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On Five Parameter Beta Lomax Distribution

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Abstract

Lomax (1954) developed Lomax Distribution (Pareto Type – II). In this paper, we have developed a new five parameter Beta Lomax Distribution from a three parameter Lomax Distribution. We have developed expressions for the r^{th} moment; Skewness and Kurtosis of three parameters Lomax, and five parameters Beta Lomax Distribution. At the end, the Maximum Likelihood Estimators (MLE) of the parameters have also been obtained.

Keywords

Lomax Distribution, Beta Lomax, r^{th} moment, Maximum likelihood estimators

1. Introduction

Lomax (1954) proposed Pareto Type – II Distribution, also known as Lomax Distribution, and used it for the analysis of the business failure life time data. The Lomax Distribution is widely applicable in reliability and life testing problems in engineering as well as in Survival Analysis as an alternative distribution. After the work of Lomax (1954), various authors studied the Lomax Distribution.

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The cumulative distribution function of the Lomax arises as a Limit Distribution for the residual life time at a great age (Balkema and De Haan, 1974). Myhre and Saunders (1982) gave application of Lomax Distribution using the right censored data. Lingappaiah (1986) proposed various procedures of estimation for the Lomax Distributions. Nayak (1987) proposed Multivariate Lomax Distribution and discussed its various properties and usefulness in reliability theory. Ahsanullah (1991), and Balakrishnan and Ahsanullah (1994) have investigated distributional properties and moments of record values from Lomax Distribution respectively. Vidondo et al. (1997) used this distribution for modeling size spectra data in aquatic ecology. Childs et al. (2001) gave order statistics from non-identical right-truncated Lomax Distributions and gave the applications for these situations. Lomax Distribution was used to obtain Discrete Poisson Lomax Distribution (Al-Awadhi and Ghitany, 2001). Bayesian method of estimation was used for estimation of Lomax Survival Function (Howlader and Hossain, 2002). Various properties of the log of the Lomax random variable were derived by Nadarajah (2005). Non-Bayesian and Bayesian estimators of the sample size in case of type I censored samples from the Lomax Distribution have been proposed by Abd-Elfattah et al. (2007). Ghitany et al. (2007) proposed the properties of a new parametric distribution which was investigated by Marshall and Olkin (1997) and comprehensively extended the distributions family which is being applied to the model of the Lomax. Hassan and Al-Ghamdi (2009) used Lomax Distribution for determination of optimal times of changing level of stress for simple stress plans under a Cumulative Exposure model. Abd-Elfattah and Alharbey (2010) estimated the parameters of Lomax Distribution Based on Generalized Probability Weighted Moments. Rajab (2013) proposed the three parameters Lomax, five parameters beta Lomax and a Kumarasawamy Generalized Lomax Distribution and discussed various properties of these distributions.

Eugene et al. (2002) proposed a generalization of Normal Distribution which is known as Beta Normal Distribution and some other generalizations were developed by Gupta and Nadarajah (2004). Closed-form expressions for the moments, asymptotic distribution of the extreme order statistics and the estimation procedure for the Beta Gumbel Distribution were proposed by Nadarajah and Kotz (2004). The Beta Exponential Distribution and its various properties were discussed by Nadarajah and Kotz (2006).

In Section 2, we have given a three parameter Lomax Distribution and discussed it various properties whereas in Section 3 a new five parameter Beta Lomax Distribution and expression for various properties have been proposed.

2. Three Parameter Lomax Distribution

A random variable y is said to be distributed as Lomax with two parameters α (shape parameter) and λ (scale parameter) if its probability density function is given by

$$f(y)=\frac{\alpha}{\lambda}\left[1+\frac{y}{\lambda}\right]^{-(\alpha+1)} \quad ; \quad 0\le y\le\infty\ ,\ \alpha>0,\lambda>0 \tag{2.1}$$

The distribution function of two parameters Lomax Distribution is

$$F(y)=1-(1+\frac{y}{\lambda})^{-\alpha} \quad ; \quad y\ge 0,\lambda>0,\alpha>0 \tag{2.2}$$

The Lomax Distribution is also known as Pareto Distribution of Type – II usually occurs in economics (Arnold, 1983) and reliability theory (Harris, 1968). In theory, Reliability Lomax Distribution usually occurs as a mixture of the single parameter Exponential Distribution.

If we use the transformation y = μ-x in (2.1) we obtain a three parameter Lomax Distribution with parameters α, λ and μ defined by the probability density function

$$f(x)=\frac{\alpha}{\lambda}\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{-(\alpha+1)} \quad ; \ x\ge\mu,\ \mu\ge 0,\alpha>0,\lambda>0 \tag{2.3}$$

The additional parameter μ, the location parameter, can be regarded as the guarantee time before which no failure occurs.

The time plot of the probability density function of the three parameters Lomax Distribution for $\mu=1,\lambda=10$ and α =1,2,3,4,5 is given in Figure 1. Figure 1 shows the decreasing shape of three parameters Lomax Distribution for various values of the shape parameter α. The shape of the three parameters resembles with the shape of Pareto Distribution.

The distribution function associated with three parameters Lomax Distribution can be defined as

$$F(x)=1-\left\{1+\left(\frac{x-\mu}{\lambda}\right)\right\}^{-\alpha} \ ; \ x\ge\mu,\ \mu\ge 0,\alpha>0,\lambda>0 \tag{2.4}$$

The time plot of the cumulative density function for $\mu = 1, \lambda = 10$ and α =1,2,3,4,5 is given in Figure 2. Figure 2 shows an increasing pattern of the unreliability function (cumulative distribution function) as time increases. Moreover, it exhibits that the shape parameter α plays an important role. As the value of α increases the cumulative probability of failure increases sharply which is a similar characteristic to Pareto Distribution.

The Reliability Function, $R(x)$ of the three parameters Lomax Distribution can be obtained by using the relation between the cumulative distribution function and the Reliability Function which is given by

$$R(x) = \left\{1 + \left(\frac{x-\mu}{\lambda}\right)\right\}^{-\alpha} \ ; \ x \geq \mu,\ \mu \geq 0, \alpha > 0, \lambda > 0 \tag{2.5}$$

The time plot of the Reliability Function of the three parameters Lomax Distribution for $\mu = 1, \lambda = 10$ and α =1,2,3,4,5 is given in Figure 3. Figure 3 exhibits the decreasing pattern of the survival curve of the three parameters Lomax Distribution. The graph also shows that as the value of α increases the probability of survival decreases sharply. Therefore, there is an inverse relationship between the shape parameter α and the Reliability Function.

The Hazard Function of the three parameters Lomax Distribution can be obtained by the relation $h(x) = \frac{f(x)}{R(x)}$ and is given by

$$h(x) = \frac{\alpha}{\lambda}\left[1 + \left(\frac{x-\mu}{\lambda}\right)\right]^{-1} \ ; \ x \geq \mu, \mu \geq 0,\ \alpha > 0,\ \lambda > 0 \tag{2.6}$$

The time plot of the Hazard Function of the three parameters Lomax Distribution for $\mu = 1, \lambda = 10$ and α =1,2,3,4,5 is given in Figure 4. Figure 4 shows the decreasing Hazard Rate of the three parameters Lomax Distribution. The graph also exhibits that rate of decrease becomes steadier as the value of the shape parameter α increases. Moreover, as Hazard Function is known as the Conditional Failure Rate, the graphs shows that for a larger value of α the risk of failure at an early stage is also high.

The expression for the Cumulative Hazard Function can be obtained by using the Reliability Function i.e. $H(x) = -\ln(R(x))$. The Cumulative Hazard Function of three parameters Lomax Distribution is

$$H(x) = \alpha \ln\left[1 + \left(\frac{x-\mu}{\lambda}\right)\right] \tag{2.7}$$

Time plot of the Cumulative Hazard Function of the three parameters Lomax Distribution for $\mu = 1, \lambda = 10$ and α =1,2,3,4,5 is given in Figure 5. Figure 5 shows the increasing pattern of Cumulative Hazard Rate as time increases. Figure 5 also exhibits that as the value of shape parameter α increases the Cumulative Hazard Rate increases more rapidly, thus it showed an inverse relationship of shape parameter α and the Cumulative Hazard Rate.

2.1 Expression for r^{th} Moment about Origin of Three Parameters Lomax Distribution:

The r^{th} moment about origin is defined as

$$\mu_r' = E(X^r) = \int_X X^r f(x)dx \tag{2.8}$$

By substituting 2.1 we have

$$\mu_r' = \int_\mu^\infty \frac{\alpha}{\lambda} x^r \left[1 + \left(\frac{x-\mu}{\lambda}\right)\right]^{-(\alpha+1)} dx$$

By using the transformation $Z = \dfrac{x-\mu}{\lambda}$ we obtain

$$\mu_r' = \alpha \int_0^\infty \frac{(\mu + \lambda Z)^r}{(1+Z)^{\alpha+1}} dZ \tag{2.9}$$

Now by considering $(\lambda Z + \mu)^r = \mu^r \left(1 + \dfrac{\lambda}{\mu} Z\right)^r$ and comparing it with the expression $(1+a)^n = \sum_{i=0}^{n} \binom{n}{i} a^i$ and after simplification we can obtain the expression of the r^{th} moment about origin

$$\mu_r' = \alpha \sum_{i=0}^{r} \binom{r}{i} \mu^{r-i} \lambda^i \beta(i+1, \alpha - i) \tag{2.10}$$

Now by using (2.10) the expressions for the first four raw moments were obtained by putting r = 1, 2, 3, 4 respectively.

$$\mu_1' = \mu + \frac{\lambda}{\alpha - 1} = \text{Mean} \tag{2.11}$$

$$\mu_2' = \mu^2 + \frac{2\mu\lambda}{\alpha(\alpha-1)} + \frac{2\lambda^2}{\alpha(\alpha-1)(\alpha-2)} \tag{2.12}$$

$$\mu_3' = \alpha\left[\frac{\mu^3}{\alpha} + \frac{3\mu^2\lambda}{\alpha(\alpha-1)} + \frac{6\mu\lambda^2}{\alpha(\alpha-1)(\alpha-2)} + \frac{6\lambda^3}{\alpha(\alpha-1)(\alpha-2)(\alpha-3)}\right] \tag{2.13}$$

$$\mu_4' = \alpha\left[\frac{\mu^4}{\alpha} + \frac{4\mu^3\lambda}{\alpha(\alpha-1)} + \frac{12\mu^2\lambda^2}{\alpha(\alpha-1)(\alpha-2)} + \frac{24\mu\lambda^3}{\alpha(\alpha-1)(\alpha-2)(\alpha-3)} + \frac{24\lambda^4}{\alpha(\alpha-1)(\alpha-2)(\alpha-3)(\alpha-4)}\right] \tag{2.14}$$

The expression for variance of three parameters Lomax Distribution can be derived from the expression of raw moments and is given

$$\mu_2 = \frac{\lambda^2}{(\alpha-1)^2(\alpha-2)} = \text{Variance} \tag{2.15}$$

$$\mu_3 = \left[\mu^3 + \frac{3\mu^2\lambda}{(\alpha-1)} + \frac{6\mu\lambda^2}{(\alpha-1)(\alpha-2)} + \frac{6\lambda^3}{(\alpha-1)(\alpha-2)(\alpha-3)}\right] - 3\left[\mu^2 + \frac{2\mu\lambda}{(\alpha-1)} + \frac{2\lambda^2}{(\alpha-1)(\alpha-2)}\right]\left[\mu + \frac{\lambda}{(\alpha-1)}\right] + 2\left[\mu + \frac{\lambda}{(\alpha-1)}\right]^3$$

$$\begin{aligned}\mu_4 = &\left[\mu^4 + \frac{4\mu^3\lambda}{(\alpha-1)} + \frac{12\mu^2\lambda^2}{(\alpha-1)(\alpha-2)} + \frac{24\mu\lambda^3}{(\alpha-1)(\alpha-2)(\alpha-3)} + \frac{24\lambda^4}{(\alpha-1)(\alpha-2)(\alpha-3)(\alpha-4)}\right] \\ &-4\left[\mu + \frac{\lambda}{(\alpha-1)}\right]\left[\mu^3 + \frac{3\mu^2\lambda}{(\alpha-1)} + \frac{6\mu\lambda^2}{(\alpha-1)(\alpha-2)} + \frac{6\lambda^3}{(\alpha-1)(\alpha-2)(\alpha-3)}\right] \\ &+6\left[\mu + \frac{\lambda}{(\alpha-1)}\right]^2\left[\mu^2 + \frac{2\mu\lambda}{(\alpha-1)} + \mu^2 + \frac{2\lambda^2}{(\alpha-1)(\alpha-2)}\right] - 3\left[\mu + \frac{\lambda}{(\alpha-1)}\right]^4\end{aligned} \tag{2.16}$$

The expressions for the coefficients of Skewness β_1 and Kurtosis β_2 are given as under

$$\beta_1 = \frac{\left(\left[\mu^3 + \frac{3\mu^2\lambda}{(\alpha-1)} + \frac{6\mu\lambda^2}{(\alpha-1)(\alpha-2)} + \frac{6\lambda^3}{(\alpha-1)(\alpha-2)(\alpha-3)}\right] - 3\left[\mu^2 + \frac{2\mu\lambda}{(\alpha-1)} + \frac{2\lambda^2}{(\alpha-1)(\alpha-2)}\right]\left[\mu + \frac{\lambda}{(\alpha-1)}\right] + 2\left[\mu + \frac{\lambda}{(\alpha-1)}\right]^3\right)^2}{\left(\frac{\alpha\lambda^2}{(\alpha-1)^2(\alpha-2)}\right)^3} \tag{2.17}$$

$$\beta_2 = \frac{\left(\begin{array}{l}\left[\mu^4 + \frac{4\mu^3\lambda}{(\alpha-1)} + \frac{12\mu^2\lambda^2}{(\alpha-1)(\alpha-2)} + \frac{24\mu\lambda^3}{(\alpha-1)(\alpha-2)(\alpha-3)} + \frac{24\lambda^4}{(\alpha-1)(\alpha-2)(\alpha-3)(\alpha-4)}\right] \\ -4\left[\mu + \frac{\lambda}{(\alpha-1)}\right]\left[\mu^3 + \frac{3\mu^2\lambda}{(\alpha-1)} + \frac{6\mu\lambda^2}{(\alpha-1)(\alpha-2)} + \frac{6\lambda^3}{(\alpha-1)(\alpha-2)(\alpha-3)}\right] \\ +6\left[\mu + \frac{\lambda}{(\alpha-1)}\right]^2\left[\mu^2 + \frac{2\mu\lambda}{(\alpha-1)} + \mu^2 + \frac{2\lambda^2}{(\alpha-1)(\alpha-2)}\right] - 3\left[\mu + \frac{\lambda}{(\alpha-1)}\right]^4\end{array}\right)}{\left(\frac{\alpha\lambda^2}{(\alpha-1)^2(\alpha-2)}\right)^2} \tag{2.18}$$

2.2 The Maximum Likelihood Estimation of Three Parameters Lomax Distribution: By considering the density function given in (2.3), then the Log-likelihood Function obtained is given as

$$\ln L(\underline{x}) = n\ln\alpha - n\ln\lambda - (\alpha+1)\sum_{i=1}^{n}\ln\left[1+\left(\frac{x-\mu}{\lambda}\right)\right] \tag{2.19}$$

Since the Parameter " μ" involves in the lower limit of the probability density function, therefore, the first order statistics $x_{(1)}$ can be regarded as the MLE of " μ". Thus (2.19) can be written as

$$\ln L(\underline{x}) = n\ln\alpha - n\ln\lambda - (\alpha+1)\sum_{i=1}^{n}\ln\left[1+\left(\frac{x-x_{(1)}}{\lambda}\right)\right] \tag{2.20}$$

Now by partially differentiating (2.20) with respect to α and λ respectively and equating to zero we have obtained the two equations as under

$$\alpha = \frac{n}{\sum_{i=1}^{n}\ln\left[1+\left(\frac{x-x_{(1)}}{\hat{\lambda}}\right)\right]} \tag{2.21}$$

$$\hat{\lambda} = \frac{-n}{\left\{\frac{n}{\sum_{i=1}^{n}\ln\left[1+\left(\frac{X-X_{(1)}}{\lambda}\right)\right]}+1\right\}\sum_{i=1}^{n}\sum_{j=1}^{\infty}(-1)^j\frac{(X-X_{(1)})^j}{\lambda^{j+1}}} \tag{2.22}$$

The MLE's of α and λ can either be obtained by solving (2.21) and (2.22) simultaneously using the Newton – Raphson iterative procedure for solution of non-linear equations.

3. A New Five Parameter Beta Lomax Distribution

Eugene et al. (2002) proposed a generalization by introducing Beta Normal Distribution.
The generalized form is given by

$$f(x) = \frac{1}{\beta(a,b)}\left[G(x)\right]^{a-1}\left[1-G(x)\right]^{b-1} g(x) \tag{3.1}$$

where G(x) is the cumulative distribution function and g(x) is probability density function of the distribution to be generalized.

Now by substituting the cumulative distribution function and probability density function proposed in (2.4) and (2.3), we have obtained a density of five parameter Beta Lomax Distribution.

$$f(x) = \frac{\alpha}{\lambda\beta(a,b)}\left[1-\left\{1+\left(\frac{x-\mu}{\lambda}\right)\right\}^{-\alpha}\right]^{a-1}\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{-(\alpha b+1)} ;x \geq \mu \ and\ (\alpha,\lambda,a,b>0) \tag{3.2}$$

The time plot of the probability density function of five parameter Beta Lomax Distribution for $\mu=1,\lambda=10,a=2,b=1\ and\ \alpha=1,2,3,4$ is given in Figure 6. Figure 6 shows various shapes of the density function of Beta Lomax Distribution. The graph shows that as α, the shape parameter, increases the peak of the density curve increases. Moreover, Figure 6 displays that the distribution is positively skewed.

The expression for the cumulative distribution function corresponding to the proposed five parameter Beta Lomax Distribution is given by

$$F(x) = \frac{\alpha}{\beta(a,b)}\sum_{i=0}^{a-1}(-1)^i\binom{a-1}{i}\frac{1}{\alpha b+i\alpha}\left[1-\left\{1+\frac{x-\mu}{\lambda}\right\}^{-(\alpha b+i\alpha)}\right] \tag{3.3}$$

The time plot of the cumulative distribution function of five parameter Beta Lomax Distribution $\mu=1,\lambda=10,a=2,b=1\ and\ \alpha=1,2,3,4$ is given in Figure 7. Figure 7 shows an increasing pattern of the unreliability function (cumulative distribution function) as time increases. Moreover, it exhibits that as the value of

α increases the cumulative probability of failure increases sharply which is a similar characteristic to Pareto distribution.

The expression for the Reliability Function associated with five parameter Beta Lomax Distribution is given by

$$R(x) = 1 - \frac{\alpha}{\beta(a,b)} \sum_{i=0}^{a-1} (-1)^i \binom{a-1}{i} \frac{1}{\alpha b + i\alpha} \left[1 - \left\{ 1 + \frac{x-\mu}{\lambda} \right\}^{-(\alpha b + i\alpha)} \right] \tag{3.4}$$

The time plot of the Reliability Function of five parameter Beta Lomax Distribution for $\mu = 1, \lambda = 10, a = 2, b = 1$ *and* $\alpha = 1, 2, 3, 4$ is given in Figure 8. Figure 8 exhibits the decreasing pattern of the Survival Curve of the five parameters Beta Lomax Distribution. The graph also shows that as the value of α increases the probability of survival decreases sharply. Therefore, there is an inverse relationship between the shape parameter α and the Reliability Function. The Hazard Function of the five parameter Beta Lomax Distribution can be obtained by the relation $h(x) = \frac{f(x)}{R(x)}$ and is given by

$$h(x) = \frac{\frac{\alpha}{\lambda\beta(a,b)} \left[1 - \left\{ 1 + \left(\frac{x-\mu}{\lambda} \right) \right\} - \alpha \right]^{a-1} \left[1 + \left(\frac{x-\mu}{\lambda} \right) \right]^{-(\alpha b + 1)}}{1 - \frac{\alpha}{\beta(a,b)} \sum_{i=0}^{a-1} (-1)^i \binom{a-1}{i} \frac{1}{\alpha b + i\alpha} \left[1 - \left\{ 1 + \frac{x-\mu}{\lambda} \right\}^{-(\alpha b + i\alpha)} \right]} \tag{3.5}$$

The time plot of the Hazard Function of five parameter Beta Lomax Distribution for $\mu = 1, \lambda = 10, a = 2, b = 1$ *and* $\alpha = 1, 2, 3, 4$ is given in Figure 9. Figure 9 shows the initially increasing then decreasing pattern of the Hazard Function of the five parameter Beta Lomax Distribution. Moreover, it also shows that as the value of the shape parameter α increases the rate of initial increase in Hazard Function becomes sharper and after reaching at a certain point it decreases gradually. Therefore, it can be said that the five parameter Beta Lomax Distribution can be used for modeling such failure times in which initial failure probability is higher and it decreases during the aging process.

The expression for the Cumulative Hazard Function corresponding to the proposed density function of five parameter Beta Lomax Distribution is given by

$$H(x)=\int h(x)\,dx=-\ln\left[R(x)\right] \tag{3.6}$$

$$H(x)=-\ln\left[1-\frac{\alpha}{\beta(a,b)}\sum_{i=0}^{a-1}(-1)^i\binom{a-1}{i}\frac{1}{\alpha b+i\alpha}\left[1-\left\{1+\frac{x-\mu}{\lambda}\right\}^{-(\alpha b+i\alpha)}\right]\right] \tag{3.7}$$

The time plot of the Cumulative Hazard Function of five parameter Beta Lomax Distribution for $\mu=1,\lambda=10,a=2,b=1\ and\ \alpha=1,2,3,4$ is given in Figure 10. Figure 10 shows the increasing pattern of Cumulative Hazard Rate as time increases. Figure 10 also exhibits that as the value of shape parameter α increases the Cumulative Hazard Rate increases more rapidly, thus it showed an inverse relationship of shape parameter α and the Cumulative Hazard Rate.

3.1 Expression for the rth Moment of the Five Parameter Beta Lomax Distribution: Assuming a=1, the probability density function of the five parameter Beta Lomax Distribution can be written as

$$f(x)=\frac{\alpha}{\lambda\beta(1,b)}\left[1+\left(\frac{x-\mu}{\lambda}\right)\right]^{-(\alpha b+1)} \quad ; x\geq\mu\ and\ (\alpha,\lambda,b>0) \tag{3.8}$$

Now by using the transformation $x-\mu=y$, we may write

$$f(y)=\frac{\alpha}{\lambda\beta(1,b)}\frac{1}{\left[1+\frac{y}{\lambda}\right]^{(\alpha b+1)}} \quad ; y\geq 0\ and\ (\alpha,\lambda,b>0) \tag{3.9}$$

Now the rth moment about origin is defined as

$$\mu_r'=E(Y^r)=\frac{\alpha}{\lambda\,\beta(1,b)}\int_0^{\infty}\frac{y^r}{\left(1+\frac{y}{\lambda}\right)^{\alpha b+1}}dy$$

Again by using the transformation $\frac{y}{\lambda}=z$ and simplifying, we get

$$\mu_r'=\frac{\alpha\,\lambda^r}{\beta(1,b)}\beta(r+1,\alpha\,b-r) \tag{3.10}$$

The expressions for the first four raw moments are given by

$$\mu_1'=\frac{\lambda}{\alpha b-1}=Mean \tag{3.11}$$

$$\mu_2' = \frac{2\lambda^2}{(\alpha b-1)(\alpha b-2)} \tag{3.12}$$

$$\mu_3' = \frac{6\lambda^3}{(\alpha b-1)(\alpha b-2)(\alpha b-3)} \tag{3.13}$$

$$\mu_4' = \frac{24\lambda^4}{(\alpha b-1)(\alpha b-2)(\alpha b-3)(\alpha b-4)} \tag{3.14}$$

The expressions for mean moments can be obtained by using the relationship between the raw moments and the mean moments and are given as under

$$\mu_1 = 0 \tag{3.15}$$

$$\mu_2 = \lambda^2\left[\frac{2(\alpha b-1)-1}{(\alpha b-1)^2\,(\alpha b-2)}\right] = Variance \tag{3.16}$$

$$\mu_3 = \frac{6\lambda^3}{(\alpha b-1)(\alpha b-2)(\alpha b-3)} - \frac{6\lambda^3}{(\alpha b-1)^2(\alpha b-2)} - \frac{2\lambda^3}{(\alpha b-1)^3} - 2\left(\frac{\lambda}{\alpha b-1}\right)^3 \tag{3.17}$$

$$\mu_4 = \frac{24\lambda^4}{(\alpha b-1)(\alpha b-2)(\alpha b-3)(\alpha b-4)} - 4\left(\frac{\lambda}{\alpha b-1}\right)\frac{6\lambda^3}{(\alpha b-1)(\alpha b-2)(\alpha b-3)} + 6\left(\frac{\lambda}{\alpha b-1}\right)^2\frac{2\lambda^2}{(\alpha b-1)(\alpha b-2)} - 3\left(\frac{\lambda}{\alpha b-1}\right)^4 \tag{3.18}$$

The expressions for the coefficients of Skewness β_1 and Kurtosis β_2 are given as under

$$\beta_1 = \frac{\left(\frac{6\lambda^3}{(\alpha b-1)(\alpha b-2)(\alpha b-3)} - \frac{6\lambda^3}{(\alpha b-1)^2(\alpha b-2)} - \frac{2\lambda^3}{(\alpha b-1)^3} - 2\left(\frac{\lambda}{\alpha b-1}\right)^3\right)^2}{\left(\lambda^2\left[\frac{2(\alpha b-1)-(ab-2)}{(\alpha b-1)^2(\alpha b-2)}\right]\right)^3} \tag{3.19}$$

$$\beta_2 = \frac{\left(\frac{24\lambda^4}{(\alpha b-1)(\alpha b-2)(\alpha b-3)(\alpha b-4)} - 4\left(\frac{\lambda}{\alpha b-1}\right)\frac{6\lambda^3}{(\alpha b-1)(\alpha b-2)(\alpha b-3)} + 6\left(\frac{\lambda}{\alpha b-1}\right)^2\left(\frac{2\lambda^2}{(\alpha b-1)(\alpha b-2)}\right) - 3\left(\frac{\lambda}{\alpha b-1}\right)^4\right)}{\left(\lambda^2\left[\frac{2(\alpha b-1)-(ab-2)}{(\alpha b-1)^2(\alpha b-2)}\right]\right)^2} \tag{3.20}$$

Figure 1: Time plot of probability density function of three parameter Lomax Distribution

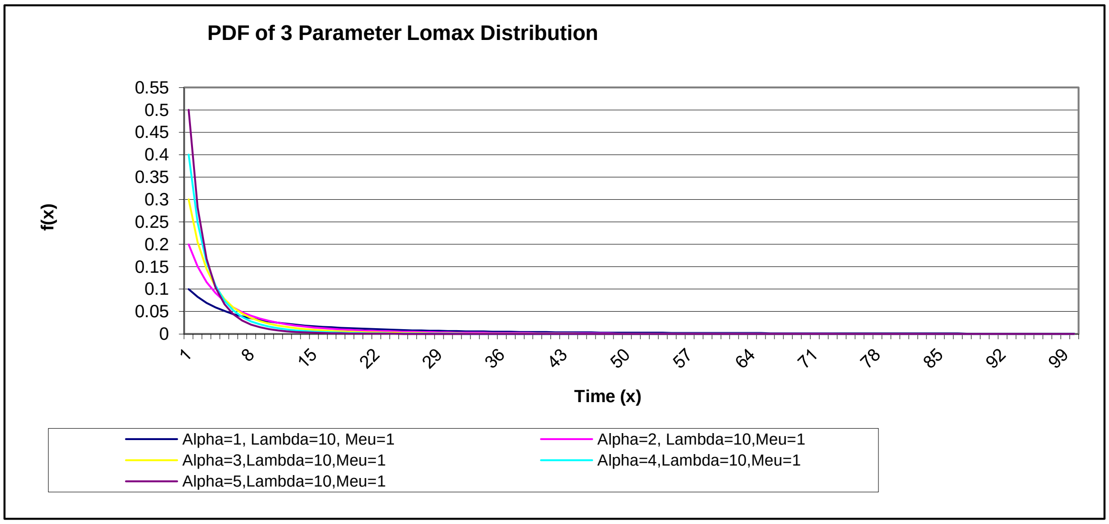


Figure 2: Time plot of cumulative distribution function of three parameter Lomax Distribution

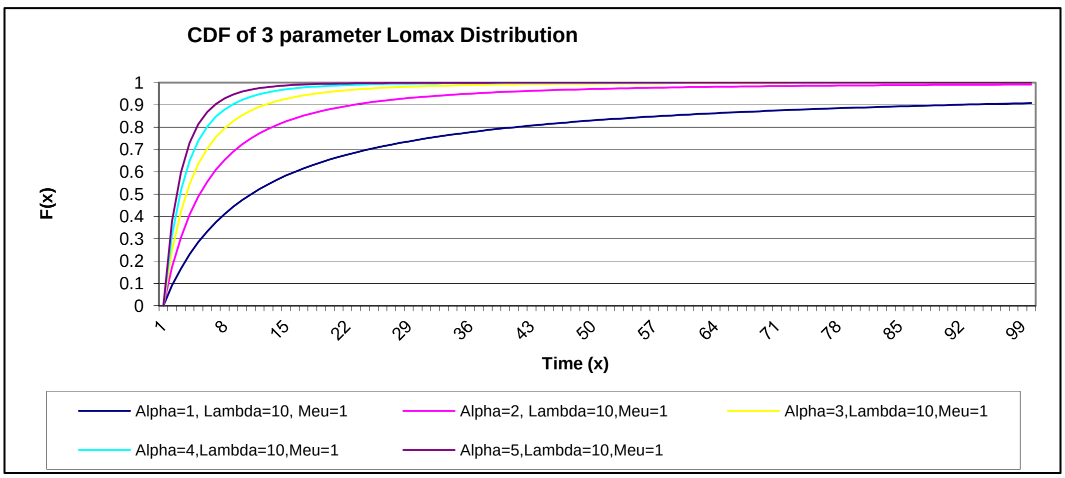


Figure 3: Time plot of Reliability Function of Lomax Distribution

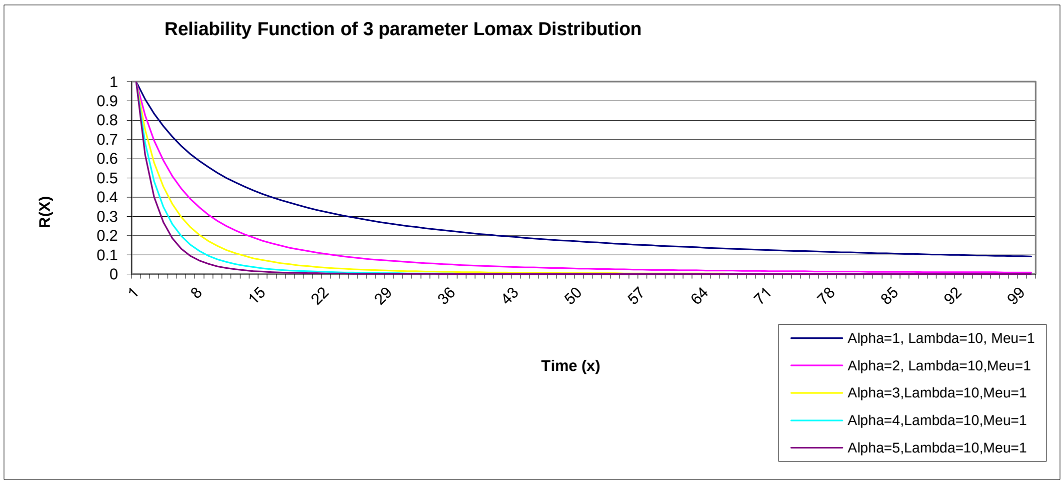


Figure 4: Time plot of Hazard Function of three parameter Lomax Distribution

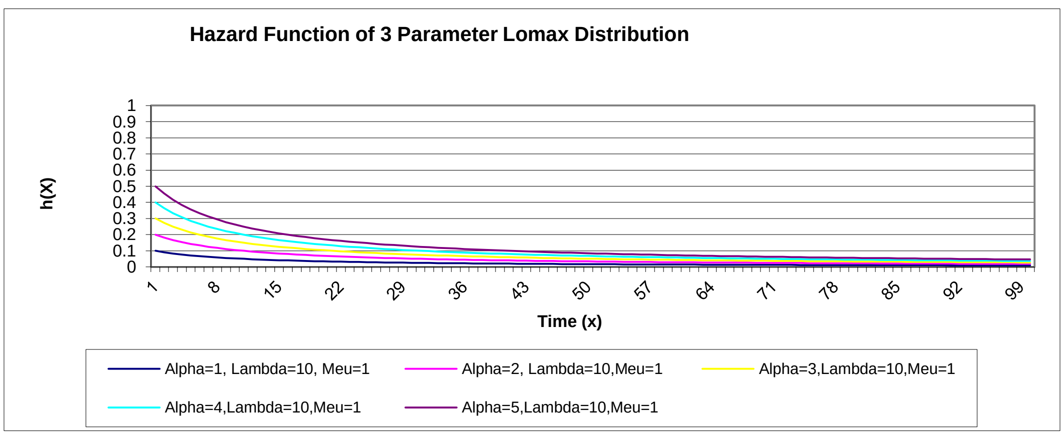


Figure 5: Time plot of Cumulative Hazard Function of Lomax Distribution

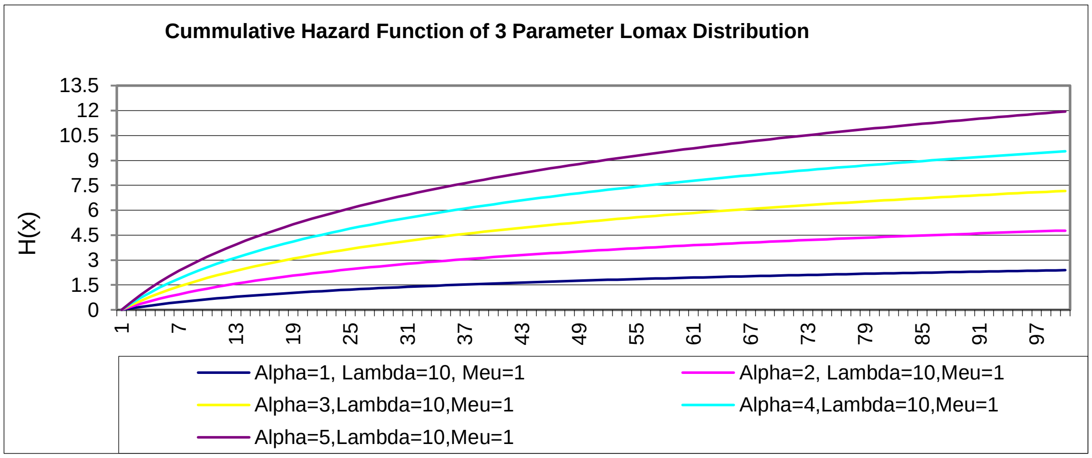


Figure 6: Time plot of probability density function of five parameter Beta Lomax Distribution

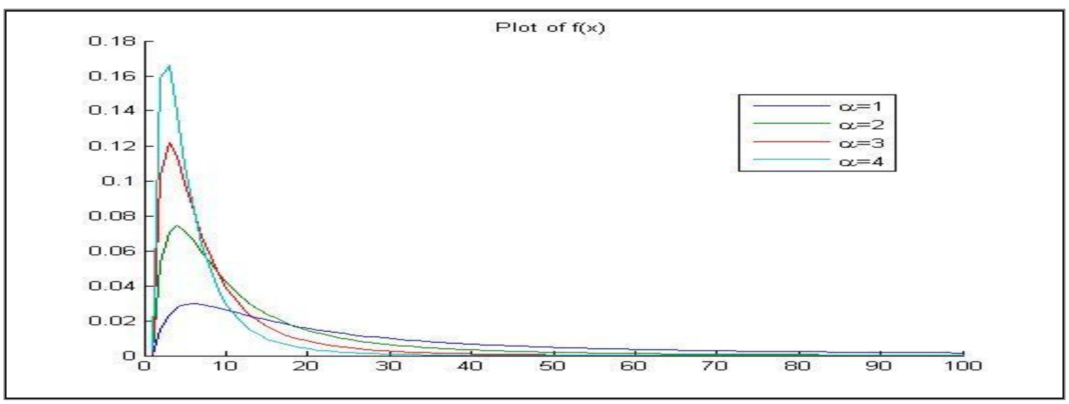


Figure 7: Time plot of cumulative distribution function of five parameter Beta Lomax Distribution

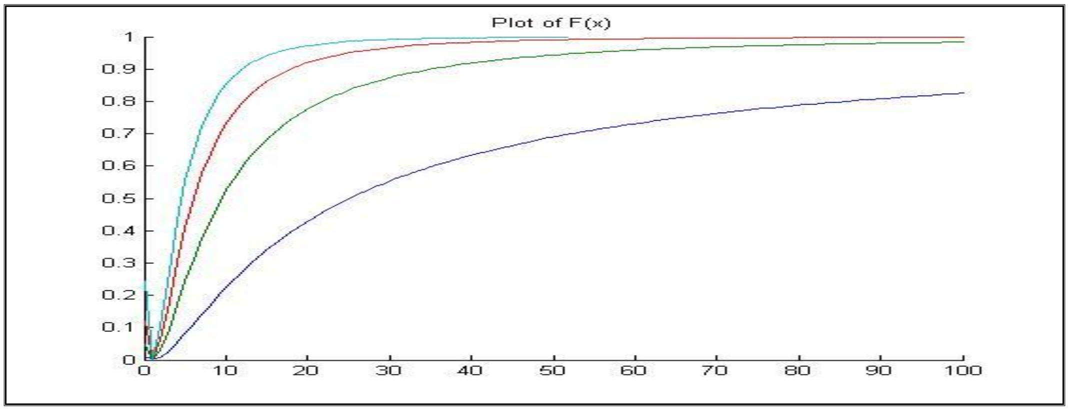


Figure 8: Time plot of Reliability Function of five parameter Beta Lomax Distribution

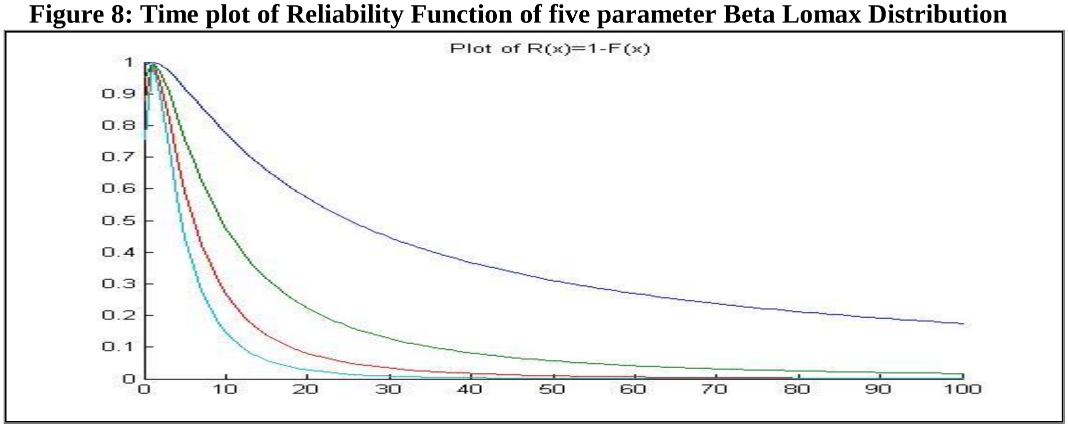


Figure 9: Time plot of Hazard Function of five parameter Beta Lomax Distribution

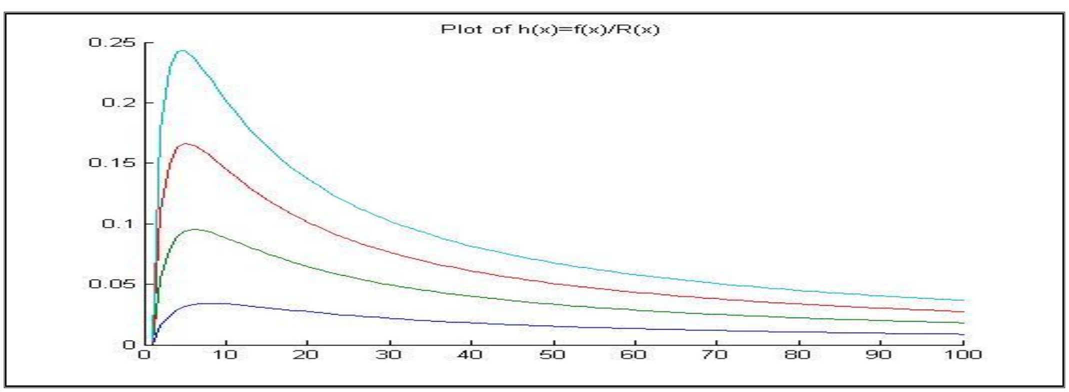
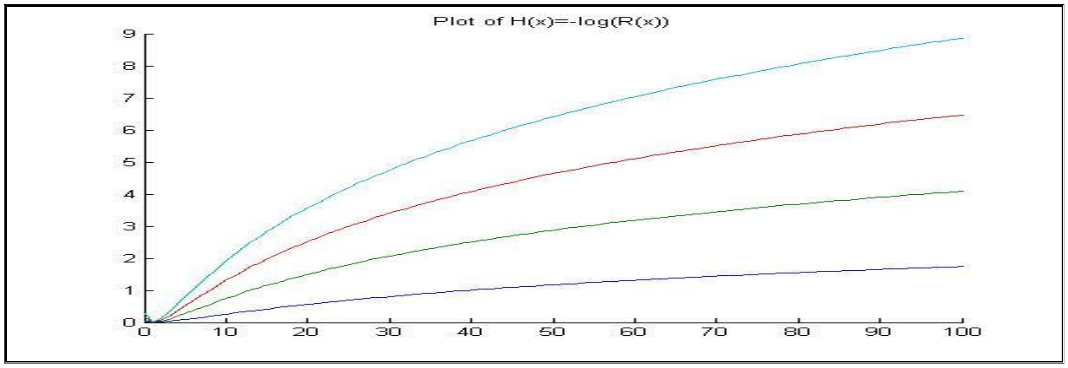


Figure 10: Time plot of Cumulative Hazard Function of five parameter Beta Lomax Distribution



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