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Reliability and Quantile Analysis of the Weibull Distribution

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Abstract

This paper presents the reliability and Quantile analysis of the Weibull distribution. We also present the properties of Quantile analysis as the percentile life used B-life in engineering terminology. The main interests are in the relationship between β and various B-lives; measure of variability for B-lives as the numerical Quantities that describe the spread of the values in a set of data. Here these Quantiles models are presented graphically and mathematically.

Keywords

Weibull distribution, Weibull quantile analysis, Percentile life

1. Introduction

The Weibull distribution is one of the most widely used probability distributions in the reliability engineering discipline. The Weibull distribution becomes a standard in reliability for modeling time-dependent failure data. This paper focuses to present the Quantile analysis as the percentile life used B-life in engineering terminology. This is the life by which the certain proportion of the population can be expected to have failed. The Weibull Probability distribution is very useful life time model for checking the failure components (Liu, 1997; Abernathy, 2004).

The Weibull model is very flexible reliability model that approaches different distributions. It is the generalization of the exponential distribution and is very

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useful in reliability theory. Nelson (1982) and Liu (1997) explain in their works that if the item consists of many parts and each part has the same failure time distribution and the item falls in the experiment when the weakest part fails, then the Weibull distribution would be an acceptable model of that failure mode.

2. Weibull Models Analysis

2.1 Weibull Probability Distribution

The Weibull probability distribution has three parameters β, η and t_0. It can be used to represent the failure probability density function (PDF) with time, so that:

$$f_w(t) = \frac{\beta}{\eta}\left(\frac{t-t_0}{\eta}\right)^{\beta-1} e^{-\left(\frac{t-t_0}{\eta}\right)^{\beta}}, \qquad \beta > 0, \eta > 0, t_0 > 0, -\infty < t_0 < t \tag{2.1}$$

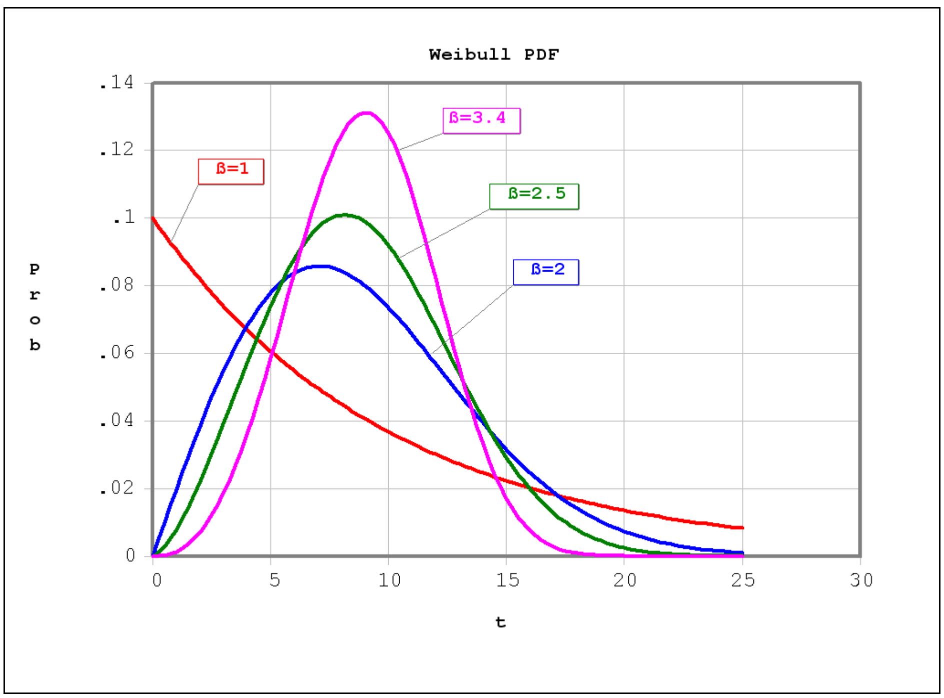

Fig. 2.1: The Weibull PDF

Here β is the shape parameter representing the different pattern of the Weibull PDF and is positive and η is a scale parameter representing the characteristic life at which 63.2% of the population can be expected to have failed and is also positive, t_0 is a location parameter. If $t_0 = 0$ then the Weibull distribution is said to be two-parameter since the restrictions in (2.1) on the values of t_0, η, β are always the same for the Weibull distribution (Kao, 1957; Dubey, 1966; Liu, 1997; Cox and Oakes, 1984; Abernathy, 2004; Pasha et al., 2007). Fig. 2.1 shows the diverse shape of the Weibull PDF with $t_0 = 0$ and value of $\eta = 10$ and β (= 1, 2, 2.5, 3.4). When β = 1, the distribution is the same as the exponential distribution for the density function. When $\beta = 2$, it is known as the Rayleigh distribution for the density function. When $\beta = 2.5$, then the shape of the density function is similar to the Lognormal shape of function. When β = 3.4 then the shape of the density function is similar to the normal shape of function. To check the validity of Figs the relevant information is provided in Tables A and B given in Appendix.

2.2 Cumulative Distribution Function

The cumulative distribution function (CDF) of the Weibull distribution is denoted by $F_w(t)$ and is defined as:

$$F_w(t) = 1 - e^{-(\frac{t-t_0}{\eta})^{\beta}} \tag{2.2}$$

When the CDF of the Weibull distribution has zero value then it represents no failure components by t_0. Using (2.2), the Weibull CDF t_0 is called minimum life. When $t = t_0 + \eta$, then $F_w(t_0 + \eta) = 1 - (1/e) =$ 0.63212 which explains as 'characteristic life' or 'characteristic value' (Gumbel 1958). Fig. 2.2 shows CDF of Weibull with t_0 = 0 and value of $\eta = 10$ and β (=1, 2, 2.5, 3.4). It is clear that all curves intersect at the point of (10, 0.632), the characteristic point for the Weibull CDF.

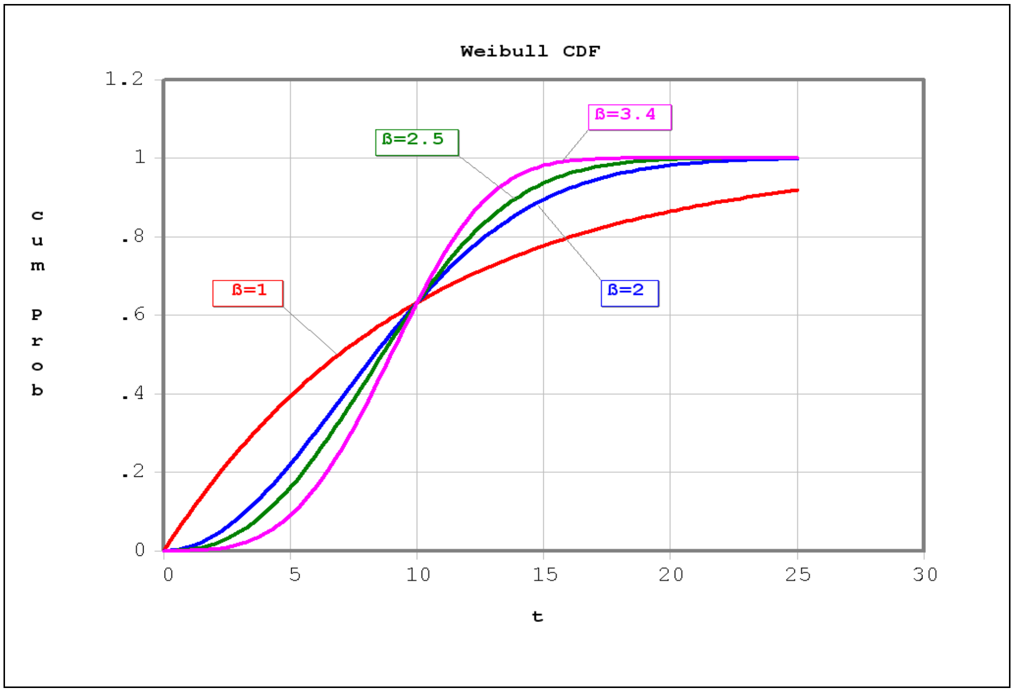

Fig. 2.2: The Weibull CDF

2.3 Reliability Function

The reliability function (RF), denoted by $R_w(t)$, also known as the survivor function, is defined as $1-F_w(t)$.

$$R_w(t) = e^{-(\frac{t-t_0}{\eta})^{\beta}} \tag{2.3}$$

We see that $R_w(t)+F_w(t)=1$. Fig. 2.3 shows RF of Weibull with $t_0 = 0$ and value of $\eta = 10$ and β (=1, 2, 2.5, 3.4). From Fig. 2.3 it is clear that all the curves intersect at the point (10, 0.368) for the characteristic point of the Weibull RF. When β = 1, the distribution is the same as the exponential distribution for a constant RF. When $\beta = 2$, it is known as the Rayleigh distribution for the RF. When β= 2.5, then the shape of the reliability function is similar to the Lognormal RF. When β= 3.4, then the shape of the reliability function is similar to the normal RF.

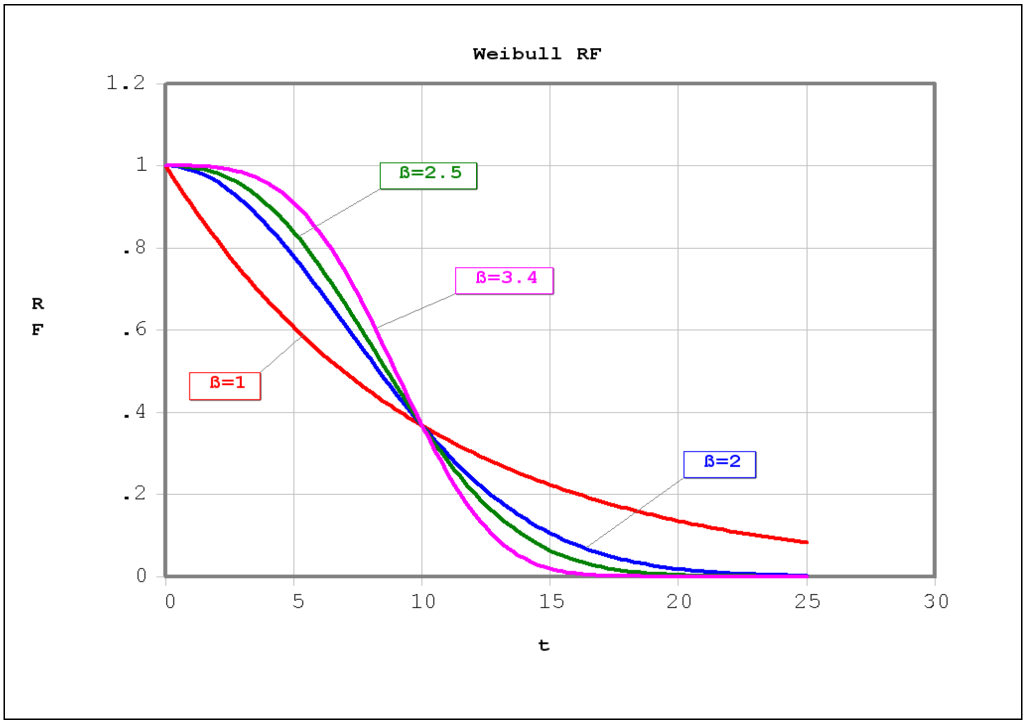

Fig. 2.3: The Weibull RF

2.4 Hazard Function

The hazard function (HF) of the Weibull distribution, also known as instantaneous failure rate, denoted by $h_w(t)$, is defined as $f_w(t)/R_w(t)$:

$$h_w(t) = \left(\frac{\beta}{\eta}\right)\left(\frac{t-t_0}{\eta}\right)^{\beta-1} \tag{2.4}$$

It is important to note that the units for $h_w(t)$ are the probability of failure per unit of time, distance or cycles.

When β = 1, the distribution is the same as the exponential distribution for constant hazard function and $h_w(t) = \dfrac{1}{\eta}$ so the exponential distribution is a special case of the Weibull distribution and the Weibull distribution can be treated as a generalization of the exponential distribution.

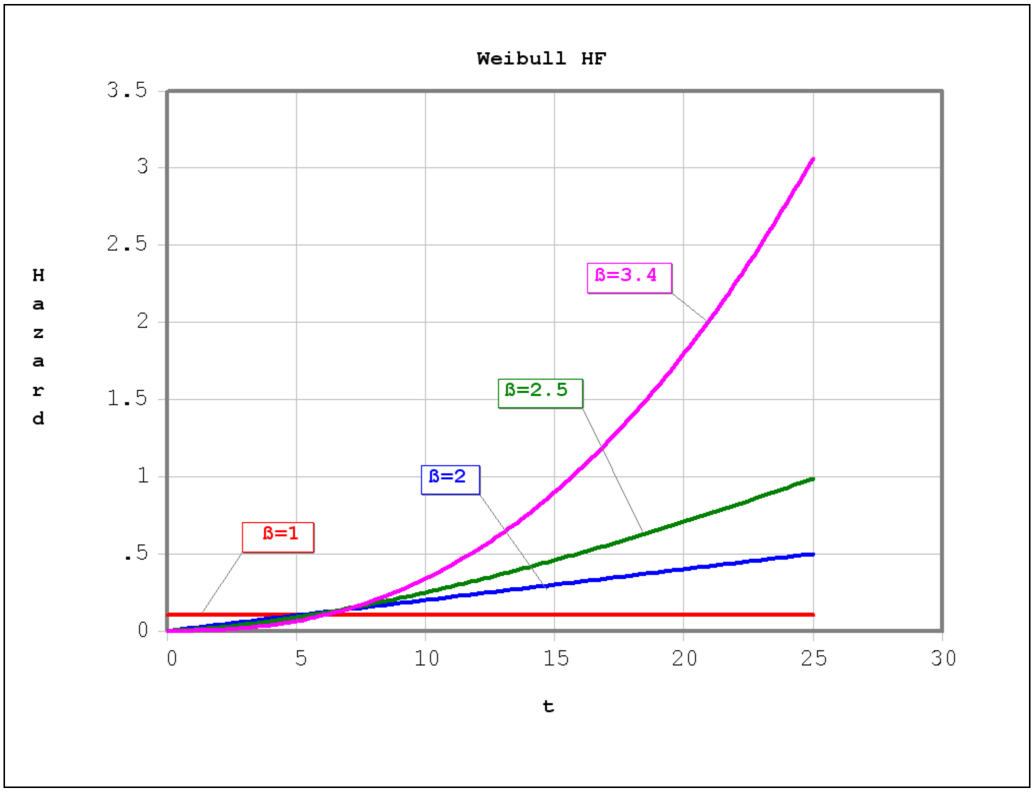

Fig. 2.4: The Weibull HF

When $\beta < 1$, the hazard function is continually decreasing which represents early failures. When $\beta > 1$, the hazard function is continually increasing which represents wear-out failures. In particular, when $\beta = 2$, it is known as the Rayleigh distribution. When $\beta = 3.4$, the shape of the PDF is similar to the normal PDF. These cases are called pseudo-symmetrical by Gumbel (1958). So the Weibull is a very flexible distribution. Fig. 2.4 shows the Weibull HF with $t_0 = 0$ and value of $\eta = 10$ and β (=1, 2, 2.5, 3.4).

3. Quantile Analysis of the Weibull Distribution

One of the important properties of the Weibull distribution is the percentile life or B-life in engineering terminology and is defined as:

$$t_p = t_0 + \eta\left(\ln\frac{1}{1-p}\right)^{\frac{1}{\beta}} \qquad (3.1)$$

$F_w(t_{0.01})$ = 0.01 mean, this is the life at which the unit will have a failure probability of 1%. Fig. 3.1 shows the relationship between β and various values of B-lives (B-l, B-2, B-3, B-4 and B-5 lives) for η = 1000. For B-2 life mean, this is the life for which the unit will have a failure probability of 2%. It is clear that larger the value of β, the longer the B-lives for the same value of η. For the case of η = 1000, all the lives, B-l to B-5, are effectively zero for $\beta \leq$ 0.3. Fig. 3.1 shows the relationship between β = 0(0.1)10 and B-lives (B-1, B-2, B-3, B-4 and B-5 lives) when η = 1000. It is interesting to note that the values of B-1, B-2, B-3, B-4 and B-5 lives for each cycle differ approximately by factors of 10 when $\beta = 1$.

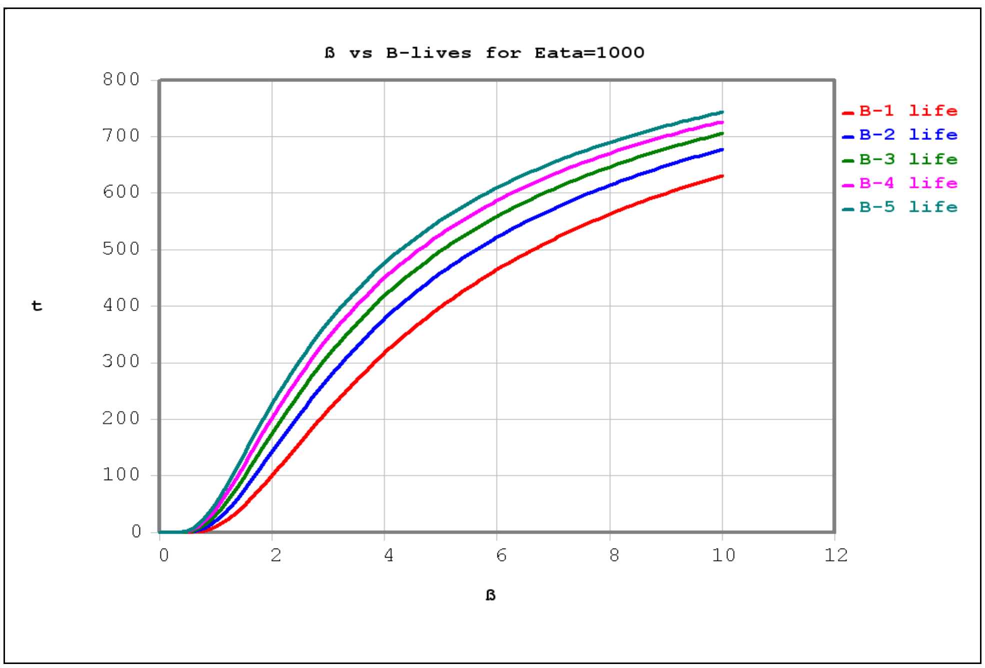

Fig. 3.1: β vs B-lives for η = 1000

The values between B-1 and B-4 lives for these two cycles differ approximately by factors of 100 when $\beta = 2$, and between B-3 and B-5 lives for these two cycles differ approximately by factors of 50 when $\beta = 2$. The values between B-1 and B-

3 lives for these two cycles differ approximately by factors of 100 when $\beta = 3$ and the values between B-2 and B-5 lives for these two cycles differ approximately by factors of 100 when $\beta = 3$. The values between B-1 and B-3 lives for these two cycles differ approximately by factors of 100 when $\beta = 3.4$, for the values between B-2 and B-5 lives. For these two cycle differ approximately by factors of 100. When $\beta = 3.4$. The values between B-1 and B-3 lives for these two cycle differ approximately by factors of 100 when $\beta = 5$ and for the values between B-2 and B-5 lives for these two cycles differ approximately by factors of 100 when $\beta = 5$. Therefore, we conclude that the values between B-lives for each 10 cycles can be approximately determined when β is the integer. We have also shown the relationship between β and various B-lives (B-l0, B-25, B-50, B-75 and B-90 lives) for $\eta = 1000$. These lives are used for censored data. For B-10 life mean this is the life for which the unit will have a failure probability of 10%.

For B-25 life mean this is the life for which the unit will have a failure probability of 25%, for B-50 life mean this is the life for which the unit will have a failure probability of 50%, for B-75 life mean this is the life for which the unit will have a failure probability of 75%. It is clear that the larger the value of β, the longer the B-lives (B-l0, B-25 and B-50 lives) for the same value of η and for larger the value of β, the smaller the B-lives (B-75 and B-90 lives) for the same value of η . For these cases of η = 1000 all the lives (B-l0, B-25, B-50, B-75 and B-90 lives) are effectively used in manufacturing technology.

The first Quartile life (25^{th} percentile) of the Weibull distribution is defined as:

$$B_{25w} = t_0 + \eta(\ln\frac{4}{3})^{\frac{1}{\beta}} \tag{3.2}$$

Quartiles are the values in the order statistics that divide the data into four equal parts. This is the life by which 25% of the units will be expected to have failed, so it is the life at which 75% of the units would be expected to still survive. We obtain that the minimum value of lower Quartile life is 0.003883 for β =0.1 and the maximum value of lower Quartile life is 882.8589 for β =10. The relationship between β and B-25 life for η = 1000 is shown in Fig. 3.2.

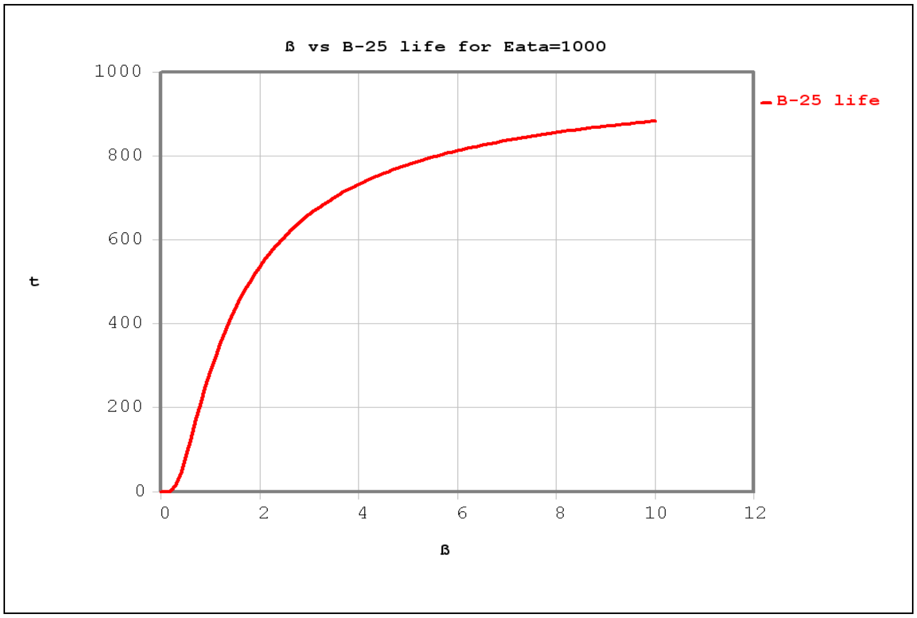

Fig. 3.2: β vs B-25 life for η = 1000

The second Quartile life (50^{th} percentile) of the Weibull distribution is defined as:

$$B_{50w} = t_0 + \eta(\ln 2)^{\frac{1}{\beta}} \tag{3.3}$$

This is the life by which 50% of the units will be expected to have failed, and so it is the life at which 50% of the units would be expected to still survive. We obtain that the minimum value of median life is 25.60086 for β=0.1 and the maximum value of median life is 964.0122 for β=10. The relationship between β and B-50 life for η = 1000 is shown in Fig. 3.3.

The upper Quartile life (75^{th} percentile) of the Weibull distribution is defined as:

$$B_{75w} = t_0 + \eta(\ln 4)^{\frac{1}{\beta}} \tag{3.4}$$

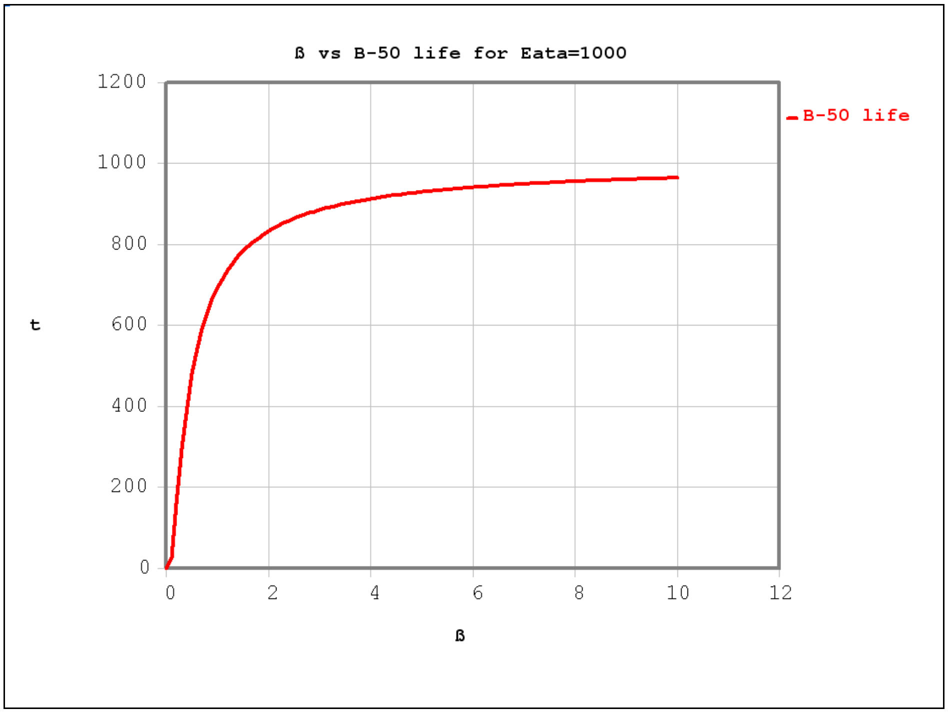

Fig. 3.3: β vs B-50 life for η = 1000

This is the life by which 75% of the units will be expected to have failed, and so it is the life at which 25% of the units would be expected to still survive. We obtain that the maximum value of upper quartile life is 26215.28 for β=0.1 and the minimum value of upper quartile life is 1033.203 for β=10. The relationship between β and B-75 life for η = 1000 is shown in Fig. 3.4.

The 90^{th} percentile life of the Weibull distribution is defined as:

$$B_{90w} = t_0 + \eta(\ln 10)^{\frac{1}{\beta}} \tag{3.5}$$

Sometimes our interest is to know the position of an observation relative to the other data set. This is the life by which 90% of the units will be expected to have failed, so it is the life at which 10% of the units would be expected to still survive. We obtain that the maximum value of the percentile life is 4189449 for β=0.1 and the minimum value of percentile life is1086.98 for β=10. The relationship between β and B-90 life for η = 1000 is shown in Fig. 3.5.

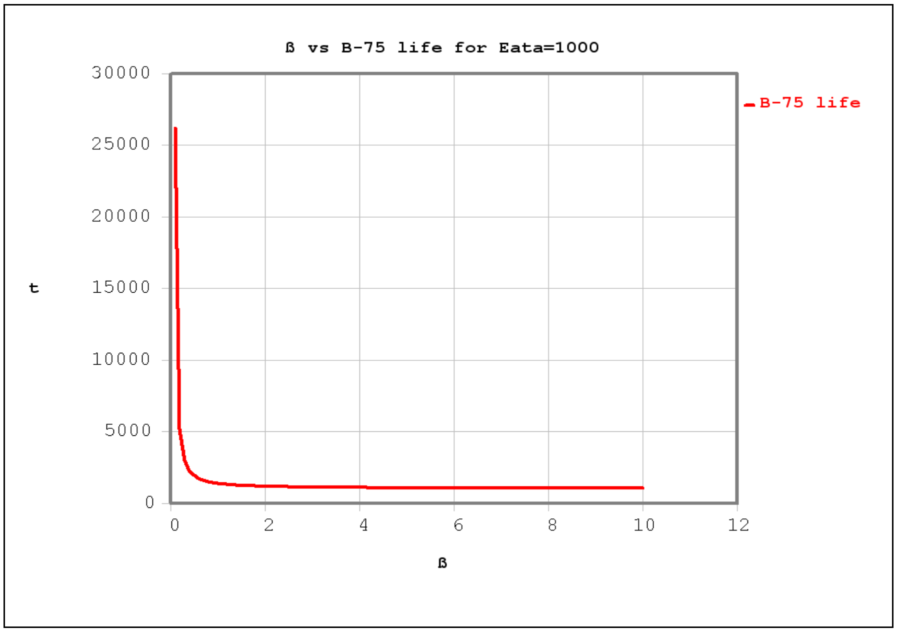

Fig. 3.4: β vs B-75 life for η = 1000

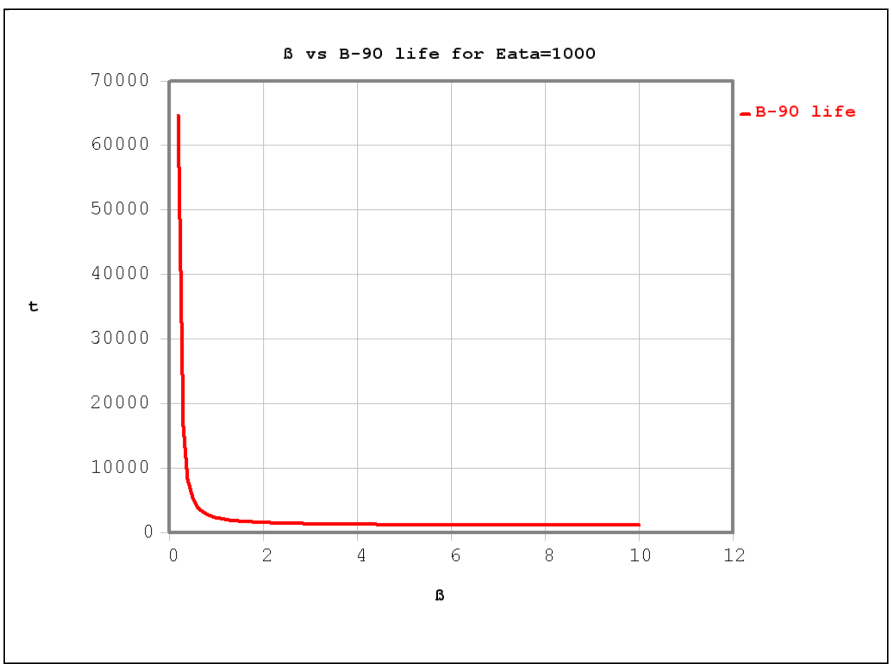

Fig. 3.5: β vs B-90 life for η = 1000

4. Measures of Variability for B-lives

A measure of variability for B-lives is defined as the numerical quantity (as percentile life) that describe the spread of the values in a set of data. It is possible that two or more set of data of the mechanical components may have the same MTTF but the vast differences in magnitude of their measure of variability would reveal that the two data sets of the components are different as MTTF and do not tell us any thing about the variation in the data set. For the percentile life the measure of variability is Quartile Deviation (Q.D). For B-10 life mean this is the life for which the unit will have a failure probability of 10%. Here are the relationships between β and various B-lives (B-l0, B-25, B-50, B-75 and B-90 lives) for η = 1000. For B-25 life mean this is the life for which the unit will have a failure probability of 25%. For B-50 life mean this is the life for which the unit will have a failure probability of 50%. For B-75 life mean this is the life for which the unit will have a failure probability of 75%. For B-90 life mean this is the life for which the unit will have a failure probability of 90%. It is clear that larger the value of β, the longer the (B-l0, B-25 and B-50) lives for the same value of η and for B-75 and B-90 larger the value of β, the smaller the B-lives for the same value of η.

The Quartile Deviation ($Q.D_W$) life of the Weibull distribution is defined as:

$$Q.D_w = \frac{B_{75w} - B_{25w}}{2} \qquad (4.1)$$

$Q.D_W$ is the measure that has positive or zero values. The minimum value of the $Q.D_W$ shows that a small amount of variability in the set of data, whereas the large values indicate the more variability in the life time data. We obtain the maximum value 13107.64 for β=0.1 and the minimum value of quartile deviation life 75.17194 for β=10. The relationship between β and $Q.D_W$ life for η = 1000 is shown in Fig. 4.1. We see that larger the value of β the smaller the value of $Q.D_W$.

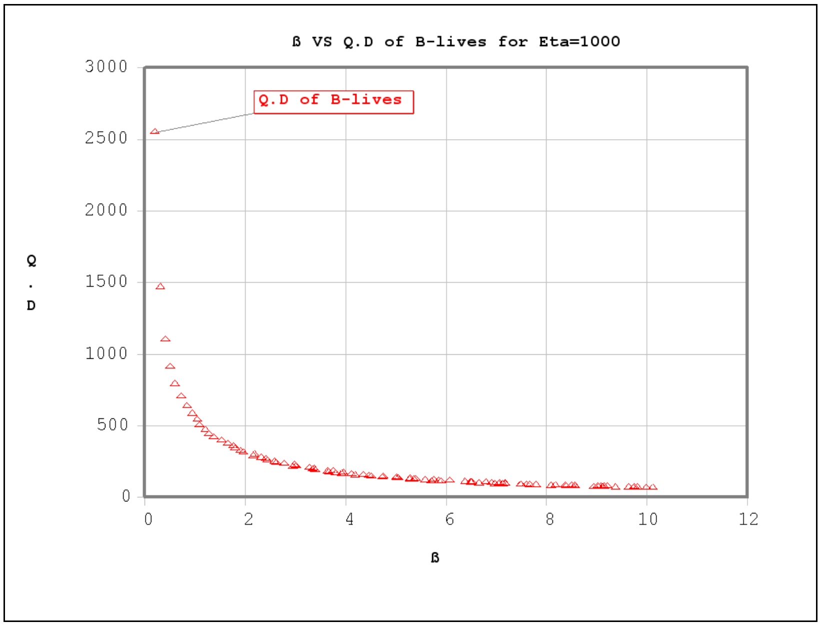

Fig. 4.1: β vs $Q.D_W$

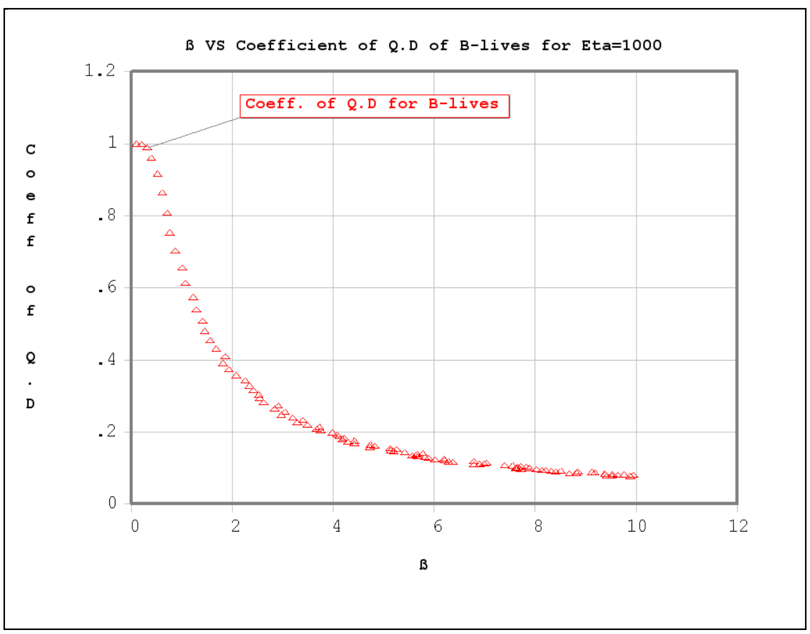

Fig. 4.2: β vs Coeff. of $Q.D_W$

The Coefficient of $Q.D_W$ life of the Weibull distribution is defined as:

$$\text{Coeff. of } Q.D_w = \frac{B_{75w} - B_{25w}}{B_{75w} + B_{25w}} \tag{4.2}$$

The Coeff. of $Q.D_W$ is the measure that is used to compare the variability of two or more set of life time data. It will take the same value for two or more populations if in each population the standard deviation is directly proportional to the mean. In these situations, we say that two or more populations are consistent. We obtain coeff. of $Q.D_W$ =1 for β=0.1 and obtain the minimum value of Coeff. of $Q.D_W$ life is 0.078465 for β=10. The relationship between β and Coeff. of $Q.D_W$ life for η = 1000 is shown in Fig. 4.2. We see that larger the value of β the smaller the value of Coeff. of $Q.D_W$.

The coefficient of skewness (SK_W) is defined as:

$$SK_w = \frac{B_{25w} + B_{75w} - 2B_{50w}}{B_{75w} - B_{25w}} \tag{4.3}$$

It is a pure number and lies between -1 and +1. For symmetrical distribution its value is zero. SK_W is the quantity used to measure the skewness of the distribution. If SK_W < 0, then the distribution is skewed to the left (Mean < Median < Mode); if SK_W = 0, then the distribution is symmetrical (Mean = Median = Mode) as in the normal distribution, and if SK_W > 0, then the distribution is skewed to the right (Mean > Median > Mode). The relationship between β and SK_W is shown in Fig. 4.3.

The Percentile coefficient of kurtosis (K_W) is defined as:

$$K_w = \frac{Q.D_w}{B_{90w} - B_{10w}} \tag{4.4}$$

K_W is the quantity which can be used to measure the kurtosis or peakedness of the symmetrical distribution. K_W = 3 represents the peakedness of the normal distribution for the percentile coefficient of kurtosis. If K_W > 3, then the Weibull PDF shape is more peaked than a normal PDF for the percentile coefficient of kurtosis.

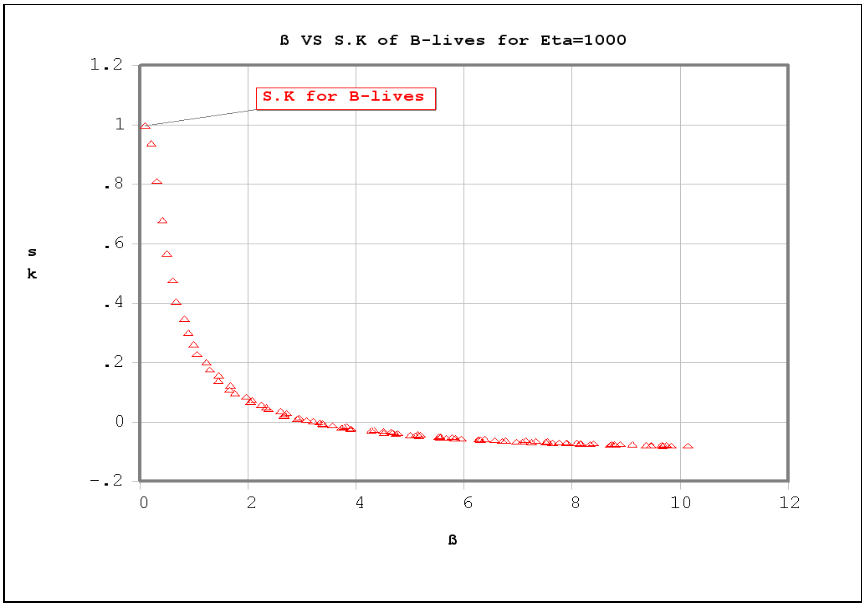

Fig. 4.3: β vs SK_W

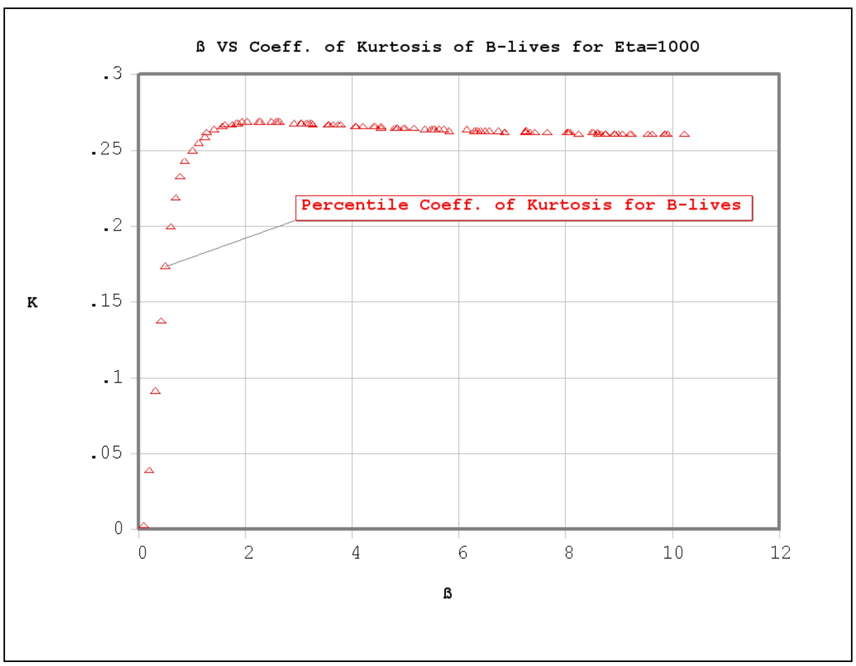

Fig. 4.4: β vs K_W

If K_W <3, then the Weibull PDF shape is flatter than the normal PDF for the percentile coefficient of kurtosis. The relationship between β and K_W is shown in Fig. 4.4. Here we note that as $\beta \to \infty$ then K_W has a maximum value asymptotically. Hence, β and K_W have a positive proportion when $\beta > 0.1$.

5. Summary and Conclusions

In this paper we have seen that the Weibull distribution is the flexible distribution that approaches to different distributions when its shape parameter changes. The Quantile comprehensive study of the Weibull Quantile modeling is predicted for finding the life time of the electrical and mechanical components. These properties of the Weibull distribution for quantile analysis are used as B-life in engineering terminology. These patterns of β and various B-lives are helpful for finding the life of components. In this paper we have also presented measure of variability for B-lives as the numerical quantities that describe the spread of the values in a set of data. Here we simulate these Quantiles models graphically and mathematically. This paper also proves the flexibility of Weibull distribution that approaches to different distributions.

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Appendix

Table A: Relationship b/w β VS B-lives for η = 1000

β	B-1life	B-2 life	B-3 life	B-4 life	B-5 life
0.1	1.05E-17	1.13E-14	6.87E-13	1.29E-11	1.26E-10
0.2	1.03E-07	3.37E-06	2.62E-05	0.000113	0.000355
0.3	0.000219	0.002246	0.008825	0.023423	0.050142
0.4	0.010126	0.058013	0.161919	0.336694	0.59587
0.5	0.101009	0.408149	0.927763	1.666435	2.631002
0.6	0.468059	1.498589	2.970738	4.839759	7.081104
0.7	1.399498	3.794517	6.82156	10.36485	14.36248
0.8	3.182187	7.616614	12.72472	18.34923	24.41045
0.9	6.02838	13.09549	20.66536	28.61206	36.87517
1	10.05034	20.20271	30.45921	40.82199	51.29329
1.1	15.26865	28.80469	41.83718	54.59798	67.19358
1.2	21.63468	38.71162	54.50448	69.56837	84.1493
1.3	29.05466	49.71288	68.17571	85.39985	101.7979
1.4	37.40986	61.59965	82.59274	101.8079	119.8436
1.5	46.57152	74.17765	97.53169	118.5576	138.0513
1.6	56.41088	87.27322	112.8039	135.4593	156.2384
1.7	66.80548	100.7354	128.2537	152.3628	174.2658
1.8	77.64264	114.4355	143.7545	169.151	192.0291
1.9	88.82109	128.2656	159.2052	185.7341	209.452
2	100.2514	142.1362	174.5257	202.0445	226.4802
2.1	111.8555	155.974	189.6537	218.0324	243.0766
2.2	123.5664	169.7195	204.5414	233.6621	259.2173
2.3	135.3265	183.3249	219.1529	248.9092	274.8886
2.4	147.0873	196.7527	233.4619	263.7582	290.085
2.5	158.8079	209.9732	247.45	278.2004	304.8065
2.6	170.4543	222.9638	261.1048	292.2325	319.0579
2.7	181.9981	235.7078	274.4191	305.8556	332.847
2.8	193.4163	248.1928	287.3895	319.0735	346.1843
2.9	204.6901	260.4105	300.016	331.8929	359.0817
3	215.8043	272.3557	312.3006	344.322	371.5525
3.1	226.7473	284.0259	324.2477	356.3702	383.6105
3.2	237.5097	295.4204	335.8629	368.048	395.2701
3.3	248.0848	306.5404	347.1529	379.3664	406.5456
3.4	258.4676	317.3884	358.1252	390.3368	417.4515
3.5	268.6547	327.9679	368.788	400.9707	428.0021

3.6	278.6443	338.2832	379.1497	411.2797	438.2112
3.7	288.4356	348.3394	389.2192	421.2753	448.0926
3.8	298.0287	358.1419	399.0052	430.9688	457.6593
3.9	307.4246	367.6966	408.5168	440.3713	466.9241
4	316.625	377.0096	417.7627	449.4936	475.8994
4.1	325.6319	386.0871	426.7517	458.3463	484.5969
4.2	334.4481	394.9354	435.4924	466.9394	493.028
4.3	343.0764	403.561	443.9933	475.2829	501.2035
4.4	351.5201	411.9702	452.2625	483.3861	509.1338
4.5	359.7826	420.1693	460.3081	491.2582	516.8289
4.6	367.8675	428.1646	468.1377	498.908	524.2982
4.7	375.7786	435.9622	475.7589	506.3439	531.5508
4.8	383.5196	443.5681	483.179	513.5739	538.5954
4.9	391.0945	450.9882	490.4049	520.6058	545.4401
5	398.5071	458.2283	497.4435	527.447	552.0928
5.1	405.7614	465.2939	504.3011	534.1046	558.5611
5.2	412.8611	472.1905	510.9841	540.5854	564.8521
5.3	419.8102	478.9233	517.4986	546.8958	570.9725
5.4	426.6124	485.4975	523.8502	553.0421	576.929
5.5	433.2715	491.9179	530.0447	559.0303	582.7276
5.6	439.7912	498.1895	536.0872	564.866	588.3743
5.7	446.1751	504.3168	541.9831	570.5546	593.8747
5.8	452.4269	510.3043	547.7371	576.1015	599.2343
5.9	458.5498	516.1564	553.3541	581.5115	604.4581
6	464.5475	521.8771	558.8386	586.7895	609.5511
6.1	470.4231	527.4706	564.1951	591.9401	614.5179
6.2	476.1799	532.9408	569.4276	596.9675	619.363
6.3	481.8211	538.2913	574.5402	601.876	624.0907
6.4	487.3497	543.5259	579.5368	606.6696	628.7051
6.5	492.7688	548.648	584.4212	611.3521	633.21
6.6	498.0811	553.6609	589.1968	615.9273	637.6093
6.7	503.2896	558.568	593.8672	620.3986	641.9064
6.8	508.3971	563.3723	598.4356	624.7694	646.1049
6.9	513.406	568.077	602.9053	629.043	650.2079
7	518.3191	572.6848	607.2792	633.2225	654.2187
7.1	523.1389	577.1986	611.5602	637.3108	658.1402
7.2	527.8677	581.6212	615.7513	641.3109	661.9752
7.3	532.508	585.9551	619.8552	645.2255	665.7267
7.4	537.062	590.2028	623.8743	649.0572	669.3972
7.5	541.5319	594.3669	627.8113	652.8086	672.9892
7.6	545.92	598.4496	631.6686	656.4822	676.5052
7.7	550.2283	602.4532	635.4485	660.0801	679.9475
7.8	554.4588	606.3799	639.1532	663.6048	683.3185

7.9	558.6135	610.2319	642.7849	667.0584	686.6201
8	562.6944	614.0111	646.3456	670.4429	689.8546
8.1	566.7032	617.7195	649.8375	673.7604	693.0239
8.2	570.6417	621.3591	653.2624	677.0128	696.1299
8.3	574.5117	624.9316	656.6221	680.2019	699.1746
8.4	578.3149	628.4389	659.9185	683.3296	702.1595
8.5	582.0529	631.8826	663.1533	686.3977	705.0865
8.6	585.7272	635.2645	666.3282	689.4076	707.9573
8.7	589.3395	638.5861	669.4448	692.3611	710.7733
8.8	592.8913	641.8491	672.5047	695.2597	713.5361
8.9	596.3839	645.0548	675.5093	698.1049	716.2473
9	599.8188	648.2047	678.4601	700.8981	718.9081
9.1	603.1973	651.3003	681.3585	703.6408	721.5201
9.2	606.5208	654.3429	684.2059	706.3342	724.0844
9.3	609.7906	657.3338	687.0036	708.9797	726.6024
9.4	613.0078	660.2743	689.7528	711.5785	729.0753
9.5	616.1738	663.1657	692.4548	714.1317	731.5043
9.6	619.2896	666.0091	695.1108	716.6407	733.8906
9.7	622.3565	668.8057	697.7219	719.1065	736.2352
9.8	625.3755	671.5566	700.2892	721.5302	738.5392
9.9	628.3477	674.2629	702.8138	723.9128	740.8036
10	631.2742	676.9256	705.2967	726.2555	743.0295

Table B: Relationship b/w β VS B-lives of Q.D., Coeff. of Q.D., Coeff. of SK & Coeff. of Kurtosis for η = 1000

β	Q.D.	Coeff.of Q.D.	CK	Kurtosis(K)
0.1	13107.64	1	0.998047	3.13E-03
0.2	2559.058	0.999231	0.938246	3.95E-02
0.3	1477.467	0.989474	0.811157	9.17E-02
0.4	1109.186	0.96152	0.679392	1.38E-01
0.5	919.5255	0.917428	0.567503	1.74E-01
0.6	799.0945	0.864388	0.477511	2.00E-01
0.7	712.9668	0.808692	0.405684	2.19E-01
0.8	646.7795	0.75429	0.347893	2.33E-01
0.9	593.52	0.703214	0.300789	2.43E-01
1	549.3061	0.656289	0.26186	2.50E-01
1.1	511.776	0.61367	0.229255	2.55E-01
1.2	479.3847	0.575175	0.201613	2.59E-01
1.3	451.0643	0.540471	0.177917	2.62E-01
1.4	426.0439	0.509178	0.157402	2.64E-01
1.5	403.748	0.480918	0.139483	2.66E-01
1.6	383.7347	0.45534	0.123707	2.67E-01

1.7	365.6577	0.432128	0.109717	2.67E-01
1.8	349.24	0.411001	0.097231	2.68E-01
1.9	334.2573	0.391715	0.086023	2.68E-01
2	320.525	0.374058	0.075908	2.69E-01
2.1	307.8899	0.357845	0.066736	2.69E-01
2.2	296.2233	0.342917	0.058382	2.69E-01
2.3	285.4163	0.329133	0.050742	2.69E-01
2.4	275.3763	0.316373	0.043729	2.69E-01
2.5	266.0234	0.304531	0.03727	2.69E-01
2.6	257.2887	0.293517	0.031303	2.69E-01
2.7	249.1124	0.283248	0.025772	2.69E-01
2.8	241.4422	0.273654	0.020634	2.68E-01
2.9	234.2323	0.264673	0.015846	2.68E-01
3	227.442	0.256248	0.011376	2.68E-01
3.1	221.0357	0.248332	0.007192	2.68E-01
3.2	214.9815	0.24088	0.003268	2.68E-01
3.3	209.2511	0.233854	-0.00042	2.68E-01
3.4	203.8189	0.227219	-0.00389	2.67E-01
3.5	198.6623	0.220943	-0.00716	2.67E-01
3.6	193.7608	0.215	-0.01026	2.67E-01
3.7	189.0957	0.209363	-0.01318	2.67E-01
3.8	184.6504	0.204009	-0.01596	2.67E-01
3.9	180.4097	0.198919	-0.01859	2.67E-01
4	176.3596	0.194074	-0.02109	2.66E-01
4.1	172.4877	0.189456	-0.02346	2.66E-01
4.2	168.7823	0.18505	-0.02573	2.66E-01
4.3	165.2329	0.180842	-0.02789	2.66E-01
4.4	161.83	0.176819	-0.02995	2.66E-01
4.5	158.5645	0.172969	-0.03192	2.66E-01
4.6	155.4283	0.169282	-0.0338	2.65E-01
4.7	152.4139	0.165747	-0.03561	2.65E-01
4.8	149.5142	0.162356	-0.03734	2.65E-01
4.9	146.7229	0.159099	-0.03899	2.65E-01
5	144.0341	0.15597	-0.04059	2.65E-01
5.1	141.442	0.15296	-0.04212	2.65E-01
5.2	138.9417	0.150063	-0.04359	2.65E-01
5.3	136.5283	0.147273	-0.045	2.64E-01
5.4	134.1973	0.144585	-0.04636	2.64E-01
5.5	131.9446	0.141992	-0.04768	2.64E-01
5.6	129.7664	0.139489	-0.04894	2.64E-01
5.7	127.6589	0.137073	-0.05017	2.64E-01
5.8	125.6188	0.134739	-0.05135	2.64E-01

5.9	123.643	0.132482	-0.05249	2.64E-01
6	121.7283	0.130299	-0.05359	2.64E-01
6.1	119.8721	0.128187	-0.05465	2.63E-01
6.2	118.0716	0.126142	-0.05568	2.63E-01
6.3	116.3245	0.12416	-0.05668	2.63E-01
6.4	114.6283	0.12224	-0.05765	2.63E-01
6.5	112.9809	0.120378	-0.05859	2.63E-01
6.6	111.3802	0.118571	-0.0595	2.63E-01
6.7	109.8242	0.116817	-0.06038	2.63E-01
6.8	108.3111	0.115115	-0.06123	2.63E-01
6.9	106.8392	0.113461	-0.06206	2.63E-01
7	105.4067	0.111854	-0.06287	2.63E-01
7.1	104.0121	0.110291	-0.06365	2.62E-01
7.2	102.654	0.108772	-0.06442	2.62E-01
7.3	101.3309	0.107293	-0.06516	2.62E-01
7.4	100.0414	0.105854	-0.06588	2.62E-01
7.5	98.78442	0.104453	-0.06658	2.62E-01
7.6	97.5586	0.103089	-0.06727	2.62E-01
7.7	96.36283	0.101759	-0.06793	2.62E-01
7.8	95.19602	0.100463	-0.06858	2.62E-01
7.9	94.05714	0.0992	-0.06921	2.62E-01
8	92.94519	0.097968	-0.06983	2.62E-01
8.1	91.85923	0.096766	-0.07043	2.62E-01
8.2	90.79835	0.095593	-0.07102	2.62E-01
8.3	89.7617	0.094449	-0.07159	2.62E-01
8.4	88.74847	0.093331	-0.07215	2.61E-01
8.5	87.75785	0.092239	-0.0727	2.61E-01
8.6	86.7891	0.091172	-0.07323	2.61E-01
8.7	85.84151	0.09013	-0.07375	2.61E-01
8.8	84.9144	0.089111	-0.07426	2.61E-01
8.9	84.0071	0.088115	-0.07476	2.61E-01
9	83.11898	0.087141	-0.07524	2.61E-01
9.1	82.24944	0.086189	-0.07572	2.61E-01
9.2	81.39792	0.085256	-0.07618	2.61E-01
9.3	80.56384	0.084344	-0.07664	2.61E-01
9.4	79.74669	0.083451	-0.07708	2.61E-01
9.5	78.94595	0.082576	-0.07752	2.61E-01
9.6	78.16113	0.08172	-0.07795	2.61E-01
9.7	77.39176	0.080881	-0.07837	2.61E-01
9.8	76.63739	0.08006	-0.07878	2.61E-01
9.9	75.89759	0.079254	-0.07918	2.61E-01
10	75.17194	0.078465	-0.07957	2.61E-01