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Economic Reliability Acceptance Sampling Plan for Generalized Rayleigh Distribution

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Abstract

In this paper, a reliability acceptance plan is developed assuming that the lifetime of a product follows the generalized Rayleigh distribution with known value of the shape parameter. We obtain test termination ratio by considering the producer's risk for given values of the sample size and the acceptance number. A comparison of proposed sampling plan has been made with an existing acceptance sampling plan. Two examples are given to illustrate the procedure.

Keywords

Acceptance sampling, Reliability sampling plan, Life test, Generalized Rayleigh distribution, Sample size

1. Introduction

The Rayleigh distribution was derived by Rayleigh (1880) to handle problems in the field of acoustics. The Rayleigh distribution has many applications in life testing of electro-vacuum devices (Polovko, 1968) and in communication engineering (Dyer and Whisenand, 1973). Tsai and Wu (2006) developed an acceptance sampling plan assuming that the life time of a product has a generalized Rayleigh distribution. They found the minimum sample size and the minimum ratio of true average life to specified average life μ/μ_0 . The cumulative distribution function (cdf) of the Rayleigh distribution is:

$$F(t;b) = 1 - \exp(-t^2/(2b^2)),\ t > 0, \tag{1.1}$$

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where b>0 is the scale parameter. The failure rate is an increasing linear function of time, which makes it suitable for modeling the lifetime of electronic components. Voda (1976) derived a generalized version of the Rayleigh distribution called the generalized Rayleigh distribution (GRD), whose cdf is given by:

$$F_k(t;\lambda) = 1 - \sum_{j=0}^{k} \frac{(t^2/\lambda)^j e^{-t^2/\lambda}}{j!}, \quad (1.2)$$

where k is a positive integer called the shape parameter and λ>0 is the scale parameter. When k=0 and $\lambda = 2b^2$, the cdf given in (1.2) reduces to (1.1). The *ith* moment of the random variable T having GRD is:

$$E[T^i] = \frac{(k+i/2+1)^{1/2}}{(k+1)^{1/2}} \lambda^{i/2}, i = 1, 2, ... \quad (1.3)$$

So the mean of GRD is given by:

$$\mu = E[T] = m\lambda^{1/2} \quad (1.4)$$

where $m = \Gamma(k+3/2)/\Gamma(k+1)$.

The quality of the product is tested on the basis of few items taken from an infinite lot. The statistical test can be stated as: Let μ be the true average life and μ_0 be the specified average life of a product. Based on the failure data, we want to test the hypotheses $H_0 : \mu \geq \mu_0$ against $H_1 : \mu < \mu_0$. A lot is considered as good if $\mu \geq \mu_0$ and bad if $\mu < \mu_0$. This hypothesis is tested using the acceptance sampling scheme as: In a life test experiment, a sample of size n selected from a lot of products is put on the test. The experiment is terminated at a pre-assigned time t_0. When we set acceptance number as c, H_0 is rejected if more than c failures are recorded before time t_0. If there are c or fewer failures before t_0, then H_0 is accepted. Producer's risk and the consumer's risk are associated with acceptance sampling. The probability of rejecting a good lot is called the producer's risk (say α) and the probability of accepting a bad lot is known as consumer's risk, say β. A well acceptance sampling plan minimizes both the risks.

Truncated life tests of this type have been developed by Sobel and Tischendrof (1959), Goode and Kao (1961) for Weibull distribution, Kantam and Rosaiah (1998) for half logistic distribution, Kantam et al. (2001) for log-logistic distribution, Baklizi (2003) for the Pareto distribution of the second kind, Rosaiah and Kantam (2005) for inverse Rayleigh distribution, Rosaiah et al. (2006) for exponentiated log-logistic distribution, Rosaiah et al. (2007) developed the reliability plans for exponentiated log-logistic distribution, Aslam (2007) for the Rayleigh distribution, Balakrishnan et al. (2007) for the generalized Birnbaum-Saunders distribution, Aslam and Shahbaz (2007) for the generalized exponential distribution, Rosaiah et al. (2008) for the inverse Rayleigh, and Aslam and Kantam (2008) for the Birnbaum-Saunders distribution. We propose a reliability acceptance sampling plan when the lifetime of a product follows the generalized Rayleigh distribution. Further, it is assumed that the shape parameter of this distribution is known. The rest of the paper is organized as: design of the proposed plan is given in Section 2, the comparative study with existing plan is given in Section 3. Some Tables are given at the end of the paper.

2. Design of Proposed Plan

We are interested in designing a sampling plan which ensures that the true average life is greater than the specified average life. We propose the following acceptance sampling plan based on truncated life test:

i. Select $n = d \times r \ \ (r = c+1,\ d = 2,3,\ldots)$ items and put them on test.
ii. Select the acceptance number c and termination time t_0.
iii. Terminate the experiment if more than c failures are recorded before termination time and reject the lot. Accept the lot if c or fewer failures occur before termination time.

Suppose that the lifetime of a product follows the GRD with cdf given in (1.2). It would be convenient to determine the termination time t_0 as a multiple of the specified average life μ_0. We assume that the lot size is large enough so that the binomial theory can be applied. Thus the acceptance or rejection criterion of the lot is equivalent to the acceptance or rejection of the hypothesis that $\mu \geq \mu_0$. For more justification about the use of the binomial distribution in proposed plan, one may refer to Stephens (2001). The lot acceptance probability is given as:

$$L(p)=\sum_{i=0}^{c}\binom{n}{i}p^{i}(1-p)^{n-i}\,, \tag{2.1}$$

where p is the probability that an item fails before termination time and is given by:

$$p=1-\sum_{j=0}^{k}\left(\frac{\left(\dfrac{am}{\mu/\mu_0}\right)^{2j}e^{-\left(\frac{am}{\mu/\mu_0}\right)^{2}}}{j!}\right). \tag{2.2}$$

for k =0, 1 and 2 m is 0.886227, 1.32934 and 1.66168, respectively.

Now, we find the minimum termination ratio for some specified producer's risk, sample size $(n=r\times d)$ and the acceptance number $(d=c+1)$ when the following inequality is satisfied:

$$\sum_{i=r}^{n}\binom{n}{i}p_0{}^{i}(1-p_0)^{n-i}\geq 1-\alpha\,. \tag{2.3}$$

Tables 1-3 represent the termination time according to various values of shape parameter, acceptance number, sample size and two values of producer's risk. It is clear from the Table 1 as the sample size increases for fixed value of r, the termination ratios decrease. If shape values increase, the termination ratio also increases. For example, when $r=4$, $n=40$, $k=0$ and α =0.05, the termination ratio is 0.213. Keeping the other quantities same and $k=1$, the termination ratio is 0.405.

3. Comparative Study

The upper entry in each cell of Table 4 corresponds to the proportion of $t/\mu_0=a$ of the proposed test plan with $k=0$. The lower entry corresponds to the similar quantity of the sampling plan of Tsai and Wu (2006). These entries reveal that the termination ratio of the proposed test plans is smaller than they are for the plans in Tsai and Wu (2006).

Example 1

Suppose that the life time of a product follows the GRD with $k=0$. An experimenter wants to run an experiment at t=1500 hours ensuring that the specified average life is at least 1000 hours. This leads to the termination ratio 1.5. Let the producer's risk be 0.05. The corresponding values of n and c from Table 1 of Tsai and Wu (2006) are 4 and 1, respectively. The sampling plan $(n=4, c=1, t/\mu_0=1.500)$ is stated as: if during 1500 hours no more than 1 failure out of 4 is recorded, the lot is accepted, otherwise rejected. For the same sampling plan the termination ratio from Table 1 is 0.362. Thus, the proposed plan will be $(n=4, c=1, t_0/\mu_0=0.362)$ which is implemented as: We reject the product if more than 1 failure is observed during the 362 hours, otherwise we accept it.

In both approaches the acceptance number, the sample size, the producer's risk and the final decision about the lot are the same. But the decision on the first approach equates to 1500^{st} hours and in the second approach, it equates to 362^{st} hours. Hence, the proposed approach is preferable to that of the first approach.

Example 2

Consider a problem associated with reliability provided by Wood (1996) and analyzed from the acceptance sampling viewpoint by Rosaiah and Kantam (2005), Rosaiah at al. (2007) and Balakrishnan et al. (2007). The ordered failure times of the release of software given in terms of hours from the starting of the execution of the software are regarded as ordered sample of size 14 with the failure times as x_i , $(i=1,2,..,14)$. The ordered data are 519, 968, 1430, 1893, 2490, 3058, 3625, 4422, 5218, 5823, 6539, 7083, 7487, and 7846.
First we check whether the GRD can be used or not for the above data. The MLE of $\hat{\lambda}$ is 13321429. The Kolmogorov-Smirnov distance between the observed and fitted distribution is 0.2447 which is less than the Kolmogorov-Smirnov Table value of 0.314. Hence, it is reasonable to assume that lifetime of this product follows GRD with k=0.

Case I:

Suppose that the specified average life of software product is 1000 hours. Let the producer's risk be 0.01. The termination time to test the product from Table 1 of Tsai and Wu (2006) is 800 hours with corresponding $c=1$. Then the acceptance plan is $(n=14, c=1, t/\mu_0=.8)$. According to existing plan the product is rejected if more than 1 failure is observed during 800 hours. The experiment is terminated if 2 failures occur before 800 hours; or end time of the experiment, whichever occurs earlier. From the data we see that only one failure occurs before 800 hours, therefore we accept this product with 99% confidence.

Case II:
From Table 1, the value of termination ratio is 0.115 for the same software data as in Example 2. Since the acceptable average life is to be 1000 hours, we get $t_0 = 0.115 \times 1000 = 115$ hours (approximately). The proposed sampling plan is $(n = 14, c = 1, t_0/\mu_0 = .115)$. We put 14 items on test. The lot of product will be accepted if no more than 1 failure occurs before 115 hours. In this approach we see that among 14 failures, there is no failure before 115 hours, therefore we accept the product.

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Table 1: Test termination ratios under GRD with $k = 0$

r	$n = 2r$	$3r$	$4r$	$5r$	$6r$	$7r$	$8r$	$9r$	$10r$
				$\alpha = 0.05$					
1	0.181	0.147	0.127	0.114	0.104	0.097	0.090	0.085	0.081
2	0.362	0.287	0.246	0.218	0.197	0.183	0.171	0.160	0.152
3	0.459	0.362	0.308	0.272	0.248	0.228	0.212	0.197	0.189
4	0.522	0.407	0.346	0.307	0.278	0.256	0.239	0.225	0.213
5	0.566	0.441	0.374	0.331	0.299	0.276	0.257	0.234	0.228
6	0.599	0.465	0.394	0.348	0.315	0.290	0.269	0.254	0.240
7	0.624	0.484	0.409	0.361	0.328	0.302	0.281	0.264	0.249
8	0.645	0.499	0.422	0.373	0.346	0.311	0.289	0.272	0.257
				$\alpha = 0.01$					
1	0.079	0.064	0.055	0.051	0.046	0.043	0.039	0.038	0.036
2	0.233	0.186	0.159	0.141	0.126	0.115	0.108	0.102	0.097
3	0.335	0.264	0.225	0.197	0.181	0.166	0.155	0.146	0.137
4	0.402	0.317	0.269	0.238	0.216	0.197	0.185	0.174	0.165
5	0.455	0.355	0.301	0.266	0.241	0.222	0.207	0.195	0.184
6	0.490	0.383	0.326	0.288	0.261	0.234	0.224	0.210	0.199
7	0.522	0.407	0.344	0.305	0.276	0.254	0.237	0.222	0.211
8	0.549	0.424	0.360	0.319	0.289	0.266	0.245	0.232	0.219

Table 2: Test termination ratios under GRD with $k = 1$

r	$n = 2r$	$3r$	$4r$	$5r$	$6r$	$7r$	$8r$	$9r$	$10r$
				$\alpha = 0.05$					
1	0.371	0.333	0.309	0.291	0.278	0.267	0.257	0.250	0.243
2	0.544	0.478	0.439	0.411	0.390	0.374	0.360	0.349	0.339
3	0.626	0.544	0.497	0.465	0.440	0.421	0.405	0.392	0.381
4	0.674	0.584	0.532	0.496	0.470	0.449	0.432	0.417	0.405
5	0.708	0.610	0.555	0.517	0.489	0.467	0.449	0.434	0.421
6	0.732	0.630	0.572	0.533	0.504	0.481	0.462	0.447	0.433
7	0.751	0.645	0.585	0.545	0.515	0.491	0.472	0.456	0.443
8	0.766	0.657	0.595	0.554	0.523	0.500	0.480	0.464	0.450

Continued									
r	$n=2r$	$3r$	$4r$	$5r$	$6r$	$7r$	$8r$	$9r$	$10r$
$\alpha=0.01$									
1	0.242	0.218	0.202	0.191	0.182	0.175	0.170	0.164	0.160
2	0.427	0.376	0.347	0.326	0.309	0.297	0.286	0.277	0.267
3	0.522	0.456	0.418	0.391	0.371	0.355	0.342	0.331	0.322
4	0.581	0.505	0.461	0.431	0.408	0.391	0.376	0.364	0.353
5	0.622	0.539	0.491	0.458	0.434	0.415	0.399	0.386	0.375
6	0.653	0.564	0.513	0.479	0.453	0.433	0.416	0.403	0.391
7	0.677	0.583	0.530	0.494	0.468	0.447	0.430	0.415	0.403
8	0.696	0.599	0.544	0.507	0.479	0.458	0.440	0.425	0.413

Table 3: Test termination ratios under GRD with $k=2$

r	$n=2r$	$3r$	$4r$	$5r$	$6r$	$7r$	$8r$	$9r$	$10r$
$\alpha=0.05$									
1	0.474	0.439	0.415	0.398	0.385	0.374	0.365	0.358	0.351
2	0.628	0.570	0.535	0.511	0.492	0.477	0.464	0.453	0.444
3	0.697	0.628	0.587	0.558	0.537	0.520	0.505	0.493	0.483
4	0.738	0.662	0.617	0.586	0.563	0.544	0.529	0.516	0.505
5	0.766	0.684	0.637	0.605	0.580	0.561	0.545	0.532	0.520
6	0.786	0.701	0.652	0.618	0.593	0.573	0.556	0.543	0.531
7	0.802	0.713	0.663	0.628	0.602	0.582	0.565	0.551	0.539
8	0.815	0.723	0.672	0.636	0.610	0.589	0.572	0.558	0.545
$\alpha=0.01$									
1	0.350	0.325	0.309	0.297	0.287	0.280	0.273	0.268	0.263
2	0.525	0.479	0.451	0.432	0.416	0.404	0.394	0.385	0.376
3	0.609	0.551	0.517	0.493	0.474	0.460	0.447	0.437	0.428
4	0.660	0.594	0.556	0.529	0.508	0.492	0.479	0.468	0.458
5	0.694	0.623	0.582	0.553	0.531	0.514	0.500	0.488	0.478
6	0.720	0.645	0.601	0.571	0.548	0.530	0.515	0.503	0.492
7	0.740	0.661	0.616	0.585	0.561	0.543	0.527	0.514	0.503
8	0.756	0.674	0.628	0.595	0.571	0.552	0.537	0.524	0.512

Table 4: Comparison of test termination ratio for k=0

r	$n = 2r$	$3r$	$4r$	$5r$	$6r$	$7r$	$8r$	$9r$	$10r$
$\alpha = 0.05$									
1					0.104 0.800				
2								0.160 0.600	
3							0.212 0.600		
4		0.407 1.000							
5						0.276 0.600			
6									
7									
8									
$\alpha = 0.01$									
1	0.079 2.000								
2						0.115 0.800			
3									
4	0.402 1.500								
5									
6									
7									
8									