (See Data set 190 and 222). We can also note the direct relationship between k and efficiency (See Data set 190 and 191).

5. Conclusions

We have concentrated on Model based approach which is actually a strategy for more than one variable; one being the study variable and the rest being auxiliary, closely correlated with the study variable. We have worked on introducing new Estimators of population mean by using weighting and transformation technique in model based approach. We have successfully improved the efficiency of estimation of population mean. Proposed Estimators are efficient under optimum as well as non-optimum weights conditions. Transformation offers even better results as the Percent Relative Efficiency of transformed Estimator, with reference to proposed Estimator without transformation, is greater than hundred.

Table 1: PRE comparison of competitive Estimators (proposed Estimator with $\bar{y}$, T_r and T_p in Data Set 1- 36

Data Set	$PRE(T_r)$	$PRE(T_p)$	$PRE(\bar{y}_{new})$	*Data Set*	$PRE(T_r)$	$PRE(T_p)$	$PRE(\bar{y}_{new})$
1	224.375	205.798	318.750	19	629.737	478.420	704.142
2	611.595	346.581	914.062	20	2216.00	913.580	2504.00
3	1266.61	432.891	1913.19	21	4853.34	1125.34	5504.01
4	167.187	167.105	225.000	22	363.105	316.325	404.733
5	351.293	268.750	515.625	23	1156.64	679.751	1304.16
6	676.974	360.042	1013.88	24	2477.39	936.230	2804.09
7	153.492	156.250	197.916	25	274.490	251.382	305.325
8	265.969	229.263	383.854	26	802.858	549.989	904.320
9	481.080	313.521	714.583	27	1683.65	805.819	1904.16
10	427.338	337.539	506.920	28	830.592	620.775	902.938
11	1416.25	627.241	1706.24	29	3013.38	1201.13	3302.94
12	3061.33	777.206	3706.16	30	6643.28	1474.31	7302.97
13	261.681	235.346	308.304	31	464.117	399.006	503.265
14	754.596	470.530	906.611	32	1557.25	890.511	1703.03
15	1577.53	645.549	1906.33	33	3376.00	1228.03	3703.02
16	207.494	195.418	243.021	34	341.888	309.441	370.258
17	534.050	385.372	640.312	35	1070.59	716.268	1169.78
18	1082.40	556.548	1306.49	36	2283.84	1056.44	2503.06

Table 2: PRE comparison of proposed Estimator with $\bar{y}$ and the Estimator proposed by Singh et al. (2006), based on data sets 37-84

Data Set	$PRE(\bar{y}_{prop})$	$PRE(\bar{y}_a)$	*Data set*	$PRE(\bar{y}_{prop})$	$PRE(\bar{y}_a)$
37	104.166	102.000	64	104.166	104.056
38	104.166	103.278	65	133.333	120.000
39	104.166	103.719	66	133.333	128.571
40	104.166	103.902	67	133.333	131.034
41	133.333	112.500	68	133.333	132.000
42	133.333	123.529	69	526.315	188.621
43	133.333	128.125	70	526.315	318.328
44	133.333	130.188	71	526.315	399.507
45	526.315	140.500	72	526.315	444.314
46	526.315	226.000	73	104.166	103.278
47	526.315	307.102	74	104.166	103.902
48	526.315	367.216	75	104.166	104.044
49	104.166	102.702	76	104.166	104.097
50	104.166	103.669	77	133.333	123.529
51	104.166	103.930	78	133.333	130.188
52	104.166	104.030	79	133.333	131.858
54	133.333	127.586	81	526.315	226.070
55	133.333	130.508	82	526.315	367.216
56	133.333	131.683	83	526.315	437.109
57	526.315	173.972	84	526.315	471.080
58	526.315	294.594			
59	526.315	378.776			
60	526.315	428.517			
61	104.166	102.907			
62	104.166	103.759			
63	104.166	103.975			

Table 3: PRE comparison of $\bar{y}_{prop}$ with r_{C1}, under optimum weights

Data Set	$PRE(\bar{y}_{prop})$	$PRE(r_{C1})$
85	104.17	100.14
86	104.17	100.40
87	104.17	100.99
88	109.89	102.55
89	109.89	105.24
90	109.89	106.24
91	119.05	108.71
92	119.05	112.97
93	119.05	114.19
94	133.33	119.59

95	133.33	125.70
96	133.33	127.06
97	156.25	139.62
98	156.25	147.78
99	156.25	148.99

Table 4: PRE comparison of $\bar{y}_{prop}$ with r_{C1}, under varying weights $(d_1 = 0.4)$

Data set	$PRE(\bar{y}_{prop})$	$PRE(r_{C1})$	*Data set*	$PRE(\bar{y}_{prop})$	$PRE(r_{C1})$
100	101.2373	89.67001	123	105.2632	90.52984
101	102.2727	81.43575	124	102.6226	108.2882
102	103.0928	66.97454	125	105.1402	111.7998
103	101.9253	94.30912	126	107.5269	103.2659
104	103.6866	89.67001	127	101.2373	100.8159
105	105.2632	75.61791	128	102.2727	95.66547
106	102.6226	99.33399	129	103.0928	83.87131
107	105.1402	99.63562	130	101.9253	105.3779
108	107.5269	86.73099	131	103.6866	104.2889
109	101.2373	97.29573	132	105.2632	94.12864
110	102.2727	91.13444	133	102.6226	110.3456
111	103.0928	78.34044	134	105.1402	114.5921
112	101.9253	101.8879	135	107.5269	107.2191
113	103.6866	99.65983	136	101.2373	99.66217
114	105.2632	88.10026	137	102.2727	94.17752
115	102.6226	106.8794	138	103.0928	82.04015
116	105.1402	109.8860	139	101.9253	104.2346
117	107.5269	100.5886	140	103.6866	102.7713
118	158.6183	151.2097	141	105.2632	92.13616
119	102.2727	92.97116	142	102.6226	109.2106
120	103.0928	80.56613	143	105.1402	113.0522
121	101.9253	103.3058	144	107.5269	105.0322
122	103.6866	101.5391			

Table 5: PRE comparison of $\bar{y}_{prop}$ with r_{C1}, under varying weights $(d_1 = 0.6)$

Data set	$PRE(\bar{y}_{prop})$	$PRE(r_{C1})$	*Data set*	$PRE(\bar{y}_{prop})$	$PRE(r_{C1})$
145	101.7812	80.51159	171	110.8033	78.35698
146	103.0928	63.92461	172	101.7812	97.62082
147	103.8961	44.85339	173	103.0928	82.88699
148	102.8278	86.73672	174	103.8961	62.53389
149	105.2632	72.50580	175	102.8278	104.3057
150	107.2386	51.32567	176	105.2632	93.11681
151	103.8961	93.76339	177	107.2386	71.44385
152	107.5269	83.55615	178	103.8961	111.9114

153	110.8033	59.88229	179	107.5269	106.1714
154	101.7812	92.03551	180	110.8033	83.28032
155	103.0928	76.51812	181	101.7812	95.77181
156	103.8961	56.36724	182	103.0928	80.76008
157	102.8278	98.58933	183	103.8961	60.44857
158	105.2632	86.23068	184	102.8278	102.4152
159	107.2386	64.43679	185	105.2632	90.82111
160	103.8961	106.0246	186	107.2386	69.07556
161	107.5269	98.66047	187	103.8961	109.9663
162	110.8033	75.14084	188	107.5269	103.6722
163	101.2373	67.20430	189	110.8033	80.53057
164	103.0928	79.06125			
165	103.8961	58.80168			
166	102.8278	100.8921			
167	105.2632	88.98464			
168	107.2386	67.20430			
169	103.8961	108.3980			
170	107.5269	101.6695			

Table 6: Comparison of $\bar{y}_{tran(f)}$ with $\bar{y}_{prop}$

Data Set	m	k	ρ	w	$PRE\left(\bar{y}_{tran(f)}\right)$	*Data Set*	m	k	ρ	w	$PRE\left(\bar{y}_{tran(f)}\right)$
190	8	0.5	-0.5	0.25	100.7086	222	8	0.5	-0.75	0.75	104.2123
191	8	1	-0.5	0.25	101.4441	223	8	1	-0.75	0.75	108.4884
192	8	1.5	-0.5	0.25	102.2047	224	8	1.5	-0.75	0.75	112.7929
193	8	2	-0.5	0.25	102.9883	225	8	2	-0.75	0.75	117.0962
194	16	0.5	-0.5	0.25	100.7086	226	16	0.5	-0.75	0.75	104.2123
195	16	1	-0.5	0.25	101.4441	227	16	1	-0.75	0.75	108.4884
196	16	1.5	-0.5	0.25	102.2047	228	16	1.5	-0.75	0.75	112.7929
197	16	2	-0.5	0.25	102.9883	229	16	2	-0.75	0.75	117.0962
198	20	0.5	-0.5	0.25	100.7086	230	20	0.5	-0.75	0.75	104.2123
199	20	1	-0.5	0.25	101.4441	231	20	1	-0.75	0.75	108.4884
200	20	1.5	-0.5	0.25	102.2047	232	20	1.5	-0.75	0.75	112.7929
201	20	2	-0.5	0.25	102.9883	233	20	2	-0.75	0.75	117.0962
202	24	0.5	-0.5	0.25	100.7086	234	24	0.5	-0.75	0.75	104.2123
203	24	1	-0.5	0.25	101.4441	235	24	1	-0.75	0.75	108.4884
204	24	1.5	-0.5	0.25	102.2047	236	24	1.5	-0.75	0.75	112.7929
205	24	2	-0.5	0.25	102.9883	237	24	2	-0.75	0.75	117.0962
206	8	0.5	-0.6	0.5	101.9195	238	8	0.5	-0.9	0.9	106.3410
207	8	1	-0.6	0.5	103.9177	239	8	1	-0.9	0.9	112.5540
208	8	1.5	-0.6	0.5	105.9800	240	8	1.5	-0.9	0.9	118.6153
209	8	2	-0.6	0.5	108.0932	241	8	2	-0.9	0.9	124.5091
210	16	0.5	-0.6	0.5	101.9195	242	16	0.5	-0.9	0.9	106.3410

211	16	1	-0.6	0.5	103.9177	243	16	1	-0.9	0.9	112.5540
212	16	1.5	-0.6	0.5	105.9800	244	16	1.5	-0.9	0.9	118.6153
213	16	2	-0.6	0.5	108.0932	245	16	2	-0.9	0.9	124.5091
214	20	0.5	-0.6	0.5	101.9195	246	20	0.5	-0.9	0.9	106.3410
215	20	1	-0.6	0.5	103.9177	247	20	1	-0.9	0.9	112.5540
216	20	1.5	-0.6	0.5	105.9800	248	20	1.5	-0.9	0.9	118.6153
217	20	2	-0.6	0.5	108.0932	249	20	2	-0.9	0.9	124.5091
218	24	0.5	-0.6	0.5	101.9195	250	24	0.5	-0.9	0.9	106.3410
219	24	1	-0.6	0.5	103.9177	251	24	1	-0.9	0.9	112.5540
220	24	1.5	-0.6	0.5	105.9800	252	24	1.5	-0.9	0.9	118.6153
221	24	2	-0.6	0.5	108.0932	253	24	2	-0.9	0.9	124.5091
254	8	0.5	-0.5	0.25	101.2757	286	8	0.5	-0.75	0.75	108.7904
255	8	1	-0.5	0.25	102.6017	287	8	1	-0.75	0.75	118.1853
256	8	1.5	-0.5	0.25	103.9762	289	8	1.5	-0.75	0.75	128.1667
257	8	2	-0.5	0.25	105.3974	290	8	2	-0.75	0.75	138.7172
258	16	0.5	-0.5	0.25	101.2757	291	16	0.5	-0.75	0.75	108.7904
259	16	1	-0.5	0.25	102.6017	292	16	1	-0.75	0.75	118.1853
260	16	1.5	-0.5	0.25	103.9762	293	16	1.5	-0.75	0.75	128.1667
261	16	2	-0.5	0.25	105.3974	294	16	2	-0.75	0.75	138.7172
262	20	0.5	-0.5	0.25	101.2757	295	20	0.5	-0.75	0.75	108.7904
263	20	1	-0.5	0.25	102.6017	296	20	1	-0.75	0.75	118.1853
264	20	1.5	-0.5	0.25	103.9762	297	20	1.5	-0.75	0.75	128.1667
265	20	2	-0.5	0.25	105.3974	298	20	2	-0.75	0.75	138.7172
266	24	0.5	-0.5	0.25	101.2757	299	24	0.5	-0.75	0.75	108.7904
267	24	1	-0.5	0.25	102.6017	300	24	1	-0.75	0.75	118.1853
268	24	1.5	-0.5	0.25	103.9762	301	24	1.5	-0.75	0.75	128.1667
269	24	2	-0.5	0.25	105.3974	302	24	2	-0.75	0.75	138.7172
270	8	0.5	-0.6	0.5	104.0090	303	8	0.5	-0.9	0.9	113.2962
271	8	1	-0.6	0.5	108.2517	304	8	1	-0.9	0.9	127.5213
272	8	1.5	-0.6	0.5	112.7183	305	8	1.5	-0.9	0.9	142.6557
273	8	2	-0.6	0.5	117.3990	306	8	2	-0.9	0.9	158.6799
274	16	0.5	-0.6	0.5	104.0090	307	16	0.5	-0.9	0.9	113.2962
275	16	1	-0.6	0.5	108.2517	308	16	1	-0.9	0.9	127.5213
276	16	1.5	-0.6	0.5	112.7183	309	16	1.5	-0.9	0.9	142.6557
277	16	2	-0.6	0.5	117.3990	310	16	2	-0.9	0.9	158.6799
278	20	0.5	-0.6	0.5	104.0090	311	20	0.5	-0.9	0.9	113.2962
279	20	1	-0.6	0.5	108.2517	312	20	1	-0.9	0.9	127.5213
280	20	1.5	-0.6	0.5	112.7183	313	20	1.5	-0.9	0.9	142.6557
281	20	2	-0.6	0.5	117.3990	314	20	2	-0.9	0.9	158.6799
282	24	0.5	-0.6	0.5	104.0090	315	24	0.5	-0.9	0.9	113.2962
283	24	1	-0.6	0.5	108.2517	316	24	1	-0.9	0.9	127.5213

Appendix-I

Comparison of the proposed Estimator with Singh's (2002) Estimator

Table A.1: Descriptive Statistics for data set 1- 9

Set values	Data 1	Data 2	Data 3	Data 4	Data 5	Data 6	Data 7	Data 8	Data 9
n	2	2	2	2	2	2	2	2	2
h	4	4	4	4	4	4	4	4	4
α	0	0	0	0	0	0	0	0	0
β	0.5	1.0	1.5	0.5	1.0	1.5	0.5	1.0	1.5
δ	1	1	1	2	2	2	3	3	3

Table A.2: Descriptive Statistics for data set 10- 18

Set values	Data 10	Data 11	Data 12	Data 13	Data 14	Data 15	Data 16	Data 17	Data 18
n	4	4	4	4	4	4	4	4	4
h	4	4	4	4	4	4	4	4	4
α	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
β	0.5	1.0	1.5	0.5	1.0	1.5	0.5	1.0	1.5
δ	1	1	1	2	2	2	3	3	3

Table A.3: Descriptive Statistics for data set 19- 27

Set values	Data 19	Data 20	Data 21	Data 22	Data 23	Data 24	Data 25	Data 26	Data 27
n	3	3	3	3	3	3	3	3	3
h	8	8	8	8	8	8	8	8	8
α	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
β	0.5	1.0	1.5	0.5	1.0	1.5	0.5	1.0	1.5
δ	1	1	1	2	2	2	3	3	3

Table A.4: Descriptive Statistics for data set 28- 36

Set values	Data 28	Data 29	Data 30	Data 31	Data 32	Data 33	Data 34	Data 35	Data 36
n	8	8	8	8	8	8	8	8	8
h	4	4	4	4	4	4	4	4	4
α	1	1	1	1	1	1	1	1	1
β	0.5	1.0	1.5	0.5	1.0	1.5	0.5	1.0	1.5
δ	1	1	1	2	2	2	3	3	3

Comparison of the proposed Estimator with Singh's (2006) Estimator

Table A.5: Descriptive Statistics for data set 37- 44

Set values	Data 37	Data 38	Data 39	Data 40	Data 41	Data 42	Data 43	Data 44
m	8	8	8	8	8	8	8	8
k	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
ρ	-0.2	-0.2	-0.2	-0.2	-0.5	-0.5	-0.5	-0.5

Table A.6: Descriptive Statistics for data set 45- 52

Set values	Data 45	Data 46	Data 47	Data 48	Data 49	Data 50	Data 51	Data 52
m	8	8	8	8	16	16	16	16
k	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
ρ	-0.9	-0.9	-0.9	-0.9	-0.2	-0.2	-0.2	-0.2

Table A.7: Descriptive Statistics for data set 53- 60

Set values	Data 53	Data 54	Data 55	Data 56	Data 57	Data 58	Data 59	Data 60
m	16	16	16	16	16	16	16	16
k	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
ρ	-0.5	-0.5	-0.5	-0.5	-0.9	-0.9	-0.9	-0.9

Table A.8: Descriptive Statistics for data set 61- 68

Set values	Data 61	Data 62	Data 63	Data 64	Data 65	Data 66	Data 67	Data 68
m	20	20	20	20	20	20	20	20
k	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
ρ	-0.2	-0.2	-0.2	-0.2	-0.5	-0.5	-0.5	-0.5

Table A.9: Descriptive Statistics for data set 69- 76

Set values	Data 69	Data 70	Data 71	Data 72	Data 73	Data 74	Data 75	Data 76
m	20	20	20	20	32	32	32	32
k	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
ρ	-0.9	-0.9	-0.9	-0.9	-0.2	-0.2	-0.2	-0.2

Table A.10: Descriptive Statistics for data set 77- 84

Set values	Data 77	Data 78	Data 79	Data 80	Data 81	Data 82	Data 83	Data 84
m	32	32	32	32	32	32	32	32
k	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
ρ	-0.5	-0.5	-0.5	-0.5	-0.9	-0.9	-0.9	-0.9

Comparison of the proposed Estimator Chakrabarty (1979) Estimator

Table A.11: Descriptive Statistics for data set 85- 91

Set values	Data 85	Data 86	Data 87	Data 88	Data 89	Data 90	Data 91	Data 92
m	8	8	8	16	16	16	20	20
k	0.5	1.0	1.5	2.0	0.5	1.0	0.5	1.0
ρ	0.2	0.2	0.2	0.3	0.3	0.3	0.4	0.4

Table A.12: Descriptive Statistics for data set 93- 99

Set values	Data 93	Data 94	Data 95	Data 96	Data 97	Data 98	Data 99
m	20	24	24	24	32	24	24
k	1.5	2.0	0.5	1.0	0.5	1.0	1.5
ρ	0.4	0.5	0.5	0.5	0.6	0.6	0.6

Note: From data set 100-144 the value of $w = 0.4.$

Table A.13: Descriptive Statistics for data set 100-106

Set values	Data 100	Data 101	Data 102	Data 103	Data 104	Data 105	Data 106
m	8	8	8	8	8	8	8
k	0.5	1.0	1.5	0.5	1.0	1.5	0.5
ρ	0.2	0.2	0.2	0.3	0.3	0.3	0.4

Table A.14: Descriptive Statistics for data set 107-113

Set values	Data 107	Data 108	Data 109	Data 110	Data 111	Data 112	Data 113
m	8	8	16	16	16	16	16
k	1.0	1.5	0.5	1.0	1.5	0.5	1.0
ρ	0.4	0.4	0.2	0.2	0.2	0.3	0.3

Table A.15: Descriptive Statistics for data set 114-120

Set values	Data 114	Data 115	Data 116	Data 117	Data 118	Data 119	Data 120
m	16	16	16	16	20	20	20
k	1.5	0.5	1.0	1.5	0.5	1.0	1.5
ρ	0.3	0.4	0.4	0.4	0.2	0.2	0.2

Table A.16: Descriptive Statistics for data set 121-127

Set values	Data 121	Data 122	Data 123	Data 124	Data 125	Data 126	Data 127
m	20	20	20	20	20	20	32
k	0.5	1.0	1.5	0.5	1.0	1.5	0.5
ρ	0.3	0.3	0.3	0.4	0.4	0.4	0.2

Table A.17: Descriptive Statistics for data set 128-135

Set values	Data 128	Data 129	Data 130	Data 131	Data 132	Data 133	Data 134	Data 135
m	32	32	32	32	32	32	32	32
k	1.0	1.5	0.5	1.0	1.5	0.5	1.0	1.5
ρ	0.2	0.2	0.3	0.3	0.3	0.4	0.4	0.4

Table A.18: Descriptive Statistics for data set 136-144

Set values	Data 136	Data 137	Data 138	Data 139	Data 140	Data 141	Data 142	Data 143	Data 144
m	20	20	20	20	20	20	20	20	20
k	0.5	1.0	1.5	0.5	1.0	1.5	0.5	1.0	1.5
ρ	0.2	0.2	0.2	0.3	0.3	0.3	0.4	0.4	0.4

Note: The data set 145-189 are same as data set 100-144, except that $w = 0.6$, in them.

References

1. Bandyopadhyay, S. (1994). On the efficiency of product method of estimation. *Sankhya – Series B*, **56**, 222-227.
2. Chakrabarty, R. P. (1979). Some Ratio Type Estimators. *Journal of the Indian Society of Agricultural Statistics*, **25**, 49- 57.
3. Durbin, J. (1959). A note on the application of Quenouille’s method of Bias reduction to the estimation of Ratios. *Biometrika*, **46**, 477-480.
4. Lui, K. J. (1991). A discussion of the limitations of B. Prasad’s Estimator in finite population sampling. *Communications in Statistics Theory and Methods*, **20**, 293-298.
5. Mohanty, S. and Sahoo, J. (1995). A note on improving Ratio method of estimation through Linear transformation using certain known population parameters. *Sankhya - Series B*, **57**, 93-102.

6. Prasad, B. (1986). Unbiased Estimators versus mean per unit and Ratio Estimators in finite population surveys. *Communications in Statistics - Theory and Methods*, **12**, 3647-3657.
7. Rao, J. N. K. and Webster, J. T. (1966). On two methods of Bias reduction in the estimation of Ratios. *Biometrika*, **53**, 571-577.
8. Singh, A. K. and Singh, H. P. (1997). A note on the efficiencies of three Product Type Estimators under a Linear Model. *Journal of Indian Society of Agricultural Statistics,* **50**, 130-134.
9. Singh, A.K., Upadhyaya, L.N. and Singh, H.P. (2006). Comparisons of three Product Type Estimators in small sample. *Statistics in Transition*, **7**, 917-928.
10. Singh, G. N. (2000). A general class of Ratio Type Estimators under super population model. *Biometrical journal*, **42**, 363-375.
11. Singh, G. N. (2002). Empirical studies of generalized classes of Ratio and Product Type Estimators under a Linear Model. *Statistics in Transition*, **5**, 701-720.
12. Srivenkataramana, T. and Tracy, D. S. T (1986). Transformation alter sampling. *Statistics*, **17**, 597-608.