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SIZE AND POWER PROPERTIES OF ASYMPTOTICALLY ROBUST TESTS FOR EQUALITY OF TWO COVARIANCE MATRICES

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ABSTRACT

Some asymptotically robust test statistics for equality of two covariance matrices are discussed. The standard error test based on combined and separate estimator of asymptotic covariance matrices of vectors of second-order sample moments, is estimated with and without transformations. The untransformed test based on separate estimator is equally good as Layard (1972, 1974) [3,4] proposed transformed test based on combined estimator. The effect of transformations on the tests is examined. The size and power performance of the untransformed tests is compared. The standard error test based on separate estimator is found reasonable for moderate size of non-normal samples.

Key words: *Robust, asymptotic covariance matrix, bivariate distribution, convergence in distribution, consistent estimator, kurtosis parameter, bias.*

1. INTRODUCTION

Layard (1972) [3] described some asymptotically robust test statistics for the equality of two covariance matrices, i.e. standard error, grouping, and jackknife for bivariate distributions. Layard (1974) [4] compared the size and power properties of these tests. He proposed transformations, i.e. Log_e transformation of variances and $\text{Tan}h^{-1}$ transformation of sample correlation co-efficient. Furthermore he assumed that the transformed vectors of second-order sample moments have same asymptotic covariance matrix and preferred to use combined estimator of it, for standard error test. Layard (1974) [4] and Pervaiz (1986) [5] concluded that standard error test based on combined estimator is better than grouping and jackknife tests as regards size and power for non-normal distributions. Like Layard (1972, 1974) [3,4] Tiku and Balakrishnan (1985) [7] treated the problem as a test for equality of mean vectors and proposed a T^2 test.

The aim of the paper is to look at:

(a) The effect of combined and separate estimates of asymptotic covariance matrix of vectors of second-order sample moments on size and power performance of standard error test, with and without transformations.

(b) The effect of transformations on size and power properties of the tests.

(c) The performance of the untransformed asymptotically robust tests.

The test statistics are defined in section 3. The sampling experiments are discussed in section 4. Simulations were carried out on ICL 2976 Computer at the University of the Southampton, United Kingdom. The random number generator used was Gϕ5DDF, Gϕ5DBF and Gϕ5CAF from the NAG library through NAG Limited. The programs were written in FORTRAN-IV.

The test statistics computed, are compared with the 5% and 1% points of the approximate null distributions. The results for the 1% case, essentially corroborate those of the 5% case, so these are not reported here. The section 5 provides discussion of empirical results. The conclusions are given in section 6.

2. PROPERTIES OF SAMPLE COVARIANCE MATRICES FOR INDEPENDENTLY AND IDENTICALLY DISTRIBUTED SAMPLES

Suppose two bivariate populations with distribution functions F and G, covariance matrices $\underline{\Sigma}_i, i = 1, 2$ and finite fourth moments.

Where

$$\underline{\Sigma}_i = \begin{bmatrix} \mu_{i,20} & \mu_{i,11} \\ \mu_{i,11} & \mu_{i,02} \end{bmatrix}$$

$$\mu_{i,rs} = \mathrm{E}\left[(X_{i1} - \mathrm{E}(X_{i1}))^r (X_{i2} - \mathrm{E}(X_{i2}))^s\right] \qquad (2.1)$$

$$X_i = (X_{i1}, X_{i2})^T$$

The sample covariance matrices $\underline{S}_i$ are

$$\underline{S}_i = \begin{bmatrix} s_{i,20} & s_{i,11} \\ s_{i,11} & s_{i,02} \end{bmatrix} \qquad (2.2)$$

Where

$$s_{i,rs} = \frac{1}{n_i} \sum_{e=1}^{n_i} \left(x_{ie1} - \bar{x}_{i.1}\right)^r \left(x_{ie2} - \bar{x}_{i.2}\right)^s \qquad (2.3)$$

$$\bar{x}_{i.1} = \frac{1}{n_i} \sum_{e=1}^{n_i} x_{ie1}$$

$$\bar{x}_{i.2} = \frac{1}{n_i} \sum_{e=1}^{n_i} x_{ie2} \qquad (2.4)$$

Let

$$\underline{S}_1^v = (s_{1,20}\ ,\ s_{1,02}\ ,\ s_{1,11})^T$$

$$\underline{S}_2^v = (s_{2,20}\ ,\ s_{2,02}\ ,\ s_{2,11})^T \qquad (2.5)$$

and $\underline{\Sigma}_i^v$ are determined similarly from (2.1) i.e. vectors of second-order population moments. Following Cramer (1946, p.365) [1], Layard (1972) [3] showed that

$$n_1^{\frac{1}{2}}\left(\underline{S}_1^* - \underline{\Sigma}_1^*\right) \xrightarrow{\xi} N_3\left(\underline{0}, \underline{\Gamma}_1\right) \quad as \quad n_1 \to \infty$$
$$n_2^{\frac{1}{2}}\left(\underline{S}_2^* - \underline{\Sigma}_2^*\right) \xrightarrow{\xi} N_3\left(\underline{0}, \underline{\Gamma}_2\right) \quad as \quad n_2 \to \infty \tag{2.6}$$

(The symbol $\xrightarrow{\xi}$ denotes convergence in distribution). Where $\underline{\Gamma}_i$ are:

$$\underline{\Gamma}_i = \begin{bmatrix} \mu_{i,40} - \mu_{i,20} & \mu_{i,22} - \mu_{i,20}\,\mu_{i,02} & \mu_{i,31} - \mu_{i,20}\,\mu_{i,11} \\ & \mu_{i,04} - \mu_{i,02}^2 & \mu_{i,13} - \mu_{i,02}\,\mu_{i,11} \\ & & \mu_{i,22} - \mu_{i,11}^2 \end{bmatrix} \tag{2.7}$$

Layard (1972, 1974) [3,4] proposed transformations to hasten convergence to normality. These are:

$$\phi\left[\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}\right] = \begin{bmatrix} ln\, v_1 \\ ln\, v_2 \\ \frac{1}{2} ln\left(\frac{1+\rho}{1-\rho}\right) \end{bmatrix} \tag{2.8}$$

Where $\ell = v_3\left(v_1 v_2\right)^{-\frac{1}{2}}$. Following Layard (1974) [4]

$$n_1^{\frac{1}{2}}\left[\phi\left(\underline{S}_1^*\right) - \phi\left(\underline{\Sigma}_1^*\right)\right] \xrightarrow{\xi} N_3\left(0, \underline{\Omega}_1\right) \quad as \quad n_1 \to \infty$$

$$n_2^{\frac{1}{2}}\left[\phi\left(\underline{S}_2^*\right) - \phi\left(\underline{\Sigma}_2^*\right)\right] \xrightarrow{\xi} N_3\left(0, \underline{\Omega}_2\right) \quad as \quad n_2 \to \infty \tag{2.9}$$

Where

$$\underline{\Omega}_i = \underline{A}_i^T\ \underline{\Gamma}_i\ \underline{A}_i \tag{2.10}$$

($\underline{A}$ is matrix of first partial of ϕ evaluated at $\underline{\mu}$)

$$\underline{A}_i = \begin{bmatrix} \mu_{i,20}^{-1} & 0 & -\frac{\rho_i}{2\mu_{i,20}}\left(1-\rho_i^2\right) \\ 0 & \mu_{i,20}^{-1} & -\frac{\rho_i}{2\mu_{i,02}}\left(1-\rho_i^2\right) \\ 0 & 0 & \frac{1}{\left(\mu_{i,20}\,\mu_{i,02}\right)^{\frac{1}{2}}}\left(1-\rho_i^2\right) \end{bmatrix} \tag{2.11}$$

and $\underline{\Gamma}_i$ as given in (2.7). $\rho_i = \mu_{i,11}\left(\mu_{i,20}\,\mu_{i,02}\right)^{\frac{1}{2}}$. Layard (1972) [3] suggested that consistent estimators of the asymptotic covariance matrices of $\phi(\underline{S}_i^v)$ can be obtained from (2.10) by substituting sample quantities $s_{i,rs}$ for $\mu_{i,rs}$ population moments.

3. TEST STATISTICS

The problem is to test:

$$H_o: F(x_1, x_2) = G(x_1 + \xi_1, x_2 + \xi_2) \quad vs \quad H_A: \underline{\Sigma}_1 \neq \underline{\Sigma}_2$$

Where ξ_1 and ξ_2 are unspecified constants. The choice of H_o ensures that the fourth moments of the distributions are equal.

The tests used in the sampling experiments are as follows:

(1) Standard error

Because we are interested in the comparison of the performance of standard error test based on combined and separate estimator, and with and without transformations, therefore the test is described with these respects.

(a) Untransformed based on Separate Estimator

From (2.6) under H_o the test statistics:

$$(\underline{S}_1^v - \underline{S}_2^v)^T \left[n_1^{-1}\hat{\underline{\Gamma}}_1 + n_2^{-1}\hat{\underline{\Gamma}}_2\right]^{-1} (\underline{S}_1^v - \underline{S}_2^v) \tag{3.1}$$

is approximately distributed as χ_3^2, provided $\hat{\underline{\Gamma}}_1$ and $\hat{\underline{\Gamma}}_2$ are consistent estimators of the asymptotic covariance matrix of $\underline{S}_1^v$ and $\underline{S}_2^v$

respectively. These can be obtained from (2.7) by using sample quantities as defined by (2.3)

(b) Untransformed based on Combined Estimator

From (2.6) the test statistic:

$$\frac{n_1 n_2}{n_1+n_2}[(\underline{S}_1^v-\underline{S}_2^v)^T \hat{\underline{\Gamma}}^{-1}(\underline{S}_1^v-\underline{S}_2^v)]$$

has the distribution of (3.1) under H_O. The $\hat{\underline{\Gamma}}$ can be obtained from (2.7) by using s_{rs} in place of $\mu_{i,rs}$, where

$$s_{rs}=\frac{1}{n_1+n_2}\sum_{i=1}^{2}\sum_{e=1}^{n_i}\left(x_{ie1}-\bar{x}_{1.1}\right)^r\left(x_{ie2}-\bar{x}_{1.2}\right)^s \tag{3.2}$$

The $\bar{x}_{1.1}$ and $\bar{x}_{1.2}$ are defined by (2.4)

(c) Transformed based on Separate Estimator

From (2.8) and (2.9), under H_O the test statistic:

$$[\phi(\underline{S}_1^v)-\phi(\underline{S}_2^v)]^T[n_1^{-1}\hat{\underline{\Omega}}_1+n_2^{-1}\hat{\underline{\Omega}}_2]^{-1}[\phi(\underline{S}_1^v)-\phi(\underline{S}_2^v)] \tag{3.3}$$

has the distribution of (3.1), provided $\hat{\underline{\Omega}}_1$ and $\hat{\underline{\Omega}}_2$ are consistent estimators of the asymptotic covariance matrices of $\phi(\underline{S}_1^v)$ and $\phi(\underline{S}_2^v)$ respectively. These can be obtained from (2.10) by using sample quantities in place of population moments.

(d) Transformed based on Combined Estimator

Under H_O, $\phi(\underline{S}_1^v)$ and $\phi(\underline{S}_2^v)$ have same asymptotic covariance matrix, i.e

$$\underline{\Omega}=\underline{A}^T\underline{\Gamma}\underline{A} \tag{3.4}$$

Therefore Layard (1974) [4], preferred the test statistic:

$$\frac{n_1 n_2}{n_1+n_2}\left[\left(\left(\phi\,\underline{S}_1^v\right)-\phi\left(\underline{S}_2^v\right)\right)^T\hat{\underline{\Omega}}^{-1}\left(\left(\phi\,\underline{S}_1^v\right)-\phi\left(\underline{S}_2^v\right)\right)\right]$$

having the distribution of (3.1). The $\underline{\hat{\Omega}}$ can be obtained from (3.4) by using s_{rs} as defined by (3.2) for population quantities in (2.7) and (2.11).

(II) Grouping

Each sample is divided randomly into $n_i^{'}\; i = 1,2;$ groups of size $L, i.e.\, n_i = Ln_i^{'}\; for\; L \geq 2$ (assumption is that n_i are divisible by L)

Let

$$\underline{S}_{i,g} = \begin{bmatrix} s_{i,20g} & s_{i,11g} \\ s_{i,11g} & s_{i,02g} \end{bmatrix} \quad g = 1,2,\ldots\ldots,n_i^{'}$$

sample variance - covariance matrices within groups. The $\underline{S}_{i,g}^{v}$, vectors of second-order sample moments from groups of first and second samples, are independent and have approximately the multivariate normal distribution with equal mean vectors and covariance matrices, under H_O, being so the test statistic:

$$(\bar{\underline{S}}_1^{v} - \bar{\underline{S}}_2^{v})^T \,[n_1^{'-1}\underline{\widetilde{\Gamma}}_1 + n_2^{'-1}\underline{\widetilde{\Gamma}}_2]^{-1}\, (\bar{\underline{S}}_1^{v} - \bar{\underline{S}}_2^{v}) \qquad (3.5)$$

has approximately Hotelling's T^2 distribution with 3 and $n_1^{'} + n_2^{'} - 2$ degrees of freedom. Where

$$\underline{\widetilde{\Gamma}}_i = \frac{1}{n_i^{'} - 1}\sum_{g=1}^{n_i^{'}} (\underline{S}_{i,g}^{v} - \bar{\underline{S}}_i^{v})\,(\underline{S}_{i,g}^{v} - \bar{\underline{S}}_i^{v})^T$$

$$\bar{\underline{S}}_i^{v} = \frac{1}{n_i^{'}}\sum_{g=1}^{n_i^{'}} \underline{S}_{i,g}^{v}$$

(III) Jackknife

Let $\underline{S}_{i-e}$, sample covariance matrices, defined as:

$$\underline{S}_{i,e} = \begin{bmatrix} s_{i,20e} & s_{i,11e} \\ s_{i,11e} & s_{i,02e} \end{bmatrix} \quad e = 1,2,\ldots\ldots,n_i$$

The elements of the matrix are estimated second-order moments from the samples by using (n_i-1) observations, with the e-th observation omitted, Let

$$\underline{S}^{v}_{i,e} = n_i \underline{S}^{v}_{i} - (n_i - 1)\underline{S}^{v}_{i-e}$$

The jackknife estimators are the average of $\underline{S}^{v}_{i,e}$; i,e,

$$\underline{S}^{*v}_{i} = n_i \underline{S}^{v}_{i} - \frac{n_i - 1}{n_i}\sum_{e=1}^{n_i}\underline{S}^{v}_{i-e}$$

The $\underline{S}^{v}_{i,e}$ are approximately independent and have, under H_O, approximately equal mean vectors and covariance matrices. Thus test statistic:

$$(\underline{S}^{*v}_{1} - \underline{S}^{*v}_{2})^T [n_1^{-1}\underline{\Gamma}^{*}_{1} + n_2^{-1}\underline{\Gamma}^{*}_{2}]^{-1} (\underline{S}^{*v}_{1} - \underline{S}^{*v}_{2})$$

has approximately Hotelling's T^2 distribution with 3 and (n_1+n_2-2) degrees of freedom under H_O: Where

$$\underline{\Gamma}^{*}_{i} = \frac{1}{n_i - 1}\sum_{e=1}^{n_i}(\underline{S}^{v}_{i,e} - \underline{S}^{*v}_{i})(\underline{S}^{v}_{i,e} - \underline{S}^{*v}_{i})^T$$

(IV) Tiku and Balakrishnan T^2

Let

$$U_{1e} = \left(x_{1e1} - \bar{x}_{1.1}\right)^2 \; and \; U_{2e} = \left(x_{1e2} - \bar{x}_{1.2-}\right)^2;$$

$$V_{1e} = \left(x_{2e1} - \bar{x}_{2.1}\right)^2 \; and \; V_{2e} = \left(x_{2e2} - \bar{x}_{2.2-}\right)^2;$$

Where

$$x_{ie2-} = x_{ie2} - \hat{b}x_{ie1},$$

$\bar{x}_{i.1}$ and $\bar{x}_{i.2-}$ are usual means, while $\hat{b}$ is the pooled regression coefficient.

Tiku and Balakrishan (1985) [7] suggested that the test statistic:

$$T^2 = \frac{n_1 n_2}{n_1 + n_2}\begin{bmatrix}\bar{w}_1 & \bar{w}_2\end{bmatrix}\begin{bmatrix}\hat{\phi}_1^2 & 0 \\ 0 & \hat{\phi}_2^2\end{bmatrix}^{-1}\begin{bmatrix}\bar{w}_1 \\ \bar{w}_2\end{bmatrix}$$

is distributed approximately as Hotelling's T^2 with 2 and (n_1+n_2-2) degrees of freedom under H_0: Where

$$\bar{w}_1 = \bar{U}_1 - \bar{V}_1$$

$$\bar{w}_2 = \bar{U}_2 - \bar{V}_2$$

$$\hat{\phi}_1^2 = \frac{\sum_{e=1}^{n_1}(U_{1e} - \bar{U}_1)^2 + \sum_{e=1}^{n_2}(V_{1e} - \bar{V}_1)^2}{n_1 + n_2 - 2}$$

$$\hat{\phi}_2^2 = \frac{\sum_{e=1}^{n_1}(U_{2e} - \bar{U}_2)^2 + \sum_{e=1}^{n_2}(V_{2e} - \bar{V}_2)^2}{n_1 + n_2 - 2}$$

4. SAMPLING EXPERIMENTS

Four hypothetical distributions, the normal, the gamma, the double exponential and the contaminated normal are sampled. For details see Layard (1974) [4] and Pervaiz (1986) [5]. Furthermore the set of covariance matrices chosen is the same as Layard (1974) [4]. The covariance matrices which represent the null hypotheses are:

(a) Both I_{2x2}

(b) Both with unit variances and correlation co-efficient 0.9, and the alternative hypotheses are:

(c) I_{2x2} and 2.25 I_{2x2},

(d) I_{2x2} and variances 4 and correlation co-efficient 0.3,

(e) I_{2x2} and the matrix of (b)

5. DISCUSSION OF EMPIRICAL RESULTS

In considering the results it should be noted that the standard deviation of the estimated binomial proportion for a true proportion of 0.05 with samples of size 1000 is 0.07 and with samples of size 500 is 0.010. Therefore for 1000 replication observed proportions lying in (3.6, 6.4)%, and for 500 replications lying in (3.0, 7.0)% do not differ significantly from a true proportion of 5% at 95% level.

Table 1 provides the proportion of rejection observed for distribution-matrix-transformation combinations by using the combined and separate estimators of the asymptotic covariance matrices. Under transformations, the standard error test based on the combined estimator produced much better sizes than the separate estimator. The effect is becoming more significant with the increase of the kurtosis co-efficient of the parent distribution. But in the untransformed case the standard error test based on the combined estimator rejected the null hypothesis too infrequently for (b) in the case of the contaminated normal distribution. The observed size was 1.3% as opposed to the nominal 5% level. While the test based on separate estimator produced reasonable sizes.

To look at the asymptotic convergence of the transformed standard error test based on separate estimator, samples of size $n_1=n_2=20,40....,100$ and 250 are considered. The proportion of rejections observed for elliptical distributions (normal and contaminated normal)-matrix combinations are recorded in Table 2. The asymptotic convergence is not very good and still does not appear to have occurred for the contaminated normal distribution. While the untransformed test based on separate estimator produced very reasonable sizes with samples of size $n_1=n_2=25$. Consequently the separate estimator is used in the untransformed case.

The proportion of rejections observed for distribution-untransformed test- matrix combinations, with samples of size 25, are recorded in Table 3. All tests produced reasonable sizes for the normal distribution. The standard error test produced reasonable sizes for the non-normal distributions as well. The test has decreasing trend in observed sizes with the increasing kurtosis parameter of the parent distribution. The observed sizes for the gamma, the double exponential and the contaminated normal distributions were (7.9, 4.3 & 3.9 %) and (8.0, 4.8 and 3.4%)

The grouping test performed well for the gamma distribution as regards observed sizes, but rejected the null hypothesis too infrequently for (b) in the case of the double exponential and for (a &b) in the case of the contaminated normal distribution. The observed sizes were [(2.8)%] and [(3.3 & 2.5)%] for the respective distributions.

Gross (1976) [2] found the jackknife disappointing in confidence interval terms. Rocke and Downs (1981) [6] empirical study concludes, the jackknife method of variance estimation produces upward bias for the contaminated normal distribution. The upward bias in jackknife variance estimation may cause two infrequent rejections of the null hypothesis in the problem. Therefore, the test rejected the null hypothesis too infrequently for the double exponential and the contaminated normal distributions. The observed sizes were [(2.3 & 2.8) %] and [(2.2 & 1.3)%] for the respective distributions. Under transformations the test was rejecting the null hypothesis too infrequently for the double exponential and the contaminated normal distributions---(cf. Layard, 1974) [4].

The Tiku and Balakrishnan T^2 test produced sizes for the normal and the non-normal distributions ranging from a minimum of 2.5% to a maximum of 5.4%. It rejected the null hypothesis too infrequently for the double exponential and the contaminated normal distributions. The observed sizes were [(3.5 & 3.3%)] and [(2.5 & 2.6%)] for the respective distributions.

The standard error test is better in power than the grouping and the jackknife tests for the normal and the non-normal distributions. The Tiku and Balakrishanan T^2 test has comparable power with the standard error test for (c) and (d), but for (e) the test in inferior in power even than the grouping test.

To analyse the effect of increase in sample size on the performance of the tests the samples of size 50 are considered. The proportions of rejections observed are recorded in Table 4.. The standard error test maintained very good nominal levels for the distributions sampled, and worst being for (b) in the case of the contaminated normal distribution. The observed size was 3.0% as opposed to the nominal 5% level, not too bad.

There is no improvement as regards observed sizes from the grouping test. For (b) for the contaminated normal distribution the situation is very poor now. The observed size had fallen down to 1.6% from 2.5% in Table 3. The jackknife test is improved, and the improvement is very significant for (b) for the contaminated normal distribution. The observed size raised upto 2.3% from 1.3% in Table 3. But the test is still unable to achieve nominal levels for the double exponential and the contaminated normal distributions. Under transformations the jackknife test rejected the null hypothesis too frequently for the contaminated normal distribution---- (cf. Pervaiz, 1986) [5]. The Tiku and Balakrishnan T^2 test produced sizes from a minimum of 2.2% to a maximum of 4.6% for the distributions sampled; no improvement with the increase in sample size.

Of course the power of the tests increased with the increase in sample size.

To be more certain about the performance of the jackknife test, with and without transformations, samples of size $n_1=n_2=250$ are considered. The observed significance levels for distribution matrix combinations are recorded in Table 5. The untransformed jackknife test was rejecting the null hypothesis too infrequently for (a) for the double exponential distribution. The observed size was 2.2%. For all other situations the test maintained very good nominal levels. Under transformations the test rejected the null hypothesis too frequently for the contaminated normal distribution. The observed sizes were (9.4 & 8.0%).

6. CONCLUSIONS

For the standard error test it is not essential to apply transformations and to use combined estimator of asymptotic covariance matrix of vectors of second-order sample moments as suggested by Layard (1972, 1974) [3,4]. The untransformed test based on separate estimator is equally good as regards size and power. Therefore a strong assumption of equal asymptotic covariance matrices can be relaxed. The transformations does not play any significant role for the jackknife test as well.

When transformations are not applied:

(a) The standard error test based on separate estimator performs better than the grouping, the jackknife, and the Tiku and Balakrishnan T^2 tests, as regards size and power, for the non-normal distributions sampled.

(b) The grouping test is the worst in power, but for (e) the Tiku and Balakrishnan T^2 test.

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APPENDIX

Table 1

Empirical size based on 1000 replications for the standard error test by using combined and separate estimators of the asymptotic covariance matrices of vectors of second-order sample moments.

I all cases, $n_1=n_2=25$

	Nominal 5% level			
	Transformed		Untransformed	
Matrix pairs-----(c.f.section 4)	(a)	(b)	(a)	(b)
Combined				
Normal	0.064	0.066	0.044	0.039
Gamma	0.082	0.099	0.045	0.054
Double exponential	0.072	0.082	0.023	0.027
Contaminated normal	0.070	0.068	0.025	0.013
Separate				
Normal	0.131	0.138	0.060	0.064
Gamma	0.172	0.207	0.079	0.080
Double exponential	0.215	0.232	0.043	0.048
Contaminated normal	0.277	0.293	0.039	0.034

Table 2

Empirical size based on 1000 replications from the standard error test using separate estimators of asymptotic covariance matrices for equality to two covariance matrices.

	Nominal 5% level			
	Normal		Contaminated normal	
Matrix pairs---(c.f.section 4)	(a)	(b)	(a)	(b)
Sample size				
$n_1=n_2=20$	0.153	0.160	0.322	0.378
40	0.102	0.103	0.234	0.256
60	0.093	0.090	0.173	0.201
80	0.073	0.088	0.161	0.170
100	0.060	0.073	0.136	0.144
250	0.059	0.063	0.094	0.078

Table 3

Empirical size and power based on 1000 replications for tests of equality of two covariance matrices.

In all cases, $n_1=n_2=25$.

Nominal 5% level

Matrix pairs---(c.f.section 4)

	(a)	(b)	(c)	(d)	(e)
	Normal				
Standard error	0.060	0.064	0.559	0.961	0.962
Grouping (L=5)	0.048	0.051	0.266	0.524	0.494
Jackknife	0.047	0.042	0.481	0.930	0.925
Tiku & Balakrishnan T^2	0.041	0.042	0.580	0.967	0.485
	Gamma				
Standard error	0.079	0.080	0.526	0.886	0.952
Grouping (L=5)	0.044	0.044	0.203	0.453	0.474
Jackknife	0.048	0.055	0.431	0.850	0.921
Tiku & Balakrishnan T^2	0.054	0.047	0.510	0.908	0.456
	Double Exponential				
Standard error	0.043	0.048	0.280	0.659	0.906
Grouping (L=5)	0.043	0.028	0.119	0.325	0.391
Jackknife	0.023	0.028	0.209	0.559	0.855
Tiku & Balakrishnan T^2	0.035	0.033	0.256	0.646	0.298
	Contaminated Normal				
Standard error	0.039	0.034	0.227	0.541	0.747
Grouping (L=5)	0.033	0.025	0.114	0.242	0.291
Jackknife	0.022	0.013	0.169	0.450	0.680
Tiku & Balakrishnan T^2	0.025	0.026	0.247	0.598	0.242

Table 4

Empirical size and power based on 1000 replications for tests of equality of two covariance matrices.

In all cases, $n_1=n_2=50$.

Matrix pairs---------,cf section 4)	Nominal 5% level				
	Size			Power	
	(a)	**(b)**	**(c)**	**(d)**	**(e)**
	Normal				
Standard error	0.051	0.050	0.908	0.999	0.999
Grouping (L=5)	0.047	0.042	0.690	0.973	0.965
Jackknife	0.041	0.040	0.893	0.999	0.999
Tiku & Balakrishnan T^2	0.039	0.040	0.933	1.000	0.887
	Gamma (replications=500)				
Standard error	0.057	0.061	0.834	0.999	1.000
Grouping (L=5)	0.037	0.032	0.583	0.947	0.964
Jackknife	0.046	0.046	0.812	0.998	0.999
Tiku & Balakrishnan T^2	0.041	0.046	0.861	1.000	0.826
	Double exponential.				
Standard error	0.044	0.039	0.512	0.947	0.994
Grouping (L=5)	0.035	0.032	0.335	0.753	0.922
Jackknife	0.029	0.029	0.469	0.930	0.991
Tiku & Balakrishnan T^2	0.022	0.033	0.583	0.961	0.594
	Contaminated normal				
Standard error	0.038	0.030	0.370	0.795	0.923
Grouping (L=5)	0.034	0.016	0.215	0.584	0.765
Jackknife	0.026	0.023	0.325	0.759	0.910
Tiku & Balakrishnan T^2	0.033	0.034	0.467	0.853	0.448

Table 5

Empirical size based on 500 replications for the jaclkknife test of equality of two covariance matrices.

In all cases, $n_1=n_2=250$.

Matrix pairs (cf.section4)	Nominal 5% level			
	Untransformed		Transformed	
	(a)	(b)	(a)	(b)
Normal	0.042	0.062	0.046	0.068
Double exponential	0.022	0.060	0.034	0.058
Contaminated normal	0.038	0.038	0.094	0.080