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Product, Ratio and Single Moments of Lower Record Values of Inverse Weibull Distribution

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Abstract:

In this paper the product, ratio and single moments of the Lower Record values are obtained from Inverse Weibull distribution.

Key Words:

Moments, Recurrence Relations, Lower Record values, Inverse Weibull distribution.

1. Introduction:

A random variable X has an Inverse Weibull distribution with pdf given by

$$f(x) = \frac{m}{\theta x^{m+1}} \exp\left(-\frac{1}{\theta x^{m}}\right) \qquad (1.1)$$

where $x>0$, $(\theta, m)>0$

And the corresponding cdf is given by

$$F(x) = \exp\left(-\frac{1}{\theta x^{m}}\right) \qquad (1.2)$$

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where x>0, (θ, m)>0

If we put m=1, it reduces to the Inverse Exponential distribution. If we put m=2, it reduces to the Inverse Rayleigh distribution. Some work has been done on Inverse Rayleigh distribution by Voda (1972), Ghanaph (1993), Mukargee & Saren (1984) and Mukarjee & Mait (1996). For distributional properties of Record values of inverse weibull distribution see Aleem and Pasha (2003), Aleem(2004). Other references are Ahsanullah (1995), Ahsanullah and Novzorov (2001).

Let v(x) = $\frac{1}{\theta x^m}$ and from (1.1) (1.2), we have

$$F(x) = \frac{1}{v^{\bullet}(x)} . f(x)$$

2. Product Moments

The lower record values are respected by $X_{L(1)}$, $X_{L(2)}$, ---, $X_{L(n)}$

The joint pdf of $X_{L(r)}$ and $X_{L(s)}$ (s>r) is

$$f_{(r).(s)}(x,y) = C_{r,s} [H(x)]^{r-1} [H(y) - H(x)]^{s-r-1} h(x) . f(y) \quad (2.1)$$

where $C_{r,s} = - \frac{1}{(r-1)!(s-r-1)!}$ $\qquad -\infty < y < x < \infty$

and $\qquad H(x) = - \text{Ln}F(x) \qquad 0 < F(x) < 1$

$$h(x) = - \frac{d}{dx} H(x)$$

If g is a Borel measurable function from R^2 to R, then

$$E\{g(X_{L(r)}, X_{L(s)})\} = C_{r,s} \iint_{0<y<x<\infty} g(x,y)[H(x)]^{r-1}[H(y) - H(x)]^{s-r-1} h(x) f(y) dxdy \quad (2.2)$$

<u>Theorem 2.1 :</u>

For the distribution function F(x) in (1.2), we have

$$\mathrm{E}\{g(X_{L(r)}, X_{L(s)})\} = E\{u(X_{L(r-1)}, X_{L(s-1)})\} - E\{u(X_{L(r)}, X_{L(s-1)})\}$$

Where $u^{\bullet}$(x,y) = $\frac{\partial}{\partial x}u(x,y) = g(x,y), v^{\bullet}\ (x)$

And $v^{\bullet}(x) = \left|\frac{\partial}{\partial x}v(x)\right|$

Proof: Using (1.3) in (2.2), we have

$$\mathrm{E}\{g(X_{L(r)}, X_{L(s)})\} = C_{r,s} \iint_{0<y<x<\infty} u^{\bullet}(x,y)[H(x)]^{r-1}[H(y)-H(x)]^{s-r-1}$$

f(y)
dxdy

Integrating RHS w.r.t "x", we get as :

$$= C_{r-1,s-1} \iint_{0<y<x<\infty} u(x,y)[H(x)]^{r-2}[H(y)-H(x)]^{s-r-1} h(x) f(y) dxdy$$

$$- C_{r,s-1} \iint_{0<y<x<\infty} u(x,y)[H(x)]^{r-1}[H(y)-H(x)]^{s-r-2} h(x) f(y) dxdy$$

and

$$\mathrm{E}\{g(X_{L(r)}, X_{L(s)})\} = E\{u(X_{L(r-1)}, X_{L(s-1)})\} - E\{u(X_{L(r)}, X_{L(s-1)})\}$$

Hence the Theorem

Theorem 2.2:

For the distribution function F(x) in (1.2) the recurrence relation for the product Moments of Inverse Weibull distribution is given by:

$$u_{(r),(s)}^{J,K} = \frac{m}{\theta(J-m)}[u_{(r-1),(s-1)}^{(J-m),K} - u_{(r),(s-1)}^{(J-m),K}]$$

Proof: Now $v(x) = \frac{1}{\theta x^m}$ and g(x,y) = $x^J y^K$ This gives

$u(x,y) = \frac{m}{\theta(J-m)} x^{J-m} y^K$, putting in Theorem (2.1), we get the required recurrence relation.

Note, For the ratio, let K= **- j**, then $u_{(r),(s)}^{J,-J} = E\left(X_{(r)} / X_{(s)}\right)^J \quad \forall J$

3. Single Moments

The lower record values are represented by X $_{L(1)}$, X $_{L(2)}$, ---, X $_{L(N)}$. The pdf of X $_{L(n)}$ $(n \geq 2)$ is

$$f_{(n)}(x) = \frac{[H(x)]^{n-1}}{(n-1)!} f(x) \qquad (3.1)$$

where H(x) = -LnF(x) 0<F(x)<1

h(x) = - $\frac{d}{dx} H(x)$

If g is a Borel measurable function from R ^{2} to R, then

$$E\{g(X_{L(n)})\} = C_n \int_{0<x<\infty} g(x)[H(x)]^{n-1} f(x) dx \qquad (3.2)$$

0<x< ∞

where $C_n = \frac{1}{(n-1)!}$

Theorem 3.1:

For the distribution function F(x) in (1.2), we have

$$E\{g(X_{L(n)})\} = E\{u(x_{L(n-1)})\} - E\{u(x_{L(n)})\}$$

Proof: Using (1.3) in (3.2), we have

$$E\{g(X_{L(n)})\} = C_n \int_{0<x<\infty} u^{\bullet}(x)[H(x)]^{n-1} F(x)dx$$

Integrating RHS, we get

$$= C_{n-1} \int_{0<x<\infty} u(x)[H(x)]^{n-2} f(x)dx - C_n \int_{0<x<\infty} u(x)[H(x)]^{n-1} f(x)dx$$

and

$$E\{g(X_{L(n)})\} = E\{u(x_{L(n-1)})\} - E\{u(x_{L(n)})\}$$

Hence the Theorem

Theorem 3.2:

For the distribution function F(x) in (1.2) the recurrence relation for the single moments of Inverse Weibull Distribution is given by

$$u_{(n)}^{J} = \frac{m}{\theta(J-m)} \left[u_{(n-1)}^{J-m} - u_{(n)}^{J-m}\right]$$

Proof: Now $v(x) = \frac{1}{\theta x^m}$ and $g(X_{L(n)}) = x^J$ this gives

$u(X_{L(n)}) = \frac{m}{\theta(J-m)} x^{J-m}$, putting in (3.1) we get the recurrence relation.

Remark: This recurrence relation in Theorem (2.2) between the moments of ratio of two lower record values, quasi-ranges, joint moment generating function, characteristic functions, whenever they exist can be obtained by setting respectively g(x,y) equal to

$$\left(x^{J}y^{-K}\right),\left(y-x\right),e^{T(X+Y)},e^{iT(X+Y)}.$$

Note: All the above results goes for single lower record values if one replace the function g(.,.) by a function of single variable g(.).

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