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Adaptive Estimation of Heteroscedastic Linear Regression Models Using Heteroscedasticity Consistent Covariance Matrix

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Abstract

For the estimation of linear regression models in the presence of heteroscedasticity of unknown form, method of ordinary least squares does not provide the estimates with the smallest variances. In this situation, adaptive estimators are used, namely, nonparametric kernel estimator and nearest neighbour regression estimator. But these estimators rely on substantially restrictive conditions. In order to have accurate inferences in the presence of heteroscedasticity of unknown form, it is a usual practice to use heteroscedasticity consistent covariance matrix (HCCME). Following the idea behind the construction of HCCME, we formulate a new estimator. The Monte Carlo results show the encouraging performance of the proposed estimator in the sense of efficiency while comparing it with the available adaptive estimators especially in small samples that makes it more attractive in practical situations.

Keywords

Adaptive estimator, Heteroscedasticity, Heteroscedasticity consistent covariance matrix estimator, Nearest neighbour regression estimator, Nonparametric kernel estimator

1. Introduction

The usual regression model assumes that the expected value of all error terms, when squared, is the same at any given point. This assumption is called homoscedasticity. If this assumption is not met, there is an indication of the existence of heteroscedasticity. The most common framework in which

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heteroscedasticity is studied in econometrics is in the context of the linear regression models. Errors of measurements, sampling strategies, model misspecifications, and presence of outliers etc. are the main causes to introduce heteroscedasticity (see Griffiths, 1999; Gujarati, 2003 for more details).

In the presence of heteroscedasticity, ordinary least squares (OLS) estimators are although unbiased and consistent but no longer efficient. In addition, the standard errors of the estimates become biased and inconsistent. This, in turn, leads to incorrect inferences. Depending on the nature of the heteroscedasticity, significance tests can be too high or too low. These effects are not ignorable as earlier noted by Geary (1966), White (1980) and Pasha (1982), among many others.

When the form of heteroscedasticity is known, using weights to correct for heteroscedasticity is very simple by using method of generalized least squares. If the form of heteroscedasticity is not known, the variance of each residual can be estimated first and these estimates can be used as weights in a second step and the resultant estimates are called the estimated weighted least squares (EWLS) estimates (see for example, Fuller and Rao, 1978; Carroll and Ruppert, 1988; Pasha and Ord, 1994; Greene, 2000). But in usual practice, the form of heteroscedasticity is seldom known that makes the weighting approach impractical. In such situations, we are required to formulate such estimators which give as adequate results as if we have known heteroscedastic errors and hence adaptive estimation procedures come in to fulfill such objectives.

Specifically, in the sense of Bickel (1982), for a linear regression model in the presence of heteroscedasticity, an estimator is said to be adaptive estimator if it has the same asymptotic distribution as that of an estimator having information about the form of heteroscedasticity. In the available literature, two adaptive estimators are popular, namely, nonparametric kernel estimator proposed by Carroll (1982) and nearest neighbour regression (NNR) given by Robinson (1987). Both of these adaptive estimators require substantially restrictive conditions for their applications.

White (1980) introduces a heteroscedasticity consistent covariance estimator (HCCME) to draw correct inferences in the presence of heteroscedasticity of unknown form, popularly known as HC0. Then in order to improve small sample properties, MacKinnon and White (1985) present other versions of HCCME, known as HC1, HC2, and HC3. These estimators are frequently used by the practitioners (e.g., see Long and Ervin, 2000; Flachaire, 2002). Since all available literature about HCCME limits up to the consistent estimation of Var-Cov matrix of the estimated regression coefficients and no progress yet has been made to construct weights based on this HCCM, this leads us to formulate weights based on HCCME and then to conduct EWLS. We present the performance of our formulation by means of Monte Carlo study.

Our study unfolds in such a way that section 2 presents the heteroscedastic linear regression model along with HCCME. In section 3, we formulate a new estimator based on HCCME. In section 4, we present empirical results based on Monte Carlo simulations. Section 5 is reserved for application of our approach and, finally, section 6 gives conclusion.

2. Heteroscedastic Linear Regression Model and HCCME

Consider a heteroscedastic linear regression model of the form

$$y = X\beta + u, \tag{2.1}$$

where y is an n x 1 vector of observations, X is an n x p ($p < n$) nonrandom matrix of covariates, β is a p x 1 vector of unknown regression parameters, and u is an n x 1 vector of random errors with zero mean and variance σ_i^2.

When the errors are homoscedastic, i.e., $\sigma_i^2 = \sigma^2$, under the other standard assumptions, the coefficients β can consistently be estimated by the OLS as follows:

$$\hat{\beta}_{OLS} = (X'X)^{-1}X'y$$

with the covariance matrix, denoted by Ψ,

$$\Psi = (X'X)^{-1}X'\Omega X(X'X)^{-1} \tag{2.2}$$

The estimation of Ψ depends on the assumptions about $\Omega = diag\{\sigma_1^2, \sigma_2^2, \cdots, \sigma_n^2\}$. In the presence of heteroscedasticity of unknown form, the OLS estimate becomes inefficient and its covariance matrix estimate inconsistent as discussed earlier. In this situation, White (1980) derives an asymptotically justified form of the

HCCME known as HC0 by letting $\hat{\Omega}$ = diag[$\hat{u}_i^2$], which results in estimation of (2.2) as $\hat{\Psi} = (X'X)^{-1} X'\hat{\Omega} X (X'X)^{-1} = (X'X)^{-1} X' diag[\hat{u}_i^2] X (X'X)^{-1}$.

As shown by White (1980) and others, HC0 is a consistent estimator of Ψ under both homoscedasticity and heteroscedasticity of an unknown form. However, it can be considerably biased in the finite samples; see, e.g., MacKinnon and White (1985); Cribari-Neto and Zarkos (1999, 2001). MacKinnon and White (1985) raise their concerns about the performance of HC0 in small samples and present three alternative estimators to improve the small sample properties of HC0. The simplest adjustment, suggested by Hinkley (1977), makes a degree of freedom correction that inflates each residual by the factor $\sqrt{n/(n-k)}$. With this correction, Mackinnon and White (1985) obtain the version of HCCME known as HC1:

$$HC1 = \frac{n}{n-k}(X'X)^{-1} X' diag[\hat{u}_i^2] X (X'X)^{-1} = \frac{n}{n-k} HC0$$

Based on work by Horn et al. (1975), MacKinnon and White (1985) propose:

$$HC2 = (X'X)^{-1} X' diag[\frac{\hat{u}_i^2}{1-h_i}] X (X'X)^{-1},$$

where $h_i = x_i (X'X)^{-1} x_i'$.

Following Efron’s (1979) jackknife estimator, MacKinnon and White (1985) give a third variant of HCCME as:

$$HC3 = (X'X)^{-1} X' diag[\frac{\hat{u}_i^2}{(1-h_i)^2}] X (X'X)^{-1}.$$

Long and Ervin (2000) explore the small sample properties of test using these four versions of HCCME in linear regression model. Their Monte Carlo simulation shows that HC0 often results in incorrect inferences when n ≤ 250, while the other three versions of HCCME, especially HC3, works well even when *n* is as small as 25.

3. HCCME-Based Adaptive Estimation

Following Horn et al. (1975), MacKinnon and White (1985) conclude that *Var* $(\hat{u}_i)$ underestimates σ_i^2 (since $1/n \leq h_i \leq 1$). By showing, $\hat{u}_i^2$ to be a biased estimator of σ_i^2, they suggest $\frac{\hat{u}_i^2}{1-h_i}$ to be less biased. Furthermore, Wu (1986) also estimates σ_i^2 by $\hat{\sigma}_i^2 = |\hat{u}_i| / \sqrt{(1-h_i)}$,
where $h_i = x_i (X'X)^{-1} x_i'$, and $\hat{u}_i = y_i - x'_i \hat{\beta}_{OLS}$.

Following these ideas, we estimate Ω and weights (wi) on the basis of $\hat{u}_i^2$ as
$\hat{\sigma}_i^2 = \frac{\hat{u}_i^2}{1-h_i}$, and $w_i = \hat{\sigma}_i^{-2}$.
The estimation of Ω is the same as used in HC2.

Then we propose the following HCCME-Based weighted least square (HWLS) adaptive estimator as

$$\hat{\beta}_{HWLS} = \left(\sum_i X_i X'_i w_i \right)^{-1} \sum_i X_i y_i w_i \tag{3.1}$$

Following the formulation of HC3, we also present another version of above adaptive estimator by setting

$$\hat{\sigma}_i^2 = \frac{\hat{u}_i^2}{(1-h_i)^2}$$

For convenience, we name both the estimators as HC2WLS-estimator (based on HC2) and HC3WLS-estimator (based on HC3), respectively.

4. Empirical Results

To evaluate the performance of the proposed estimators, we calculate bias, mean square error (MSE) and relative efficiency (R.E). For this purpose, we conduct a Monte Carlo study and use the same scheme as carried out by Carroll (1982) so that our proposed estimator can directly be compared with already existing estimators. The heteroscedastic linear regression model, under study, is:

$$y_i = \beta_0 + \beta_1 x_i + \sigma_i \varepsilon_i, \qquad i = 1, 2, \ldots, n, \tag{4.1}$$

where, $\beta_0 = 50$, $\beta_1 = 60$, ε_i are i.i.d. standard normal variables and x_i are i.i.d. uniform on the interval (-0.5, 0.5). Furthermore, it is assumed that ε_i's are observed independent to x_i. Three different variance models (VMs) are considered to generate σ_i.

The first variance model (VM-I), given by Jobson and Fuller (1980), is

$$\sigma_i^2 = a_1 + a_2 \tau_i^2, \qquad \tau_i^2 = \beta_0 + \beta_1 x_i$$

We choose $a_1 = 100$ and $a_2 = 0.25$.

The second variance model (VM-II), a model with more severe heteroscedasticity, used by Box and Hill (1974), is

$$\sigma_i = a_1 \exp(a_2 |\tau_i|),$$

where, $a_1 = 0.25$ and $a_2 = 0.04$.

The third model (VM-III), a model of severe heteroscedasticity, used by Bickel (1978), is

$$\sigma_i = a_1 \exp(a_2 \tau_i^2),$$

where, $a_1 = 1/4$ and $a_2 = 1/3200$.

To evaluate the performance of the proposed adaptive estimators with the change in sample size, we use different sample sizes by taking n = 10, 25, 50, 100, and 250. For each variance model and for each sample size, we run 1,000 simulations as done by Carroll (1982).

To estimate the model (4.1), we use the following estimators to highlight their comparative performances under different data generating schemes:

1. Ordinary least square (OLS) estimator
2. Adaptive kernel estimators:
   Following Carroll (1982), we use adaptive kernel estimator. Since it is well known in the nonparametric literature that the choice of the kernel is not crucial as long as it satisfies certain regularity conditions (Roy, 1999), so unlike Carroll (1982), we use normal kernel. We refer this estimator by KWLS estimator for the next discussion.
3. Nearest neighbor regression estimator:
   Following Robinson (1987), we use nearest neighbor regression estimator. We choose the number of nearly neighborhood, $K = n^{4/5}$ (following Härdle, 1994). We refer this estimator by NWLS estimator.
4. HCCME-based adaptive estimator:
   We use our proposed adaptive estimator (3.1). We use both, above discussed, versions of this estimator and refer them as HC2WLS and HC3WLS estimators, respectively.

For each VM and for each sample size, we calculate biases and mean square errors (MSEs) of the estimated coefficients obtained by 1,000 simulations in each case. We compute relative efficiency to compare the results as follows:

$$\text{R.E} = \text{MSE (OLS)/MSE (Estimator under consideration)}$$

We carry out all the computational work by using the software, E-Views 3.0 for the said simulations. Table 1 shows the empirical results of the estimators about bias and MSE and R.E when the sample size, n = 10. These results show that with small sample and mild heteroscedasticity (VM-I), OLS performance is not so bad in estimating both of the coefficients. When estimating β_0, OLS and H3WLS are almost equally efficient showing relatively minimum MSEs while comparing with rest of the estimators. Then KWLS and H2WLS show equivalent efficient behaviour while NWLS remains the worst so far.

On the other hand, when estimating β_1, NWLS again remains least efficient while the gain in efficiency of our proposed HC3WLS becomes about 11% as compared to that of OLS and 10% as compared to that of the other nonparametric estimator KWLS.

Although there is no question of bias in OLS under heteroscedasticity, but in order to compare this end with respect to our proposed estimators, we present the comparisons. In case of a small sample of size 10, HC2WLS remains a little bit more biased while computing the intercept and similar is the case for KWLS while estimating β_1, as compared to all other estimators.

Under VM-II, OLS performs quite badly as expected. Our proposed estimators outperform the two nonparametric estimators, NWLS and KWLS. The proposed estimators become 3 to 6 times more efficient as compared to the OLS. Here, we also see that the performances of NWLS and KWLS are also similar to our estimators to some extent. We note that HC3WLS performs better than HC2WLS while NWLS performs better than KWLS. As far as biasedness is concerned, OLS gives least differences. Under VM-III, the bad performance of OLS continues but the other estimators do not remain as much efficient as they are under VM-II. In this case (under VM-III), HC3WLS again holds the leading role while all the other estimators perform in quite similar manner in the terms of relative efficiency.

Table 2 (for n = 25) shows almost the same picture as Table 1 does under VM-I. However, under VM-II, the bad performance of OLS declines with increase in the sample size from 10 to 25 relative to the figures given in Table 1. The performance of our proposed estimators is quite encouraging for the variance models, VM-II and VM-III. It is also noted that HC3WLS and HC2WLS remain equally efficient under VM-II and VM-III.

Through Table 3, we confirm the Carroll's (1982) findings in terms of relative efficiency of the estimators. Carroll (1982) reports the gain in efficiency using the KWLS to be 19%, 188% and 125%, respectively, for VM-I, VM-II and VM-III while estimating β_1. We find almost the same figures for gain in efficiency using the KWLS and they are 15%, 177% and 108% for VM-I, VM-II and VM-III, respectively. We also find similar results while estimating β_0. We report the gain in efficiency to be 30% for HC2WLS and 32% for HC3WLS while comparing with OLS and this gain is almost double than the gain for the other two adaptive estimators while estimating β_1. Our both proposed estimators show excellent performance for severe heteroscedasticity (VM-II) where the gain reaches to about 261% (see HC3WLS for β_1). The situation for VM-III, is also quite encouraging. Moreover, the bias reduces expectedly, with the increase in sample size for all the cases.

Similar findings are obtained for n = 100 and 250 (Tables 4 and 5).). Our proposed estimators perform adequately well in all the cases. For larger samples, the nonparametric estimators and our proposed estimators become quite identical in performance.

Table 1: Empirical Results of the Estimators about MSE and Bias (n = 10)

Estimators	$\hat{\beta}_0$ (True value = 50)			$\hat{\beta}_1$ (True value = 60)		
	Bias	MSE*	R.E	Bias	MSE*	R.E
VM-I						
OLS	0.114	7.74		-0.059	119.19	
KWLS	0.148	8.68	0.89	0.125	117.72	1.01
NWLS	0.156	9.02	0.86	0.096	122.47	0.97
HC2WLS	0.181	8.77	0.88	0.064	117.31	1.02
HC3WLS	0.113	7.72	1.00	-0.072	107.02	1.11
VM-II						
OLS	0.002	19.72		0.007	281.51	
KWLS	0.013	6.34	3.11	0.039	47.81	5.89
NWLS	0.013	6.24	3.16	0.040	46.73	6.02
HC2WLS	0.011	6.51	3.03	0.042	46.88	6.00
HC3WLS	0.008	5.96	3.31	0.032	44.81	6.28
VM-III						
OLS	0.002	7.83		0.001	85.54	
KWLS	0.002	6.46	1.21	0.001	42.96	1.99
NWLS	0.002	6.43	1.22	0.001	43.12	1.98
HC2WLS	0.001	6.70	1.17	0.000	43.11	1.98
HC3WLS	0.001	5.89	1.33	-0.002	37.89	2.26

*The actual MSEs for VM-II are the figures presented in the Table divided by 10^2 while by 10^3 for VM-III

Table 2: Empirical Results of the Estimators about MSE and Bias (n = 25)

Estimators	$\hat{\beta}_0$ (True value = 50)			$\hat{\beta}_1$ (True value = 60)		
	Bias	MSE*	R.E	Bias	MSE*	R.E
VM-I						
OLS	0.014	4.50		-0.048	35.82	
KWLS	-0.014	4.83	0.93	-0.040	34.79	1.03
NWLS	-0.013	4.83	0.93	-0.037	34.73	1.03
HC2WLS	0.012	4.44	1.01	0.003	32.19	1.11
HC3WLS	0.023	4.35	1.03	-0.007	31.20	1.15
VM-II						
OLS	-0.002	4.42		-0.030	64.02	
KWLS	0.007	2.90	1.53	0.008	22.75	2.81
NWLS	0.007	2.88	1.54	0.008	22.56	2.84
HC2WLS	0.006	2.59	1.71	0.012	19.10	3.35
HC3WLS	0.004	2.58	1.71	0.005	19.33	3.31
VM-III						
OLS	0.000	2.50		-0.002	26.29	
KWLS	0.000	2.25	1.11	0.000	19.96	1.32
NWLS	0.000	2.25	1.11	0.000	19.98	1.32
HC2WLS	0.001	2.07	1.21	0.002	17.86	1.47
HC3WLS	0.001	2.04	1.22	0.002	17.76	1.48

* The actual MSEs for VM-II are the figures presented in the Table divided by 10^2 while by 10^3 for VM-III.

Table 3: Empirical Results of the Estimators about MSE and Bias (n = 50)

Estimators	$\hat{\beta}_0$ (True value = 50)			$\hat{\beta}_1$ (True value = 60)		
	Bias	MSE*	R.E	Bias	MSE*	R.E
VM-I						
OLS	0.046	17.55		0.055	191.05	
KWLS	0.026	20.04	0.88	0.134	166.68	1.15
NWLS	0.026	20.04	0.88	0.133	166.58	1.15
HC2WLS	0.043	17.63	1.00	0.065	147.50	1.30
HC3WLS	0.044	17.30	1.01	0.068	145.19	1.32
VM-II						
OLS	0.002	12.95		0.034	355.69	
KWLS	0.001	11.42	1.13	0.009	128.24	2.77
NWLS	0.001	11.39	1.14	0.009	127.79	2.78
HC2WLS	0.000	9.64	1.34	0.004	104.34	3.41
HC3WLS	-0.001	9.44	1.37	0.000	98.65	3.61
VM-III						
OLS	-0.001	12.76		-0.003	260.27	
KWLS	-0.001	10.61	1.20	-0.002	124.92	2.08
NWLS	-0.001	10.61	1.20	-0.002	125.12	2.08
HC2WLS	-0.001	9.54	1.34	-0.002	108.61	2.40
HC3WLS	-0.001	9.68	1.32	-0.002	108.77	2.39

* The actual MSEs for VM-I are the figures presented in the Table divided by 10, by 10^3 for VM-II while by 10^4 for VM-III

Table 4: Empirical Results of the Estimators about MSE and Bias (n = 100)

Estimators	$\hat{\beta}_0$ (True value = 50)			$\hat{\beta}_1$ (True value = 60)		
	Bias	MSE*	R.E	Bias	MSE*	R.E
VM-I						
OLS	0.004	7.61		0.104	114.71	
KWLS	0.015	8.95	0.85	0.151	109.26	1.05
NWLS	0.015	8.95	0.85	0.151	109.26	1.05
HC2WLS	0.006	7.60	1.00	0.117	92.69	1.24
HC3WLS	0.007	7.62	1.00	0.127	92.62	1.24
VM-II						
OLS	-0.005	7.47		-0.019	134.92	
KWLS	-0.006	6.62	1.13	-0.022	57.57	2.34
NWLS	-0.006	6.62	1.13	-0.022	57.52	2.35
HC2WLS	-0.006	5.86	1.28	-0.022	48.45	2.78
HC3WLS	-0.006	5.85	1.28	-0.022	48.02	2.81
VM-III						
OLS	-0.001	5.62		-0.004	128.15	
KWLS	-0.001	5.32	1.06	-0.002	63.64	2.01
NWLS	-0.001	5.31	1.06	-0.002	63.58	2.02
HC2WLS	-0.001	4.63	1.21	-0.002	54.33	2.36
HC3WLS	-0.001	4.60	1.22	-0.002	53.57	2.39

* The actual MSEs for VM-I are the figures presented in the Table divided by 10, by 10^3 for VM-II while by 10^4 for VM-III

Table 5: Empirical Results of the Estimators about MSE and Bias (n = 250)

Estimators	$\hat{\beta}_0$ (True value = 50)			$\hat{\beta}_1$ (True value = 60)		
	Bias	MSE*	R.E	Bias	MSE*	R.E
VM-I						
OLS	0.017	3.33		-0.062	44.49	
KWLS	0.012	3.81	0.87	-0.086	39.69	1.12
NWLS	0.012	3.81	0.87	-0.086	39.70	1.12
HC2WLS	0.017	3.30	1.01	-0.072	34.55	1.29
HC3WLS	0.018	3.31	1.01	-0.066	34.52	1.29
VM-II						
OLS	-0.001	3.10		-0.003	55.07	
KWLS	-0.002	2.45	1.27	-0.007	20.22	2.72
NWLS	-0.002	2.45	1.27	-0.007	20.21	2.73
HC2WLS	-0.001	2.11	1.47	-0.005	17.12	3.22
HC3WLS	-0.001	2.06	1.51	-0.005	16.37	3.37
VM-III						
OLS	-0.001	2.63		-0.001	47.93	
KWLS	-0.001	2.38	1.10	-0.001	23.83	2.01
NWLS	-0.001	2.38	1.10	-0.001	23.82	2.01
HC2WLS	-0.001	2.02	1.30	-0.001	19.49	2.46
HC3WLS	-0.001	2.01	1.31	-0.002	19.29	2.49

* The actual MSEs for VM-I are the figures presented in the Table divided by 10, by 10^3 for VM-II while by 10^4 for VM-III.

5. Applications

To illustrate the performance of our proposed adaptive estimators and to compare them with the estimators already available in the literature, we take the example of compensation per employee ($) in Nondurable Manufacturing Industries of US Department of Commerce as quoted by Gujarati (2003, pp. 392). We take this example just to compare our findings for practical data with the findings already available in the literature.

In Table 6, we report the performance of all the estimators discussed above. First of all, we apply OLS to the data and find it to be heteroscedastic by White's Test of Heteroscedasticity (1980) with p-value 0.07. Then we report the estimates, their standard errors and respective values of *t*-statistic. We note that our proposed estimators bear lower standard errors among all the remaining estimators presenting an adequate reliability for their adaptation. We also compute HCCME for giving the correct standard errors for the estimates in the presence of heteroscedasticity. We note that our estimators give better R^2 and much improved standard errors of regression confirming the adequacy of the fitted model and this finding can be viewed in Fig. 1 about fitted values and residuals. Similarly, the proposed adaptive estimators give lowest Akaike Information Criteria (AIC) values that indicate the right specification of the weighting mechanism by using our proposed estimators.

Table 6: Comparative Statistics

Estimators	Estimation of β_0			Estimation of β_1			$R2$	S.E. of Regression	AIC
	$\hat{\beta}_0$	SE	*t*-statistic	$\hat{\beta}_1$	SE	*t*-statistic			
OLS	3417.70	81.04	42.17	148.81	14.40	10.33	0.9385	111.56	12.46
HCCME	----	106.94	31.96	----	16.85	8.83	----	----	----
EWLS	3406.20	80.86	42.13	154.24	16.93	9.11	0.9645	126.54	12.71
KWLS	3444.73	89.84	38.34	142.14	14.27	9.96	0.9962	93.61	12.11
NWLS	3453.14	63.42	54.45	139.32	11.94	11.66	0.9998	60.52	11.24
HC2WLS	3468.10	62.77	55.25	136.24	11.93	11.42	0.9999	56.61	11.10
HC3WLS	3482.89	62.01	56.17	133.21	11.88	11.21	0.9999	52.87	10.97

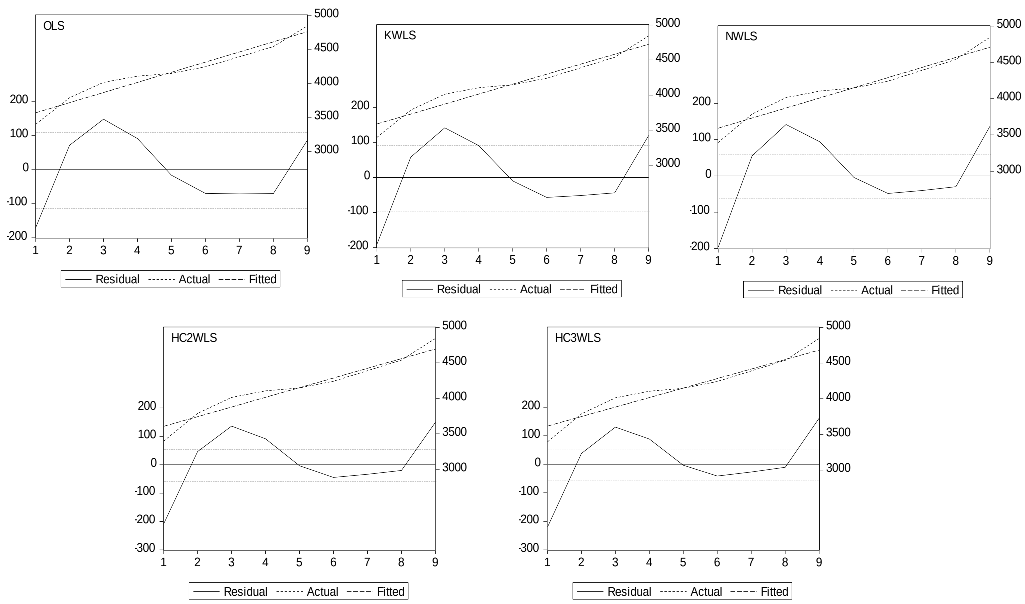

Fig. 1: Fitted Values and Residuals

6. Conclusion

In most of the practical situations, we seldom know anything about the form of heteroscedasticity so it becomes a question to have correct analysis of regression problems in the presence of heteroscedasticity. The available literature on this issue leads us to use adaptive estimators that give desirable results that are not possible when just OLS estimation is taken into account. Usually, nonparametric approaches are used for adaptive estimation. In these available approaches, nearest neighbour regression estimators give better results as compared to adaptive kernel estimators, in the presence of heteroscedasticity of unknown form. But we propose new estimators that are based on the idea as used in the formulation of HCCME. We report that our proposed HCCM-based adaptive estimators outperform in all the situations, from small to large samples and from mild to severe heteroscedasticity. These estimators become 3 to 6 times more efficient as compared to OLS and about one and half times more efficient as compared to both already available adaptive estimators. Specifically, our formulation becomes more attractive in small samples.

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