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A Hybrid Group Acceptance Sampling Plan for Lifetimes Having Generalized Pareto Distribution

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Abstract

In this paper, a Hybrid Group Acceptance Sampling Plan (HGASP) is proposed for a Truncated Life Test if lifetimes of the items follow the Generalized Pareto Distribution. The Designed Parameters such as minimum number of testers and acceptance number are found when the Consumer’s Risk, test termination time and Group Size are pre-specified. The Operating Characteristic values, minimum ratios of the true average (mean) life for the given Producer’s Risk are also determined. A comparative study of conventional plan and existing plan is elaborated.

Keywords

Group sampling plan, Generalized pareto distribution, Consumer’s risk, Producer’s risk. Operating characteristics

1. Introduction

An Acceptance Sampling is a scheme, which consists of sampling, observation and inference in determining the acceptance or rejection of a lot of items submitted by the vendor. It is very important and useful tool if the quality of an item is explained by its lifetime. It has been an important decision to choose an appropriate Probability Distribution in describing the lifetime of the testing items.

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The selection of a Moderate Life Test Sampling Plan is a crucial decision because a good plan not only can help producer but is also very necessary for the consumer. Acceptance Sampling Plan is used when testing is destructive; examining every item is not possible and large number of items are inspected in a short interval of time. In an ordinary Acceptance Sampling Plan, it is assumed that a single item is observed in a tester but in practice more than one item can be examined by the availability of the testers. The items put in a tester can be considered as a group and the number of items in a group is called Group Size. Any Acceptance Sampling Plan which follows such type of pattern is called Group Acceptance Sampling Plan (GASP). The technique of obtaining minimum number of testers for a specified number of groups is called Hybrid Group Acceptance Sampling Plan (HGASP).

An ordinary Acceptance Sampling Plan based on Truncated Life Test for a variety of Lifetime Distributions were developed by Baklizi (2003), Balakrishnan et al. (2007), Epstein (1954), Kantam et al. (2006), Rosaiah et al. (2008) and Tsai and Wu (2006). More recently, Aslam et al. (2010b), Aslam et al. (2010c), Aslam and Jun (2009), Mughal (2011a), Mughal et al. (2010), Mughal et al. (2011b), Mughal et al. (2011c), and Rao (2011) have developed the Group Acceptance Sampling Plan based on Truncated Life Test. The objective of this study is to develop a Hybrid Group Acceptance Sampling Plan (HGASP) based on Truncated Life Test when the lifetime of an item follows the Generalized Pareto Distribution (GPD) introduced by Abd Elfattah et al. (2007). The Probability Density Function (p.d.f.) and Cumulative Distribution Function (c.d.f.) of GPD are given respectively

$$f(t;\alpha,\beta,\lambda,\delta)=\frac{\delta\alpha}{\beta}\left(\frac{t-\lambda}{\beta}\right)^{\delta-1}\left[1+\left(\frac{t-\lambda}{\beta}\right)^{\delta}\right]^{-(\alpha+1)} \quad (1.1)$$

where $\lambda<t<\infty$, $\beta>0, \alpha>0$, $\delta>0$, λ is the Location Parameter, β is the Scale Parameter and (α , δ) are Shape Parameters.

$$F(t;\alpha,\beta,\lambda,\delta)=1-\left[1+\left(\frac{t-\lambda}{\beta}\right)^{\delta}\right]^{-\alpha} \quad (1.2)$$

Aslam et al. (2010a) studied Group Acceptance Sampling Plan based on Generalized Pareto Distribution (GPD). The mean and variance of GPD are

$$\mu = \beta \frac{\Gamma\left(\alpha - \frac{1}{\delta}\right)\Gamma\left(1 + \frac{1}{\delta}\right)}{\Gamma(\alpha)} + \lambda \tag{1.3}$$

$$\sigma^2 = \beta^2 \left[\frac{\Gamma\left(1 + \frac{2}{\delta}\right)\Gamma\left(\alpha - \frac{2}{\delta}\right)}{\Gamma(\alpha)} - \left(\frac{\Gamma\left(1 + \frac{1}{\delta}\right)\Gamma\left(\alpha - \frac{1}{\delta}\right)}{\Gamma(\alpha)} \right)^2 \right] \tag{1.4}$$

For the existence of mean and variance, we assume the following conditions namely, $\alpha > 1/\delta$ and $\alpha > 2/\delta$ respectively.

2. Hybrid Group Acceptance Sampling Plans (HGASP)

Consider μ denotes the true mean life and μ_0 represents the specified mean life of an item which follows Generalized Pareto Distribution (GPD). It is wished to propose a HGASP if the true mean life is higher than the specified mean life i.e. $H_1 = \mu > \mu_0$. The HGASP also develop the mutual agreement of the both Producer's and the Consumer's Risk. The chance of rejecting a good item is the Producer's risk whereas the chance of accepting a bad item is called the Consumer's risk denoting by γ and β respectively. Now, we propose the HGASP followed by Truncated Life Tests:

Step 1: Find the number of testers 'r' and allocate the 'r' items to each pre-specified 'g' groups. The required sample size for a lot in the Truncated Life Test is n = gr.

Step 2: Specify the acceptance number 'c' for every group and the termination time t_0.

Step 3: Terminate the experiment and reject the lot if more than 'c' failures are found in each group.

If $r = 1$, the proposed HGASP convert to the ordinary Acceptance Sampling Plan and we can say that the proposed HGASP is an extension of the ordinary plans available in the literature. Our concern is to find the number of testers 'r', required for Generalized Pareto Distribution and different values of acceptance number 'c' whereas the number of groups 'g' and the test truncation time t_0 are considered to be pre-specified. For convenience, we will consider that the test termination time as a multiple of the specified value of μ_0 which is written as $t_0 = a\mu_0$. The lot Acceptance Probability for the proposed HGASP is given by

$$L(p) = \left[\sum_{i=0}^{c} \binom{r}{i} p^i (1-p)^{r-i} \right]^g \tag{2.1}$$

where ‘p’ is the probability that a bad item is selected in the experimental time and the probability ‘p’ for the Generalized Pareto Distribution with $\lambda = 0, \alpha = 2$ and $\delta = 2$ is given by

$$p = F(t) = 1 - \left[1 + \left(\frac{a\Gamma\left(\alpha - \frac{1}{\delta}\right)\Gamma\left(1 + \frac{1}{\delta}\right)}{\left(\frac{\mu}{\mu_0}\right)\Gamma(\alpha)} \right)^{\delta} \right]^{-\alpha} \tag{2.2}$$

Here we considered only the case when $\lambda = 0, \alpha = \beta = 2$, so under these restrictions

$$\mu = \beta \frac{\Gamma(3/2)\Gamma(3/2)}{\Gamma(2)} + 0 = \beta(\pi/4)$$

$$\beta = 4(\mu/\pi)$$

$$F(t) = 1 - \left[1 + (t/\beta)^2 \right]^{-2}$$

Now equation (2.2) should be read as,

$$p = F(t_0) = F(a\mu_0) = 1 - \left[1 + \left(\frac{\pi a \mu_0}{4\mu} \right)^2 \right]^{-2}$$

The minimum number of testers can be evaluated by assuming the Consumer’s Risk when the true average life equals the specified life $(\mu = \mu_0)$ through the following inequality (2.3) is satisfied

$$L(p_0) \le \beta^* \tag{2.3}$$

Tables 1 shows for the pre-specified number of groups, test termination time, Consumer’s Risk and acceptance number to find the minimum number of testers.

Table 2 indicates the minimum number of testers for the proposed HGASP for the Shape Parameters $(\alpha,\delta)=2$. The frequently used values of test termination time a=0.7, 0.8, 1.0, 1.2, 1.5, 2.0, number of groups $g=2(1)9$, acceptance number $c=0(1)7$ and Consumer's Risk β^*=0.25, 0.10, 0.05, 0.01 are given in Table 1. The selections of the shape of Parameters $(\alpha,\delta)=2$ are used for the comparison purpose.

On the other hand, Operating Characteristic (OC) involves a system of principles, techniques and their purpose is to construct decision rules to accept or reject the lot through numerically or graphically based on the sample information. The Operating Characteristic (OC) curve indicates the Probability of Acceptance for various levels of submitted lot quality. If the minimum number of testers is found, one can be delighted to obtain the lot Acceptance Probability when the quality of an item is highly good. As discussed earlier, an item is assumed to be bad or poor quality if $\mu<\mu_0$. For $(\alpha,\delta)=2$, the probabilities of acceptance are showed in Table 3 based on Equation 3.4 for given Design Parameters. From Table 3, we observe that OC values decreases as quickly as the mean ratio decreases. For example, when β^*=0.05, g=4, c=2 and a=1.0, the number of testers required is r=3. If the true mean lifetime is twice the specified mean lifetime $(\mu/\mu_0=2)$, the Producer's Risk is approximately γ=1-0.6796=0.3204 and γ=0.0003 when the true value of mean is 8 times the specified value.

The producer can be concerned in enlarging the quality level of an item so that the Acceptance Probability should be higher than the pre-assumed level. When the Producer's Risk is given, the minimum ratio (μ/μ_0) can be found by solving the following inequality (2.4),

$$L(p_1)\geq 1-\gamma \tag{2.4}$$

Table 4 shows the minimum ratio for Generalized Pareto Distribution with $(\alpha,\delta)=2$, at the Producer's Risks of γ=0.05 based on the Designed Parameters given in Table 1 and one may see that the mean ratio decreased as the Test Termination Ratio decreased. For example, when β^*=0.25, g=4, c=2, r=3 and a=0.8 for determining a Producer's Risk γ=0.05 will be increased the true value of mean μ to 2.51 times the specified value of μ_0.

2.1 Example: Suppose that the lifetimes of an item follows the Generalized Pareto Distribution with (α , δ) =2. It is wished to propose a HGASP if the mean life is higher than the specified life, μ_0 =1000 hours based on a testing time of 700 hours and using 4 groups. It is assumed that β^* = 0.05 and c =2, this leads to the test termination time a =0.7. From Table 2, the minimum number of testers is r =4. So, draw a sample of 16 items and allocate 4 items to each of 4 groups. Truncate the Life Test and reject the lot if more than 2 failures are observed in 700 hours in each of 4 groups. The OC values for the proposed HGASP (*g, r, c, a*) = (4, 4, 2, 0.7) are as follows:

μ/μ_0	**2**	**4**	**6**	**8**	**10**	**12**
$\mathbf{P_a}$	0.7683	0.9922	0.9992	0.9999	1.0000	1.0000

This shows that, if the true mean life is 8 times of 1000 hours, the Producer's Risk is 0.0001. So, a lot of submitted items shall be accepted with probability 0.7683 if the true mean life is 2 times the specified mean life. The Accepting Probability of submitted lot is increased up to 0.9999 if the true mean life of an item in a lot is 8 times the specified mean life.

3. Comparative Study

It can be easily observed from Table 5 that proposed HGASP perform better than the existing plan developed by Rao (2011).

We compared the proposed HGASP with the existing HGASP for the Generalized Pareto Distribution with Shape Parameters $\alpha = 2, \delta = 2$. From Table 5, we can see that for specific value of β^* and various values of termination time, the proposed HGASP provides the less number of testers as compared to the existing HGASP. So, the proposed HGASP is better than existing HGASP to reach at the same decision as in existing HGASP with less number of items to be inspected. At the end, in this research article, a HGASP is developed for the Generalized Pareto Distribution based on Truncated Life Test. The minimum number of testers, OC values and the minimum ratio of the true mean life to the specified mean life are found when the other Designed Parameters are pre-assumed.

It is concluded that the proposed HGASP is more economical and beneficial than the existing HGASP in terms of minimum sample size, cost, test truncation time and labor.

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Table 1: Group size (g), test termination time (a), Consumer's Risk (β^*) and acceptance number (c)

β*	g	c	a
0.25	2	0	0.5
0.10	3	1	0.7
0.05	4	2	0.8
0.01	5	3	1.0
	6	4	1.2
	7	5	1.5
	8	6	
	9	7	

Table 2: Number of testers required for the proposed plan for the Generalized Pareto Distribution α=2, δ=2

			a					
β	**g**	**c**	**0.5**	**0.7**	**0.8**	**1.0**	**1.2**	**1.5**
0.25	**2**	**0**	2	1	1	1	1	1
	3	**1**	3	2	2	2	2	2
	4	**2**	5	4	3	3	3	3
	5	**3**	6	5	4	4	4	4
	6	**4**	8	6	6	5	5	5
	7	**5**	10	7	7	6	6	6
	8	**6**	11	8	8	7	7	7
	9	**7**	13	10	10	8	8	8
0.10	**2**	**0**	2	2	1	1	1	1
	3	**1**	4	3	3	2	2	2
	4	**2**	6	4	4	3	3	3
	5	**3**	7	5	5	4	4	4
	6	**4**	9	7	6	5	5	5
	7	**5**	11	8	7	6	6	6
	8	**6**	12	9	8	7	7	7
	9	**7**	14	10	9	9	8	8
0.05	**2**	**0**	3	2	2	1	1	1
	3	**1**	5	3	3	2	2	2
	4	**2**	6	4	4	3	3	3
	5	**3**	8	6	5	4	4	4
	6	**4**	10	7	6	6	5	5
	7	**5**	11	8	7	7	6	6
	8	**6**	13	9	9	8	7	7
	9	**7**	15	11	10	9	8	8
0.01	**2**	**0**	4	3	2	2	2	1
	3	**1**	5	4	3	3	2	2
	4	**2**	7	5	4	4	3	3
	5	**3**	9	6	6	5	4	4
	6	**4**	11	8	7	6	5	5
	7	**5**	13	9	8	7	6	6
	8	**6**	14	10	9	8	7	7
	9	**7**	16	11	10	9	9	8

Table 3: Operating Characteristics values of the group sampling plan for Generalized Pareto Distribution α=2, δ=2, g=4 and c=2

β	**r**	**a**	**2**	**4**	**6**	**8**	**10**	**12**
0.25	**5**	**0.5**	0.8894	0.9971	0.9997	0.9999	1.0000	1.0000
	4	**0.7**	0.7683	0.9922	0.9992	0.9999	1.0000	1.0000
	3	**0.8**	0.8606	0.9956	0.9996	0.9999	1.0000	1.0000
	3	**1.0**	0.6796	0.9853	0.9984	0.9997	0.9999	1.0000
	3	**1.2**	0.4657	0.9627	0.9956	0.9991	0.9998	0.9999
	3	**1.5**	0.2081	0.8942	0.9853	0.9969	0.9991	0.9997
0.10	**6**	**0.5**	0.8104	0.9945	0.9995	0.9999	1.0000	1.0000
	4	**0.7**	0.7683	0.9922	0.9992	0.9999	1.0000	1.0000
	4	**0.8**	0.6258	0.9838	0.9983	0.9997	0.9999	1.0000
	3	**1.0**	0.6796	0.9853	0.9984	0.9997	0.9999	1.0000
	3	**1.2**	0.4657	0.9627	0.9956	0.9991	0.9998	0.9999
	3	**1.5**	0.2081	0.8942	0.9853	0.9969	0.9991	0.9997
0.05	**6**	**0.5**	0.8104	0.9945	0.9995	0.9999	1.0000	1.0000
	4	**0.7**	0.7683	0.9922	0.9992	0.9999	1.0000	1.0000
	4	**0.8**	0.6258	0.9838	0.9983	0.9997	0.9999	1.0000
	3	**1.0**	0.6796	0.9853	0.9984	0.9997	0.9999	1.0000
	3	**1.2**	0.4657	0.9627	0.9956	0.9991	0.9998	0.9999
	3	**1.5**	0.2081	0.8942	0.9853	0.9969	0.9991	0.9997
	7	**0.5**	0.7180	0.9907	0.9991	0.9998	1.0000	1.0000
	5	**0.7**	0.5795	0.9816	0.9981	0.9996	0.9999	1.0000
	4	**0.8**	0.6258	0.9838	0.9983	0.9997	0.9999	1.0000
	4	**1.0**	0.3275	0.9488	0.9940	0.9988	0.9997	0.9999
	3	**1.2**	0.4657	0.9627	0.9956	0.9991	0.9998	0.9999
	3	**1.5**	0.2081	0.8942	0.9853	0.9969	0.9991	0.9997

Table 4: Minimum ratio of true average life to specified life for the Producer's Risk of 0.05, for Generalized Pareto Distribution $\alpha = 2, \delta = 2$

			a					
β	**g**	**c**	**0.5**	**0.7**	**0.8**	**1.0**	**1.2**	**1.5**
0.25	**2**	**0**	7.35	7.26	8.3	10.38	12.46	15.57
	3	**1**	2.92	3.07	3.51	4.4	5.29	6.6
	4	**2**	2.35	2.81	2.51	3.13	3.76	4.69
	5	**3**	1.86	2.25	2.06	2.57	3.08	3.86

			a					
β	**g**	**c**	**0.5**	**0.7**	**0.8**	**1.0**	**1.2**	**1.5**
0.25	**6**	**4**	1.76	1.93	2.21	2.25	2.7	3.37
	7	**5**	1.69	1.73	1.98	2.03	2.44	3.05
	8	**6**	1.53	1.58	1.81	1.88	2.26	2.82
	9	**7**	1.5	1.66	1.89	1.76	2.11	2.64
0.10	**2**	**0**	7.35	10.3	8.3	10.38	12.46	15.57
	3	**1**	3.49	4.08	4.67	4.4	5.29	6.6
	4	**2**	2.64	2.81	3.2	3.13	3.76	4.69
	5	**3**	2.08	2.25	2.57	2.57	3.08	3.86
	6	**4**	1.92	2.22	2.21	2.25	2.7	3.37
	7	**5**	1.81	1.97	1.98	2.03	2.44	3.05
	8	**6**	1.63	1.79	1.81	1.88	2.26	2.82
	9	**7**	1.59	1.66	1.68	2.1	2.11	2.64
0.05	**2**	**0**	9.01	10.3	11.76	10.38	12.46	15.57
	3	**1**	3.96	4.08	4.67	4.4	5.29	6.6
	4	**2**	2.64	2.81	3.2	3.13	3.76	4.69
	5	**3**	2.28	2.61	2.57	2.57	3.08	3.86
	6	**4**	2.07	2.22	2.21	2.76	2.7	3.37
	7	**5**	1.81	1.97	1.98	2.47	2.44	3.05
	8	**6**	1.73	1.79	2.05	2.26	2.26	2.82
	9	**7**	1.67	1.82	1.89	2.1	2.11	2.64
0.01	**2**	**0**	10.4	12.61	11.76	14.71	17.66	15.57
	3	**1**	3.96	4.87	4.67	5.83	5.29	6.6
	4	**2**	2.92	3.3	3.2	4.01	3.76	4.69
	5	**3**	2.46	2.61	2.98	3.21	3.08	3.86
	6	**4**	2.2	2.47	2.54	2.76	2.7	3.37
	7	**5**	2.03	2.18	2.25	2.47	2.44	3.05
	8	**6**	1.83	1.97	2.05	2.26	2.26	2.82
	9	**7**	1.75	1.82	1.89	2.1	2.52	2.64

Table 5: Comparisons of sample size (n) when g=7 and c=5

a	β*	Existing HGASP	Proposed HGASP
0.7	**0.05**	105	56
0.8		91	49
1.0		70	49
1.2		63	42
1.5		56	42

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