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# A Class of Tests for New Better than Used Life Distributions

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## Abstract

A class of test statistics for testing exponentiality against new better than used (NBU) alternatives is considered. The proposed class is based on a sub sample minima. The distributional properties of the class of test statistics are studied. . The properties of the test procedure such as asymptotic normality, consistency are established. The performance of the class of test statistics with respect to the existing tests in the literature is studied in terms of Pitman asymptotic relative efficiency (ARE).

## Keywords

 NBU, Pitman ARE, Exponentiality, U-statistic

## 1.  Introduction

Statistical theory and its applications have played a significant role in advancement of methodology and techniques for solving various problems in reliability and survival analysis. The exponential distribution is a popular model which is useful wherever the 'no aging' phenomenon is evident. 'No aging' means that the probability distribution of the life time of a unit does not change with the knowledge that the unit has already survived for a given time. As against this many units exhibit the 'positive aging' phenomena. The term 'positive aging' is used to denote the situation where the performance of a unit deteriorates with its age. Classes of life distributions based on notion of aging have been introduced

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in the literature. Some of the classes of life distributions based on aging are increasing failure rate (IFR), increasing failure rate average (IFRA), new better than used (NBU). The chain of implication of these notions is given by IFR $\Rightarrow$ IFRA $\Rightarrow$ NBU.

In practical situations it has been noticed that life time of units possess one or the other aging property. Hence it is of interest to the statisticians to propose tests for the null hypothesis of exponentiality against the alternative of positive aging. In this paper, we consider one of the important classes of life distribution, namely, new better than used (NBU) which is defined below.

***Definition 1.1:***  A life distribution with distribution function F is said to be new better than used (NBU) if

$$\overline{F}(x+y) \leq \overline{F}(x)\,\overline{F}(y), \qquad x,y \geq 0$$

The new better than used of distributions is important in replacement policies (see Barlow and Proschan, 1981). Many tests have been proposed for testing exponentiality against the alternative of NBU by researchers. Testing exponentiality against the new better than used alternatives are discussed by Hollander and Proschan (1972), Koul (1977), Kumazawa (1983), Pandit and Anuradha (2007a, b, c), and Ahmad (1994, 2001, 2004) among others. Abdel-Aziz (2007) considered the problem of testing against RNBRUE altermatives and Ahmad (1998) considered the problem of testing against NBU-$t_0$ alternatives. In this paper, we consider a class of test statistics for the problem of testing exponentiality against new better than used class of alternatives.

In section 2, we propose a class of test statistics and have studied their asymptotic distributions. The section 3 is devoted to asymptotic relative efficiency comparisons. In section 4, we present the remarks and conclusions.

## 2. Proposed Test Statistics

Let $X_1$, $X_2$, …, $X_n$ be a random sample from a distribution with distribution function F. Here, we wish to consider the problem of testing

$$H_0 : \overline{F}(x) = e^{-\lambda x}, \lambda > 0, x > 0 \text{ Versus } \quad H_1 : \overline{F}(x+y) \leq \overline{F}(x)\overline{F}(y), \qquad x,y > 0$$

Since the property NBU implies $\overline{F}(mx) \leq \{\overline{F}(x)\}^m$, for all $x \geq 0$ and for every integer $m \geq 1$, we consider a measure of discrepancy between $H_0$ and $H_1$ which is defined as , for c>0

$$\gamma(F) = \frac{1}{cm+1} - \int_0^\infty \overline{F}^c(mx)dF(x).$$

Here it is to be noted that $\gamma(F) > 0$ when the distribution function F belongs to NBU class.

Based on a random sample of size n, we propose a test statistic which is an unbiased estimate of $\gamma(F)$ as

$$T(c,m) = \frac{1}{cm+1} - \frac{1}{\binom{n}{c+1}} \sum_A \psi(x_1, x_2, \ldots, x_c),$$

where c is a positive integer, $\psi(.)$ is the symmetrized version of the kernel

$$\psi^*(x_1, x_2, \ldots, x_c) = \begin{cases} 1 & \text{if } \min(x_1, x_2, \ldots, x_c) > mx_{c+1} \\ 0 & \text{otherwise} \end{cases}$$

and A is the set of $\{i_1, i_2, \ldots, i_{c+1}\}$ integers taken without replacement from a set of integers $\{1, 2, \ldots, n\}$. It can be seen that under $H_0$, $E[T(c,m)] = 0$ and under $H_1$, $E[T(c,m)] = \gamma(F) > 0$. The test criterion is to reject $H_0$ for large values of T(c,m).

*Asymptotic Normality and Consistency:*

The statistic $T(c,m)$ is the U-statistic corresponding to the kernel $\psi(.)$. Using the results of Hoeffding (1948), the asymptotic distribution of $\sqrt{n}[T(c,m) - \gamma(F)]$ is normal with mean zero and variance $(c+1)^2 \varsigma_1$, where $\varsigma_1 = E[\psi_1^2(X_1)] - [\gamma(F)]^2$

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and

$$\psi_1(x_1) = E\{\psi(x, X_2,...,X_{c+1})\}$$

$$= \frac{1}{c+1}\{cP(Min(x, X_2,...,X_c) > mX_{c+1}) + P(Min(X_2,...,X_{c+1}) > mx)\}$$

$$= \frac{1}{c+1}\{c.A + B\}.$$

Here,

$$A = P(Min(x, X_2,...,X_c) > mX_{c+1})$$

$$= P(x > Min(X_2,...,X_c) > mX_{c+1}) + P(Min(X_2,...,X_c) > x > mX_{c+1})$$

$$= \int_0^x \left[\overline{F}^{c-1}(my) - \overline{F}^{c-1}(x)\right]dF(y) + \overline{F}^{c-2}(x).F\left(\frac{x}{m}\right)$$

and

$$B = P(Min(X_2,...,X_{c+1}) > mx)$$

$$= \overline{F}^c(mx).$$

Under the null hypothesis,

$$(c+1)^2\varsigma_1 = \frac{1}{2mc+1} + \frac{2c}{(m(c-1)+1)(mc+1)} - \frac{2cm}{(m(c-1)+1)(mc(m+1)+1)} - \frac{(c+1)^2}{(mc+1)^2}$$

$$+ \frac{c^2}{(m(c-1)+1)^2}\left[\frac{m}{m(2c-1)+2} - \frac{2m}{mc+1} + c^2\right]$$

Since $\gamma(F) > 0$ the class of test statistics is consistent against new better than used alternatives by Lehmann (1951).

## 3.  Asymptotic Relative Efficiency

For asymptotic relative efficiency comparisons, we have considered three parametric families of distributions, namely, Weibull, Makeham and Gamma distributions.  These depend upon a real parameter $\theta$ in such a way that $\theta = \theta_0$

yields a distribution belonging to null hypothesis whereas $\theta > \theta_0$ yields distribution from the alternative. These are

(i)        Weibull Distribution

$$\overline{F}_\theta(x) = \exp(-x^\theta), \quad x > 0, \quad \theta \geq 1, \quad \theta_0 = 1$$

(ii)        Makeham Distirbution

$$\overline{F}_\theta(x) = \exp[-x + \theta(x + e^{-x} - 1)], \quad x > 0, \quad \theta \geq 0, \quad \theta_0 = 0$$

(iii)        Gamma Distribution

$$\overline{F}_\theta(x) = \int_0^x \frac{1}{\Gamma(\theta)} u^{\theta-1} e^{-u} du, \quad x > 0, \quad \theta \geq 0, \quad \theta_0 = 1$$

The Pitman ARE's of $T(c,m)$ for (m=2(1)5 and c = 2) with respect to Kumazawa (1983) test are given in Table 1.

**Table 1**

| m   ➝ | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| Weibull | 0.6526 | 0.6104 | 0.5705 | 0.5392 |
| Makeham | 6.9723 | 1.9591 | 0.9963 | 0.7058 |
| Gamma | 1.2203 | 1.2067 | 1.1917 | 1.1762 |

## 4. Some Remarks

1. The asymptotic relative efficiencies (AREs) of the proposed test with respect to the test due to Kumazawa (1983) is computed for the alternatives Weibull Makeham and Gamma distributions. The AREs of the proposed test with respect to the member of Kumazawa (1983) class of tests with maximum efficacy are computed.

2. It is observed that the proposed test performs better for the alternatives Makeham and Gamma distributions.

3. It is observed that the proposed test performs better than the test due to Hollander and Proschan (1972) when the underlying distribution is Makeham or Gamma.

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4. Based on the ARE values, the optimum value of m to use the proposed two sample test is 2.

5. Hence, if the data under consideration is exactly NBU, the new test proposed would be a better choice.

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