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Comparison of Least Square Estimators with Rank Regression Estimators of Weibull Distribution - A Simulation Study

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Abstract

We estimated the parameters of two-parameter Weibull Distribution with the Least Square method from an optimally constructed grouped sample and compared the efficiencies of these estimators with the parameters estimated with Rank Regression method from an ungrouped (complete) sample, though the estimators from both the methods are in closed form, we resorted to Monte-Carlo simulation for computing Bias, Variance and Mean Square Error. The Least Square Estimation method for optimally grouped sample will give an efficient shape parameter than the Rank Regression Estimation method for complete sample. A numerical example is presented to illustrate the methods proposed here.

Keywords

Weibull parameters, Asymptotically optimal grouped sample, Asymptotically relative efficiency, Least Square method, Rank Regression method.

1. Introduction

It is well known that the two-parameter Weibull model is a widely used model in life testing experiments because it has increasing, decreasing or constant failure rate according to its shape parameter being greater than or less than or equal to one, respectively.

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The statistical inference for the Weibull Distribution under censoring schemes has been investigated by several authors such as Herd (1960), Johnson (1964), Mann (1969), Cohen (1975), Cacciari and Montanari (1987), Wong (1993) and Viveros and Balakrishna (1994). In two-parameter Weibull Distribution, Vasudeva Rao et al. (1994) studied the problem of asymptotically optimum grouping for Maximum Likelihood Estimation (MLE) of the parameters for grouped samples and they constructed optimal interval lengths of an equi-class grouped sample. In the same way, Srinivas (1994) studied the problem of optimum grouping for Maximum Likelihood Estimation of the parameters in the case of un-equi-class grouped samples and constructed the class limits of an optimal un-equi-class grouped sample. He tabulated the optimal class limits of a grouped sample for k=3(1) 10, 15 groups which are presented in Table 1. Shuo-Jye (2002), Balakrishnan et al. (2004) and Kantam et al. (2005) constructed the optimum group limits for un-equi-spaced grouped sample with k=3(1) 15 groups for Maximum Likelihood Estimation in scaled Log-logistic Distribution. Further, they constructed the optimum interval lengths for equi-spaced grouped sample with k=3(1) 10, 15 and 20 groups. The Rank Regression Estimation method is quite good for functions that can be linearized. Olteann and Freeman (2009) have studied in their technical report, the Median Rank Regression and Maximum Likelihood Estimation techniques for a two-parameter Weibull Distribution. Srinivas (1994) had constructed the optimal group limits of a grouped sample from two-parameter Weibull Distribution. But in practice the use of the optimal group limits constructed by him requires a prior knowledge or guess value of the parameters and hence the optimal group limits constructed by him have a limited use in practical applications. In this context, we develop a practical procedure to construct an optimally grouped sample even when there is no prior knowledge or guess values of the parameters for constructing this optimal grouped sample.

The research topic can be considered and discussed under the following headings:

- Least Squares Estimation of the parameters from the optimally constructed grouped sample.
- Rank Regression Estimation of the parameters from ungrouped sample.
- Comparison of the efficiencies of Least Squares Estimators obtained from the optimally constructed grouped sample with Rank Regression Estimators obtained from ungrouped (complete) sample using Monte Carlo simulation based on 1000 samples of sizes.

2. Least Squares Estimation of the Parameters from the Optimally Constructed Grouped Sample

The p.d.f. of two-parameter Weibull Distribution is given by

$$f(x; b, c) = \left(\frac{c}{b}\right)\left(\frac{x}{b}\right)^{c-1} \exp\left[-\left(\frac{x}{b}\right)^{c}\right], \quad x > 0, \quad b, c > 0 \tag{1.1}$$

where b and c are respectively the scale and shape parameters. Its cumulative distribution function is

$$F(x; b, c) = 1 - \exp\left[-\left(\frac{x}{b}\right)^{c}\right], \quad x > 0, \quad b, c > 0 \tag{1.2}$$

To construct an optimally grouped sample when there is no a priori knowledge or guess values of the parameters suppose N be the number of sampling units put under a life-testing experiment which assumes the Weibull model (1.1) and suppose the experimenter wishes to obtain the grouped life-time data with k classes. Then from Table 1 optimal group limits y_i's, i=1,2,……k-1. can be used to compute the expected number of sampling units to be failed in the time interval (t_{i-1}, t_i) for i=1,2,……k and is given by

$$f_i = Np_i \qquad for\ i = 1,2\ldots\ldots k \tag{1.3}$$

where

$$p_i = \exp(-y_{i-1}) - \exp(-y_i)$$

f_i is expected number of failures in the i^{th} interval
Y_i's are optimal group limits obtained from Table 1
K is number of groups
N is total frequency

and f_i's may be rounded to the nearest integers so that $N=f_1+ f_2+\ldots+f_k$. Here, it may be noted that the experimenter has to observe the random time instants t_1, t_2, …, t_{k-1} so as the optimum pre-fixed number of units f_i to be failed in the time interval (t_{i-1}, t_i) for i=1, 2,…, k taking $t_o=0$ and $t_k=\infty$. In other words, he/she has to record a random time instant after failure of first f_1 sampling units, but before the failure of $(f_1+1)^{th}$ sampling unit and to record a random time instant after failure of first $f_1+ f_2$ sampling units, but before the failure of $(f_1+ f_2 +1)^{th}$ sampling unit and so on. Further, it may be noted that it is difficult to record all exact failure times of the individual units but it is not so difficult to note a random time instant between the failure times of two consecutive sampling units, $t_1, t_2, \ldots, t_{k-1}$, the group limits of the optimally constructed grouped sample using the procedure explained above, are the observed values of the true asymptotic optimal group limits $x_1, x_2, \ldots, x_{k-1}$ whereas their expected (estimated) values are given by

$$\hat{x}_i = \hat{b}(y_i)^{1/\hat{c}} \tag{1.4}$$

where $\hat{b}$ and $\hat{c}$ are obtained by using the principle of Least Squares method that is by minimizing the Error Sum of Squares and y_i's are asymptotically optimal group limits are taken from Table 1 as fixed values.

$$\sum_{i=1}^{k-1} \varepsilon_i^2 \,, \qquad \varepsilon_i = logt_i - log\hat{x}_i \tag{1.5}$$

where ε_i is the difference between the log values of the observed and estimated values of the optimal group limits. According to the principle of Least Squares method the estimates of b and c may be obtained by regressing log t_i's on log(y_i's) that is by fitting the model (that is the power curve $x=by^{1/c}$).

$$logt_i = log\hat{b} + \left(\frac{1}{\hat{c}}\right) logy_i + log\varepsilon_i \qquad for\ i = 1,2,\ldots,k-1 \tag{1.6}$$

and are given by

$$\hat{c} = \frac{\sum_{i=1}^{k-1} log^2 y_i - \frac{\left[\sum_{i=1}^{k-1} logy_i\right]^2}{k-1}}{\sum_{i=1}^{k-1} logy_i logt_i - \left[\sum_{i=1}^{k-1} logy_i\right]\left[\sum_{i=1}^{k-1} logt_i\right]/(k-1)} \tag{1.7}$$

$$\hat{b} = exp\left[\frac{1}{k-1}\left\{\sum_{i=1}^{k-1} logt_i - \left(\sum_{i=1}^{k-1} logy_i/\hat{c}\right\}\right] \tag{1.8}$$

The rationale for applying Least Squares method is that for a given k, y_i's are fixed values and can be borrowed from Table 1 whereas t_i's are random values and are obtained as observations from the procedure explained as above.

3. Rank Regression Estimation of the Parameters from Ungrouped (Complete) Sample

Rank Regression Estimates of b and c from a complete sample $x_1, x_2, \ldots, x_N$ drawn from the above Weibull population (1.1) is that to bring the above function (1.2) into a linear form.

$$Ln\{-Ln(1-F(xi))\} = cLn\left(\frac{xi}{b}\right) = -cLnb + cLnxi \tag{2.1}$$

Now denote $y = Ln\{-Ln(1-F(xi))\}$ and Ln x_i=T_i

This results in the following linear equations

$$y_i = (-cLnb) + cT_i \tag{2.2}$$

where i = 1,2 …………N

$$T_i = \text{Ln } b + (1/c)\, Y_i \tag{2.3}$$

Now let us denote $\alpha = Lnb, \quad \beta = \frac{1}{c}$

By simplification, we have

$$\tilde{c} = \frac{\frac{1}{N}\sum Y_i^2 - \overline{Y}^2}{\frac{1}{N}\sum T_i Y_i - \overline{T}\overline{Y}} \tag{2.4}$$

$$\tilde{b} = e^{\overline{T}_i - \overline{Y}_i/\tilde{c}} \tag{2.5}$$

In the Rank Regression method for parameter estimation we replace the unknown $F(x_i)$ with median rank, namely

$$\hat{F}(x_i) = \frac{i-0.3}{N+0.4} \quad ; \ i = 1,2,\text{----}N$$

Among Y_i and T_i values we may treat Y_i as independent variable (since it is fixed) and T_i can be treated as dependent variable (since it is a random variable). But in the above (2.2) equation, the fixed variable Y is expressed as a function of the dependent variable T which is not reasonable form for applying Least Square method. In view of this important assumption, we rewrite equation (2.2) as (2.3).

4. Comparison of Least Square Estimators Obtained from Optimal Grouped Sample with Rank Regression Estimates Obtained from Ungrouped Samples

The Least Square Estimates of $\hat{b}$ and $\hat{c}$ obtained from an optimal grouped sample are compared with the Rank Regression Estimates $\tilde{b}$ and $\tilde{c}$ obtained from ungrouped sample. Though the estimates from both the methods in closed form, it is very difficult to obtain the bias and variance of estimates mathematically, since the estimates are in non-linear form. Hence, we have resorted to Manto Carlo simulation to compute bias, variance and Mean Square Error of $\hat{b}$ and $\hat{c}$ as well as $\tilde{b}$ and $\tilde{c}$. We simulate these values based on 1000 samples of size N=50, 100, 150, 200, 250, 300 generated from standard Weibull Distribution with shape parameter C=2 to 5 (1). Here to assess the performances of $\hat{b}$ and $\hat{c}$, we compare these estimates with the corresponding rank regression estimates obtained from ungrouped sample.

Results from the simulation results:

- The Least Square Estimators are less biased when compared to the Rank Regression Estimators, in particularly the scale parameter.
- Least Square Estimator of scale parameter is having slightly larger variance than Rank Regression Estimator of scale parameter; on the other hand, as Least Square Estimator of shape parameter is having smaller variance than Rank Regression Estimator of shape parameter
- The trace of the variance-covariance matrix of Least Square Estimators is relatively small than the trace of the variance- covariance matrix of Rank Regression Estimators.
- If our interest is to find an efficient estimator for shape parameter c, Least Square method may be applied instead of Rank Regression Estimation method.

5. Conclusion

The Least Square Estimators are obtained from an optimal grouped sample whereas Rank Regression Estimators are obtained from complete sample. Thus Least Square method applied for an optimal grouped sample is yielding more efficient estimators than the Rank Regression method applied to a complete sample, especially when sample is large. This is an interesting application of Least Square method.

An illustration

A random sample of 200 observations is generated from a two-parameter Weibull Distribution with the parameters b=100 and c=2.5 using MINITAB package and the ordered sample is given below:

20.393	15.115	16.835	18.516	25.093	26.197	28.730	29.819	29.915
30.253	31.163	31.461	31.732	32.610	33.664	35.517	36.066	37.025
37.539	37.734	37.777	37.897	40.576	41.296	41.547	42.951	43.934
44.251	46.182	46.438	47.429	47.696	47.724	48.800	49.133	49.184
49.341	49.777	49.953	50.687	51.483	51.528	51.645	52.303	53.399
54.387	54.645	55.233	55.608	56.212	56.491	58.455	59.046	59.751
59.782	61.024	61.762	62.605	62.917	64.975	66.976	67.090	67.144
67.389	67.732	67.759	68.659	69.350	69.748	70.309	70.456	70.744
71.125	71.657	72.124	72.905	73.104	73.294	74.151	74.160	74.768
74.881	75.735	76.445	77.273	77.391	77.686	78.098	78.387	78.396
78.577	79.868	80.126	80.933	80.969	81.261	82.313	82.387	82.408
83.223	83.431	84.851	84.949	85.229	85.308	85.779	86.087	86.098
86.213	86.890	86.927	87.546	88.126	88.333	88.794	90.038	90.753
90.956	91.295	91.576	92.649	93.069	94.160	95.144	95.794	96.032
96.133	96.605	96.677	97.436	98.138	99.763	100.713	100.898	101.740
101.843	102.814	102.849	104.344	105.910	106.650	108.114	108.175	110.553
111.443	111.654	111.891	111.998	112.496	112.646	112.898	113.013	114.022
114.169	115.132	116.082	119.297	119.984	120.985	121.432	121.537	122.616
122.757	125.039	125.815	126.095	128.360	128.787	130.107	130.172	130.447
132.115	132.706	133.505	134.846	135.335	136.332	137.926	138.248	138.257
138.417	139.445	142.873	143.703	146.090	146.141	151.264	152.379	153.720
155.579	157.108	160.668	160.711	164.649	165.699	168.145	168.963	172.468
181.222	207.136							

The computation of $\hat{b}$ and $\hat{c}$ from the optimal grouped sample of k=6 and size N=200 using (1.8), (1.7) and the computation of $\tilde{b}$ and $\tilde{c}$ from an ungrouped sample of size N=200 using (2.5) and (2.4) is carried out using MS-Excel and computation Table is presented in Tables 3 and 4, respectively.

Table 1: Asymptotically optimal group limits X_i in the form $Y_{i=}(X_i/B)^C$, (I=1,2,…,K-1) of Weibull scale (b) and shape(c) parameters from a grouped sample with k groups

k	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}
3	0.273	2.6067												
4	0.210	1.3979	3.413											
5	0.104	0.5123	1.959	3.860										
6	0.077	0.3649	1.226	2.572	4.409									
7	0.050	0.2319	0.675	1.719	2.992	4.794								
8	0.037	0.1739	0.483	1.190	2.204	3.428	5.205							
9	0.027	0.1269	0.343	0.782	1.602	2.571	3.767	5.527						
10	0.021	0.0987	0.263	0.577	1.180	1.993	2.927	4.102	5.847					
15	0.007	0.0343	0.091	0.187	0.339	0.573	0.938	1.442	2.011	2.637	3.353	4.216	5.342	7.050

Table 2: Computation of $\hat{b}$ and $\hat{c}$ from the optimal grouped sample using Least-Squares method

t	y	Ln(t)	ln(y)	ln^2(y)	ln(t)ln(y)
34.59	0.0772	3.5436	-2.5614	6.5605	-9.0764
67.12	0.3649	4.2064	-1.0081	1.0163	-4.2406
108.14	1.2268	4.6835	0.2044	0.0418	0.9573
146.12	2.5724	4.9844	0.9448	0.8927	4.7095
176.85	4.4096	5.1753	1.4838	2.2016	7.6790
		22.5932	-0.9365	10.7130	0.0288

	Optimal Grouped Sample	**Complete Sample**
Parameter	**Estimates using Least Squares**	**Estimates using Rank Regression**
c (shape)	2.4735	2.4632
b (scale)	98.9240	97.9284

TR_{RR} =Trace of the variance –covariance matrix of Rank Regression Estimators

$= V(\tilde{c}) + V(\tilde{c})$

T_{LS} = Trace of the variance –covariance matrix of Least Square Estimators.

$= V(\hat{b}) + V(\tilde{c})$

Table 3: Performance of Least Square Estimates of scale (b) and shape (c) parameters of Weibull Distribution obtained from ungrouped sample as compared with the Rank Regression Estimates obtained from complete sample. ($ARE_{RR}(\hat{b})$: ARE of ($\hat{b}$) as compared with $\tilde{b}$, ARE_{RR} ($\hat{c}$): ARE of ($\hat{c}$) as compared with $\tilde{c}$)

K	N	C	Least Square Estimate $\hat{b}$			Rank Regression Estimate $\tilde{b}$			Rank Regression Estimate $\tilde{c}$			Least Square Estimate $\hat{c}$			TR_{RR}	TR_{LS}	ARE_{RR} ($\hat{b}$)	($\hat{c}$)
			BIAS/b (1)	MSE/b^2 (2)	VAR/b^2 (3)	BIAS/b (4)	MSE/b^2 (5)	VAR/b^2 (6)	BIAS (7)	MSE (8)	VAR (9)	BIAS (10)	MSE (11)	VAR (12)	(13)	(14)	(15)	(16)
6	50	2	-.0121	.0062	.0060	.0015	.0057	.0057	.0079	.0779	.0779	-.0306	.2085	.2076	.0836	.0840	94.5	99.8
		3	-.0091	.0028	.0027	.0003	.0025	.0025	.0119	.1754	.1752	.0241	.0230	.0225	.1778	.1786	93.9	99.6
		4	-.0072	.0016	.0015	.0000	.0014	.0014	.0159	.3118	.3115	.0362	.0519	.0506	.3130	.3145	93.6	99.5
		5	-.0060	.0010	.0010	-.0001	.0009	.0009	-.0199	.4872	.4868	.1302	.0950	.0780	.4877	.4904	93.4	99.5
	100	2	-.0061	.0033	.0033	.0005	.0029	.0029	-.0029	.0386	.0386	.1945	.2137	.1759	.0414	.0365	87.1	116.1
		3	-.0046	.0015	.0015	.0000	.0013	.0013	-.0044	.0868	.0867	.2588	.3800	.3130	.0880	.0762	86.6	116.1
		4	-.0037	.0008	.0008	-.0001	.0007	.0007	-.0058	.1542	.1542	.3231	.5939	.4894	.1549	.1337	86.4	116.1
		5	-.0030	.0005	.0005	-.0002	.0005	.0005	-.0073	.2410	.2410	-.0125	.0334	.0332	.2414	.2081	86.2	116.1
	150	2	-.0066	.0021	.0021	.0000	.0018	.0018	-.0051	.0266	.0266	-.0185	.0751	.0747	.0284	.0245	89.3	118.3
		3	-.0047	.0009	.0009	-.0002	.0008	.0008	-.0077	.0598	.0598	-.0246	.1334	.1328	.0606	.0515	88.9	118.2
		4	-.0037	.0005	.0005	-.0002	.0005	.0005	-.0102	.1063	.1062	.0482	.0922	.0899	.1067	.0904	88.7	118.2
		5	-.0030	.0003	.0003	-.0002	.0003	.0003	-.0128	.1662	.1660	.0603	.1441	.1405	.1663	.1408	88.5	118.2
	200	2	-.0095	.0017	.0016	.0002	.0014	.0014	-.0122	.0206	.0205	.0319	.0185	.0175	.0219	.0191	87.3	117.0
		3	-.0066	.0008	.0007	.0000	.0006	.0006	-.0183	.0464	.0461	.0478	.0417	.0394	.0467	.0401	86.7	117.0
		4	-.0050	.0004	.0004	-.0001	.0003	.0003	-.0244	.0825	.0819	.0637	.0741	.0700	.0823	.0704	86.4	117.0
		5	-.0040	.0003	.0003	-.0001	.0002	.0002	-.0305	.1289	.1280	.0796	.1157	.1094	.1282	.1097	86.2	117.0
	250	2	.0015	.0014	.0014	-.0004	.0011	.0011	-.0121	.0170	.0169	.0168	.0143	.0140	.0180	.0154	82.5	120.5
		3	.0008	.0006	.0006	-.0004	.0005	.0005	-.0182	.0384	.0380	.0252	.0322	.0316	.0385	.0322	82.7	120.5
		4	.0005	.0003	.0003	-.0004	.0003	.0003	-.0242	.0682	.0676	.0337	.0573	.0561	.0679	.0565	82.8	120.4
		5	.0004	.0002	.0002	-.0003	.0002	.0002	-.0303	.1066	.1056	.0421	.0895	.0877	.1058	.0879	82.8	120.4

Table 4: Performance of Least Square Estimates of scale (b) and shape (c) parameters of Weibull Distribution obtained from ungrouped sample as compared with the Rank Regression Estimates obtained from complete sample. ($ARE_{RR}(\hat{b})$: ARE of ($\hat{b}$) as compared with $\tilde{b}$, ARE_{RR} ($\hat{c}$): ARE of ($\hat{c}$) as compared with $\tilde{c}$)

K	N	C	Least Square Estimate $\hat{b}$			Rank Regression Estimate $\tilde{b}$			Rank Regression Estimate $\tilde{c}$			Least Square Estimate $\hat{c}$			TR_{RR}	TR_{LS}	ARE_{RR} ($\hat{b}$)	ARE_{RR} ($\hat{c}$)
			BIAS/b (1)	MSE/b^2 (2)	VAR/b^2 (3)	BIAS/b (4)	MSE/b^2 (5)	VAR/b^2 (6)	BIAS (7)	MSE (8)	VAR (9)	BIAS (10)	MSE (11)	VAR (12)	(13)	(14)	(15)	(16)
8	100	2	-.0160	.0034	.0031	.0005	.0029	.0029	-.0121	.0170	.0169	.1023	.0498	.0394	.0414	.0425	91.2	97.9
		3	-.0113	.0015	.0014	.0000	.0013	.0013	-.0182	.0384	.0380	.1526	.1120	.0887	.0880	.0901	90.2	97.8
		4	-.0087	.0009	.0008	-.0001	.0007	.0007	-.0029	.0386	.0386	.2030	.1990	.1578	.1549	.1586	89.7	97.7
		5	-.0070	.0006	.0005	-.0002	.0005	.0005	-.0044	.0868	.0867	.2533	.3109	.2467	.2414	.2472	89.4	97.7
	150	2	-.0150	.0023	.0021	.0000	.0018	.0018	-.0058	.1542	.1542	-.0098	.0245	.0244	.0284	.0265	86.5	108.9
		3	-.0104	.0011	.0010	-.0002	.0008	.0008	-.0073	.2410	.2410	-.0151	.0551	.0549	.0606	.0559	85.7	108.8
		4	-.0079	.0006	.0005	-.0002	.0005	.0005	-.0051	.0266	.0266	-.0203	.0981	.0977	.1067	.0982	85.3	108.8
		5	-.0064	.0004	.0003	-.0002	.0003	.0003	-.0077	.0598	.0598	-.0256	.1533	.1526	.1663	.1530	85.0	108.8
	200	2	-.0010	.0018	.0018	.0002	.0014	.0014	-.0102	.1063	.1062	-.0149	.0189	.0187	.0219	.0205	77.2	109.4
		3	-.0010	.0008	.0008	.0000	.0006	.0006	-.0128	.1662	.1660	-.0224	.0426	.0421	.0467	.0429	77.2	109.4
		4	-.0009	.0005	.0005	-.0001	.0003	.0003	-.0122	.0206	.0205	-.0299	.0758	.0749	.0823	.0754	77.2	109.3
		5	-.0007	.0003	.0003	-.0001	.0002	.0002	-.0183	.0464	.0461	-.0373	.1185	.1171	.1282	.1174	77.2	109.3
	250	2	-.0125	.0015	.0014	-.0004	.0011	.0011	-.0244	.0825	.0819	.0195	.0159	.0155	.0180	.0169	82.0	108.9
		3	-.0085	.0007	.0006	-.0004	.0005	.0005	-.0305	.1289	.1280	.0291	.0358	.0349	.0385	.0355	81.3	108.9
		4	-.0065	.0004	.0003	-.0004	.0003	.0003	-.0242	.0682	.0676	.0388	.0636	.0621	.0679	.0624	81.0	108.9
		5	-.0052	.0002	.0002	-.0003	.0002	.0002	-.0303	.1066	.1056	.0484	.0994	.0970	.1058	.0973	80.7	108.9
	300	2	-.0058	.0012	.0012	.0000	.0010	.0010	-.0124	.0142	.0141	.0097	.0125	.0124	.0151	.0136	83.5	113.5
		3	-.0041	.0005	.0005	-.0001	.0004	.0004	-.0187	.0320	.0317	.0145	.0281	.0279	.0321	.0284	83.1	113.5
		4	-.0031	.0003	.0003	-.0001	.0002	.0002	-.0249	.0569	.0563	.0193	.0500	.0496	.0566	.0499	82.9	113.5
		5	-.0025	.0002	.0002	-.0001	.0002	.0002	-.0311	.0890	.0880	.0241	.0782	.0776	.0882	.0778	82.8	113.4

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APPENDIX-I

Symbols: Least Square Estimators of scale(b) and shape(c) are $\hat{b}$ and $\hat{c}$. The Rank Regression Estimators of scale(b) and shape(c) are $\tilde{b}$ and $\tilde{c}$ Asymptotically relative efficiency of Least Square Estimators scale ($\hat{b}$) and shape($\hat{c}$) as compared with Rank Regression Estimators scale($\hat{b}$) and shape($\tilde{c}$) are $ARE_{RR}(\hat{b})$ and $ARE_{RR}(\hat{c})$, Trace of variance – covariance matrix of Least Square Estimators is TR_{LS} and Trace of variance – covariance matrix of Rank Regression Estimators is TR_{RR}, Asymptotically optimal group limits are y_i's where i=1,2,……k-1.