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# An Algorithm to Generate Neighbor Balanced Binary Block Designs

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## Abstract

Ahmed and Akhtar (2007) developed some algorithms to generate neighbor balanced designs. Here, a new algorithm is developed to generate circular neighbor balanced binary block designs for $v$ = 2k in 2($v$-1) blocks when blocks are well separated. A catalogue of circular neighbor balanced binary block designs for $v$ up to 50 is presented.

## Keywords

Complete block neighbor designs, Second order neighbor designs, Circular neighbor designs, Binary block designs

## 1. Introduction

A neighbor design is a collection of circular blocks in which any two treatments appear as neighbors equally often. Rees (1967) constructed complete block neighbor designs for every odd $v$ (number of treatments). He also constructed neighbor designs for incomplete block designs. These designs were initially used in Serology. Lawless (1971) and Hwang (1973) constructed incomplete block neighbor designs. Azais *et al.* (1993) considered designs in linear blocks with border plots in which a treatment may affect the response on the two adjacent plots. Three series of designs developed are: (i) neighbor balanced designs in complete blocks, (ii) neighbor balanced designs in blocks each of which lacks one treatment, and (iii) partially neighbor balanced designs in few complete blocks.

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Mettei (1996) developed an algorithm to construct neighbor designs for $v$=2$m$ and k = $m$ in 2($v$-1) blocks but the designs generated through this algorithm are not binary. Jacroux (1998) constructed incomplete block neighbor balanced designs in linear blocks, for all $v$, taking blocks of size 3. Muller and Pazman (2003) developed optimal designs in case of correlated observations. Bailey and Druilhet (2004) studied optimality of circular neighbor balanced block designs when neighbor effects are present in the model. They showed that for total effects, circular neighbor balanced designs are universally optimal among designs with no self neighbor. They also presented efficiency factors of these designs and discussed some situations where a design with self neighbors is preferable to a neighbor balanced design. Filipiak and Rozanski (2005) constructed E-optimal designs under an interference model and expressed that circular neighbor balanced designs are universally optimal in a circular model of such experiment. Iqbal *et al*. (2006) constructed second order neighbor designs using method of cyclic shifts. Ahmed and Akhtar (2007) developed some algorithms to generate neighbor balanced designs which are presented in Section 2.

Binary block design is a design in which each treatment occurs at most once in a block. In Section 3, we have presented an algorithm to generate circular binary block neighbor balanced designs for $v$= 2$m$, k = $m$ in b = 2($v$-1) blocks when blocks are well separated. A catalogue of such designs for $14 \leq v \leq 50$ is presented in Section 4.

## 2. An Algorithm Developed by Ahmed and Akhtar (2007) to Generate Neighbor Balanced Designs

(i) An algorithm to generate complete block neighbor designs for odd $v$ = 2$m$+1 which are useful for one sided neighbor effects: These designs are obtained by developing the following base block modulo 2$m$ keeping $y$ invariant

[0, 1, -1, 2, -2,…,-($m$-1), $m$, $y$] modulo 2$m$

(ii) An algorithm to generate complete block neighbor designs for $v$ multiple of four in b = $v$ -1 blocks: Let $v$ = 2$m$ +2 for $m$ = 1, 3, 5 … then these designs are obtained by developing the following base block modulo 2$m$ keeping $y$ invariant

[0, 1, 3, 6,…, ($m$-1)$m$/2, $m$ ($m$+1)/2, $m$($m$+3)/2,…, $m$($m$+1) -1, $y$] modulo (2$m$+1)

(iii) An algorithm to generate neighbor balanced designs for odd $v = 2m+1$ and $k = v-1$: These designs are obtained by developing the following base block modulo $2m$ keeping $y$ invariant. Required design is obtained by augmenting the block $((v-2), (v-3),\ldots,2, 1, 0)$ with the resultant $(v-1)$ blocks obtained through base block

$[0, 1, -1, 2, -2, \ldots, -(m-1), y]$ modulo $2m$, where $y = 2m$.

(iv) An algorithm to generate neighbor balanced designs for $v$ multiple of four and $k = v -1$: Let $v = 2m +2$, $(m = 1, 3, 5, \ldots)$. These designs are obtained by developing the following base block modulo $2m+1$ keeping y invariant. Required design is obtained by augmenting the block $(0, 1, 2\ldots, k-1)$ with the resultant $(v-1)$ blocks obtained through the base block.

$[0, 1, 3, 6, \ldots, (m-1)m/2, m(m+1)/2, m(m+3)/2, \ldots, m(m+1) -1, y]$ modulo $(2m+1)$

## 3. An algorithm to Generate Neighbor Balanced Binary Block Designs for *v*=2*m*, k =*m* in 2(*v*-1) Blocks

Let $v=2m$ $(m =1,2,3, \ldots)$. Consider a series: $1, 2, 3,\ldots, (m-1), m$. (3.1)
If the sum of any 2, 3, …, or $(m+1)$ successive numbers of series (3.1) is not 0 modulo $(v-1)$ then base block I is:

$$[0,1,3,6, \ldots, (m-1)m/2, m(m+1)/2] \quad \text{modulo } (v-1)$$

Otherwise, rearrange the numbers in series (3.1) in such a way that the sum of any 2, 3… or $m$ successive rearranged numbers is not 0 modulo $(v-1)$ to obtain the binary block neighbor designs. If the rearranged numbers are $a_1, a_2, ..., a_m$ then the base block I will be:

$$[0, a_1, (a_1 + a_2), \ldots, (a_1 + a_2 + ... + a_m)] \quad \text{modulo } (v-1)$$

Base block II is constructed as: Let $s = m(m+1)/2$ modulo $(v-1)$.

a) If $s < m$, consider the numbers $1,2,3,\ldots, (m-1), (m+1)$, then exclude the number equal to $s$.
b) If $s > m$, consider the numbers $1,2,3,\ldots, (m-1), m$, then exclude the number equal to $(2m+1-s)$.
c) If $s = m$, consider the numbers $1,2,3,\ldots,(m-2),(m+2)$, then exclude the number equal to $s$.

If the sum of any 2, 3, …, or $(m-1)$ successive numbers of above three cases is 0 modulo $(v-1)$ then to obtain the binary block neighbor designs, rearrange the numbers in such a way that the sum of any 2, 3, …, or ($m$-1) successive rearranged numbers is not 0 modulo ($v$-1). If the rearranged numbers are $b_1, b_2, \ldots, b_{m-1}$ then the base block II will be:

$$[0,\ b_1, (b_1 + b_2),\ \ldots, (b_1, b_2, \ldots, b_{m-1}), y] \text{ modulo } (v\text{-}1)$$

Then ($v$-2) blocks are obtained cyclically modulo ($v$-1) from the base block I. Similarly ($v$-2) blocks are obtained through base block II where $y$ remains unchanged. In final design $y$ is replaced by $v$-1.

*Example 1:* For $v$=14 and k=7, the base blocks I and II are (0,1,3,6,10,2,8) and (0,1,4,6,10,3, $y$) respectively. Complete design is in Table 1.

**Table 1:** The Base Blocks I and II

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **B1:** | (0, | 1, | 3, | 6, | 10, | 2, | 8) | **B14:** | (0, | 1, | 4, | 6, | 10, | 3, | $y$) |
| **B2:** | (1, | 2, | 4, | 7, | 11, | 3, | 9) | **B15:** | (1, | 2, | 5, | 7, | 11, | 4, | $y$) |
| **B3:** | (2, | 3, | 5, | 8, | 12, | 4, | 10) | **B16:** | (2, | 3, | 6, | 8, | 12, | 5, | $y$) |
| **B4:** | (3 | 4, | 6, | 9, | 0, | 5, | 11) | **B17:** | (3, | 4, | 7, | 9, | 0, | 6, | $y$) |
| **B5:** | (4, | 5, | 7, | 10, | 1, | 6, | 12) | **B18:** | (4, | 5, | 8, | 10, | 1, | 7, | $y$) |
| **B6:** | (5, | 6, | 8, | 11, | 2, | 7, | 0) | **B19:** | (5, | 6, | 9, | 11, | 2, | 8, | $y$) |
| **B7:** | (6, | 7, | 9, | 12, | 3, | 8, | 1) | **B20:** | (6, | 7, | 10, | 12, | 3, | 9, | $y$) |
| **B8:** | (7, | 8, | 10, | 0, | 4, | 9, | 2) | **B21:** | (7, | 8, | 11, | 0, | 4, | 10, | $y$) |
| **B9:** | (8, | 9, | 11, | 1, | 5, | 10, | 3) | **B22:** | (8, | 9, | 12, | 1, | 5, | 11, | $y$) |
| **B10:** | (9, | 10, | 12, | 2, | 6, | 11, | 4) | **B23:** | (9, | 10, | 13, | 2, | 6, | 12, | $y$) |
| **B11:** | (10, | 11, | 0, | 3, | 7, | 12, | 5) | **B24:** | (10, | 11, | 14, | 3, | 7, | 0, | $y$) |
| **B12:** | (11, | 12, | 1, | 4, | 8, | 0, | 6) | **B25:** | (11, | 12, | 15, | 4, | 8, | 1, | $y$) |
| **B13:** | (12, | 0, | 2, | 5, | 9, | 1, | 7) | **B26:** | (12, | 0, | 16, | 5, | 9, | 2, | $y$) |

where $y = 13$.

## 4. A Catalogue of Circular Neighbor Balanced Binary Block Designs for *v* = 2 and k = *m*

Such designs are already generated by Iqbal *et al.* (2006) for $v$ up to 12. Here we have presented a catalogue for $14 \le v \le 50$. Following are the two base blocks in Table 2. Remaining each of ($v$-2) blocks are obtained cyclically modulo ($v$-1) from the base block I and II keeping $y$ unchanged.

**Table 2:** Base Blocks for $14 \leq v \leq 50$

| *v* | k | b | Base Block I | Base Block II |
|---|---|---|---|---|
| 14 | 7 | 26 | (0,1,3,6,10,2,8) | (0,1,4,6,10,3, *y*) |
| 16 | 8 | 30 | (0,1,3,6,10,2,8,13) | (0,1,4,8,14,6,11, *y*) |
| 18 | 9 | 34 | (0,1,3,6,10,15,4,11,2) | (0,3,4,8,13,2,9,1, *y*) |
| 20 | 10 | 38 | (0,1,3,6,10,15,2,9,17,7) | (0,3,4,6,10,15,2,12,1, *y*) |
| 22 | 11 | 42 | (0,4,5,7,10,15,2,9,18,3,13) | (0,4,5,8,10,15,1,7,16,6, *y*) |
| 24 | 12 | 46 | (0,1,3,6,10,15,21,5,13,22,9, 20) | (0,1,3,7,12,18,2,10,19,6,17, *y*) |
| 26 | 13 | 50 | (0,2,5,6,10,15,21,3,11,20,7,18,8) | (0,2,5,6,10,15,21,3,12,24,9,20, *y*) |
| 28 | 14 | 54 | (0,3,4,6,10,15,21,1,9,18,2,12,24,11) | (0,3,4,6,10,15,21,1,9,18,5,17,27 ,*y*) |
| 30 | 15 | 58 | (0,1,3,6,10,15,21,28,7,16,26,8,20,4, 18) | (0,1,3,6,10,15,21,28,7,16,26,9,22, 8, *y*) |
| 32 | 16 | 62 | (0,1,3,6,10,15,21,28,5,14,24,4,16,29 ,12,27) | (0,1,3,6,11,17,24,2,10,20,4,15,27, 9,23, *y*) |
| 34 | 17 | 66 | (0,4,5,7,10,15,21,28,3,12,22,1,14,25 ,6,24,8) | (0,4,6,9,10,15,21,28,5,14,25,7,19, 32,13, 29, *y*) |
| 36 | 18 | 70 | (0,2,3,6,10,15,21,28,1,11,20,31,8,22 ,4,17, 32,13) | (0,15,31,33,34,2,6,11,17,24,32, 7, 16,27,4, 18,1, *y*) |
| 38 | 19 | 74 | (0,1,3,6,10,15,21,28,36,8,18,29,4,17 ,31,9, 25,5,23) | (0,1,3,6,10,15,21,28,36,8,18,29, 4, 17,32, 12,30,9, *y*) |
| 40 | 20 | 78 | (0,1,3,6,10,15,21,28,36,7,16,27,2,14 ,29,8, 25,38,18,34) | (0,1,3,6,10,16,23,31,2,11,22,34, 8, 24,38, 14,32,12,29, *y*) |
| 42 | 21 | 82 | (0,1,3,6,10,15,21,28,36,4,14,25,37,9 ,23,38,13,30,7,26,5) | (0,2,3,6,10,16,23,31,40,9,20,32, 4, 18,33, 8,29,7,24,1, *y*) |
| 44 | 22 | 86 | (0,1,3,6,10,15,21,28,36,2,12,23,35,5 ,19,34,7,24,42,18,38,16) | (0,3,4,6,10,15,21,28,36,2,12,23, 35,5,19,34,13,30,7,25,1, *y*) |
| 46 | 23 | 90 | (0,2,6,11,17,24,32,41,7,19,29,42,12, 26,43, 14,33,8,30,3,4,25,28) | (0,44,1,5,10,16,23,31,40,6,18,28,41,11,25,43,14,33,8,29,32,9, *y*) |
| 48 | 24 | 94 | (0,1,3,6,10,15,21,28,36,45,8,19,31,44,11,26,42,12,30,2,22,43,18,41) | (0,1,5,7,10,15,22,30,39,2,13,25, 38,6,20,36,8,26,43,16,37,12,35, *y*) |
| 50 | 25 | 98 | (0,1,3,7,10,15,21,28,36,45,6,17,29,42,8,22,38,9,26,44,14,37,12,33,11) | (0,2,3,7,10,15,21,28,36,45,6,18, 31,46,11,27,44,13,32,4,24,47,25,1, *y*) |

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