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Economic Reliability Test Plans using the Generalized Exponential Distribution

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Abstract

Economic Reliability Test Plans (ERTP) are developed assuming that the life time follows the Generalized Exponential Distribution (GED). For a given termination number, sample size and producer's risk waiting time to terminate the experiment are computed. Comparisons of these reliability plans have been made with reliability plans of the Log-Logistic Distribution (Kantam et al., 2006). On the basis of the Kolmogorov-Smirnov test for both distributions it is found that the reliability plans obtained from GED are more economic in saving cost, energy and time. The results are explained with the help of tables and examples.

Keywords

Life Tests, Reliability Test Plans, Generalized Exponential Distribution, Log-logistic Distribution

1. Introduction

Reliability study plays a vital role in the quality control analysis. On the basis of this study, an experimenter can save his time and cost to reach result which is to accept the submitted lot or to reject it. If the genuine products are rejected on the basis of sample information, this error is called type-1. On the other hand, if the genuine products are not accepted by the consumer, this error is type-2 error. If a decision to accept or reject the lot are subjected to the risks associated with the two types of errors, this procedure is termed as 'reliability test plan' or

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‘acceptance sampling based on life test’; for more details, see Duncan (1986 and Stephens (2001).

Probability distributions/models have been in use to develope the reliability test plans. Under these plans we can find the termination time of experiment. These distributions can be used to find the best reliability sampling plans which are more economical for the experimenter. The log-logistic distribution has been studied in detail by Quigley and Struthers (1982). Kantam et al. (2001) used the log-logistic distribution to develop the acceptance sampling plans. Kantam et al. (2006) also studied the reliability test plans using the log- logistic distribution. In reliability test plan we terminate the experiment if the termination time t ends or the rth failure occurs if we put the rn sample units on test, whichever occurs first.

In this article, economic reliability plans are constructed using the generalized exponential distribution. Comparison of these plans has been made with the reliability test plans of log-logistic distribution (Kantam et al., 2006).

2. The Generalized Exponential Distribution

This distribution is proposed by Mudholker et al. (1995) and has the probability density function:

$$f(x;\alpha,\lambda,\mu)=\frac{a}{\lambda}(1-e^{-\frac{(x-\mu)}{\lambda}})^{\alpha-1}e^{-\frac{(x-\mu)}{\lambda}} \quad ; x;\mu,\lambda,\alpha>0 \tag{2.1}$$

The corresponding distribution function is:

$$F(x;\alpha,\lambda,\mu)=(1-e^{-\frac{(x-\mu)}{\lambda}})^{\alpha} \quad ;\ x;\mu,\lambda,\alpha>0 \tag{2.2}$$

The cumulative distribution function for the log-logistic distribution is:

$$F(x)=\frac{x/\sigma}{\left[1+(x/\sigma)^{\beta}\right]}; \quad x>0,\sigma>0,\beta>1 \tag{2.3}$$

This distribution can be used effectively to analyze the positive life time data. The density function and the distribution function of $GE(\alpha,\lambda,\mu)$ family distributions are similar to the density and the distribution function of Weibull and gamma distributions. If α =1, then all three distributions coincide with the two parameters exponential distribution (Gupta and Kundu, 1999 & 2001). If $X \sim GE(\alpha, \lambda, \mu)$, the survival and hazard functions are given as:

$$S(x;\alpha,\lambda,\mu) = 1 - F(x;\alpha,\lambda,\mu) = 1-(1-e^{-\frac{(x-\mu)}{\lambda}})^{\alpha} \quad (2.4)$$

$$h(x;\alpha,\lambda,\mu) = \frac{\alpha(1-e^{-\frac{(x-\mu)}{\lambda}})^{\alpha-1} e^{-\frac{(x-\mu)}{\lambda}}}{1-(1-e^{-\frac{(x-\mu)}{\lambda}})^{\alpha}} \quad (2.5)$$

In the following section the economic reliability test plans for $GE(\alpha, \lambda, \mu)$ have been discussed.

3. An Economic Reliability Test Plan for Generalized Exponential Distribution

Let n be the number of items put on test taken from an infinite lot and r be the terminating number for the experiment. The experiment will be stopped if r failures occur from n sample items in termination time t or the termination time is ended. In former case, the lot is rejected otherwise it is accepted. Sample size selection is an important factor in life test experiments. The sample size is selected keeping in view the cost of experiment and the expected waiting time to reach the decision, for more detail see Kantam et al. (2006). In this situation, the binomial distribution is suitable. The probability of acceptance is given in this case:

$$L(p) = \sum_{r=1}^{r-1} \binom{n}{r} p^{i} q^{n-r} \quad (3.1)$$

where $p = F(x;\alpha,\lambda,\mu)$ and $F(x;\theta)$ is *CDF* of random variable X with parameter θ.

If α^{*} is producer's risk then equation (3.1) can be written as:

$$\sum_{r=1}^{r-1} \binom{n}{r} p^{i} q^{n-r} = 1-\alpha \quad (3.2)$$

Given the values of n (where $n = rk$), r and k equation (3.2) can be solved for p using cumulative probabilities of binomial distribution. Then the values of p can be used in equation (2.2.) for α =1.25 to find the values of t/λ. These values for different values of r and n are given in Tables 1 and 2, respectively.

Table 1: Reliability test plan for generalized exponential distribution $(\alpha = 1.25, \mu = 0)$ for α^*=0.05

r\n	2r	3r	4r	5r	6r	7r	8r	9r	10r
1	0.0543	0.0390	0.0307	0.0259	0.0223	0.0197	0.0177	0.0161	0.0148
2	0.1689	0.1154	0.0896	0.0737	0.0622	0.0554	0.0495	0.0445	0.0410
3	0.2520	0.1691	0.1293	0.1059	0.0906	0.0790	0.0705	0.0622	0.0584
4	0.3121	0.2059	0.1573	0.1289	0.1097	0.0959	0.0852	0.0773	0.0707
5	0.3573	0.2349	0.1786	0.1457	0.1238	0.1082	0.0961	0.0826	0.0794
6	0.3927	0.2569	0.1946	0.1587	0.1347	0.1176	0.1044	0.0945	0.0862
7	0.4217	0.2745	0.2074	0.1686	0.1436	0.1253	0.1113	0.1005	0.0919
8	0.4459	0.2886	0.2184	0.1775	0.1506	0.1316	0.1169	0.1056	0.0965

where α is shape parameter and α^* is type-1 error.

Table 2: Reliability test plan for generalized exponential distribution $(\alpha = 1.25, \mu = 0)$ for α^*=0.01

r\n	2r	3r	4r	5r	6r	7r	8r	9r	10r
1	0.0145	0.0101	0.0080	0.0070	0.0060	0.0053	0.0048	0.0043	0.0040
2	0.0822	0.0567	0.0440	0.0363	0.0303	0.0263	0.0238	0.0216	0.0197
3	0.1491	0.1007	0.0774	0.0626	0.0542	0.0474	0.0425	0.0384	0.0346
4	0.2012	0.1358	0.1039	0.0849	0.0724	0.0623	0.0565	0.0509	0.0467
5	0.2478	0.1635	0.1249	0.1021	0.0869	0.0759	0.0677	0.0612	0.0560
6	0.2806	0.1859	0.1421	0.1158	0.0985	0.0861	0.0767	0.0693	0.0633
7	0.3121	0.2054	0.1558	0.1273	0.1081	0.0945	0.0841	0.0761	0.0695
8	0.3396	0.2204	0.1680	0.1371	0.1165	0.1017	0.0892	0.0817	0.0744

3.1 Comparison of Reliability Plans

For the comparison purpose, our measurements are consistent with Kantam et al. (2006). For example, for a lot with specified average life of 15707, the termination time values for log-logistic (borrowed from the paper of Kantam et al. (2006)) and generalized exponential distribution are placed in Tables 3 and 4, respectively.

Table 3: Termination Time for log-logistic with r=5, α^* =0.01

n	$\frac{t}{\lambda_0}$	$1.5707\,\sigma_0$ =15,707	Termination time (t_0)
2 r	0.5349	10,000	5,348.7
3 r	0.4063	10,000	4,063.4
4 r	0.3412	10,000	3,412.1
5 r	0.2997	10,000	2,996.8
6 r	0.2705	10,000	2,704.6
7 r	0.2488	10,000	2,487.9
8 r	0.2313	10,000	2,312.7
9 r	0.2170	10,000	2,169.6
10 r	0.2005	10,000	2,005.03

Table 4: Termination Time for *GED* with r=5, α^* =0.01

n	$\frac{t}{\lambda_0}$	$1.5707\,\sigma_0$ =15,707	Termination Time (t_0)
2 r	0.3573	10,000	3,573
3 r	0.2347	10,000	2,347
4 r	0.1786	10,000	1,786
5 r	0.1457	10,000	1,457
6 r	0.1238	10,000	1,238
7 r	0.1082	10,000	1,082
8 r	0.0961	10,000	961
9 r	0.0826	10,000	826
10 r	0.0794	10,000	794

Suppose that we want to construct a life test sampling plan with specified average life of 15,707. Select 15 items from the lot and put them on test. If the 3rd failure occurs during the termination time 2,347 hours we reject the lot, otherwise accept it. Terminate the experiment as soon as 3rd failure released or the 2,347th hour is reached. At the same measurements, termination time using the log-logistic distribution is 4,063.4 hours. The terminating points from *GE* distribution are smaller than the termination time plan of Kantam et al. (2006). Therefore, the terminating values from *GE* distribution would result in saving the time, cost and also energy in order to accept or to reject the submitted lot on the basis of life time taken from the lot.

3.2 A Live Data Example

In this section, we consider one real dataset to see the effectiveness of generalized exponential and log-logistic distribution in terms of saving cost and time by using the Kolmogorov- Smirnov test. The data (Lawless, 1982 and Gupta and Kundu, 1999) given here arose in tests on endurance of deep groove ball bearing. The data are the number of million revolutions before failure for each of the 23 ball bearing in the life test and these are:

17.88, 28.92, 33.00, 41.52, 42.12, 45.60, 48.60, 51.84, 51.96, 54.12, 55.56, 67.80, 68.64, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04 and 173.40.

The results of Kolmogorov- Smirnov test are given in Table 5.

Table 5 Kolmogorov- Smirnov test for GED and LLD

Interval	f	$F(X)$	$F_0(X)$ GED	D_0	$F_1(X)$ LLD	D_1
0-40	3	0.130435	0.369817	0.239	0.147196	0.016
40-80	12	0.652174	0.797872	0.145	0.608369	0.043
80-120	5	0.869565	0.938177	0.068	0.811856	0.057
120-160	2	0.956522	0.981299	0.024	0.894264	0.062
160-200	1	1	0.994361	0.005	0.933248	0.066

We state null and alternative hypothesis as:

H_0: The population distribution is generalized exponential or log-logistic distribution or both.

H_1: The population distribution is not the above distributions.

The estimated values of unknown parameters are λ =33.33 and σ =48.14 for the data. These values are calculated using (3.3) and (3.4), respectively.

$$Estimated\ \beta = \frac{n}{\sum_{i=1}^{n} x_i - (\alpha-1)\sum_{i=1}^{n}\left(\frac{x_i\, e^{-x\beta}}{1-e^{-x\beta}}\right)} \tag{3.3}$$

In (3.2), shape parameter α =1.25 and $\beta = 1/\lambda$.

$$Estimated \quad \sigma = \frac{E(x) = mean\ of\ data}{\sqrt{(1+\frac{1}{\beta})}\sqrt{(1-\frac{1}{\beta})}} \tag{3.4}$$

In (3.3), shape parameter β =2. $F(X)$ is the cumulative relative frequency for 23 sample values. $F_0(X)$ and $F_1(X)$ are cumulative probabilities using (2.2) and (2.3). The maximum absolute difference D_0 and D_1 for both distributions are 0.239 and 0.062. These differences are less than the Table value 0.275. Therefore, we conclude that both distributions are fitted to data. But the preference will be given to generalize exponential distribution because the reliability plans using this distribution for above data is economical in terms of cost and time.

4. Concluding Remarks

We developed the reliability test plans using the *GED* as a lifetime distribution with shape parameter 1.25. The results are compared with the log-logistic distribution with shape parameter 2. It is concluded that the reliability test plans obtained from the *GED* with shape parameter 1.25 are economically best as compared to the plans obtained from the log-logistic distribution with shape parameter 2 in terms of saving time and energy.

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