

## FORECAST MODELS FOR IMPORTS AND EXPORTS USING MULTIVARIATE TIME SERIES

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**ABSTRACT:** In this paper we have obtained the models to forecast imports and exports of Pakistan. These models have been obtained by using Multivariate Auto-regressive Integrated Moving Average (MARIMA) technique. The suitability of models has been decided on the basis of autocorrelations and partial autocorrelations. Finally the conclusions and recommendations have been given.

**Key Words:** Imports, Exports, Autocorrelations, Multivariate Auto-regressive Integrated Moving Average (MARIMA).

### 1. INTRODUCTION

The economies of India and Pakistan were complementary before independence. The areas that came in the share of Pakistan were supplying raw material to India. There was a trade deadlock with India in September, 1949 when Pakistan refused to devalue its currency and diversifying its exports to other countries. The assessment of a decade shows that exports increased from about 7% in 1950-51 to about 14% in 1960-61. In decade 1961-1970, exports were less as compared to imports. During 1965-66, the value of Pakistan's visible exports raised, in spite of Indo-Pak war. There would be a further increase in the country's exports during 1966-67. Imports during 1965-67 declined. Political disturbances and strikes the position in the remaining period of 1968-69 was not as good.

In 1972-73, Pakistan had a surplus balance of trade after 21 successive years of deficit. This surplus was achieved partly as a result of a sharp increase in the volume and value of exports and partly due to slower increase in imports. The export performance in the last five years had been satisfactory, in spite of international recession in the developed countries. The value of imports had been growing fast during the last five years, due to world wide inflation. During decades of 80's and 90's exports and imports almost showed same behavior. Positive change in exports brought good impact on economy but due to increased Imports no remarkable

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improvement in the balance of payment witnessed. A study about the market of Pakistan has also been conducted by Ehsan and Rosser (1995).

We have used data from Economic surveys of Pakistan for year 2001.

## 2. Multivariate Auto-regressive Integrated Moving Average (VARIMA) Models

The multivariate form of the Box-Jenkins (1970) univariate models has sometimes called the ARMAV model, for Auto-Regressive Moving Average Vector or simply vector ARMA process.

The ARMAV model for a stationary multivariate time series, with a zero mean vector, is of the form

$$\mathbf{x}_t = \phi_1 \mathbf{x}_{t-1} + \phi_2 \mathbf{x}_{t-2} + \dots + \mathbf{a}_t - \theta_1 \mathbf{a}_{t-1} - \theta_2 \mathbf{a}_{t-2} - \dots - \theta_q \mathbf{a}_{t-q} \quad (2.1)$$

The estimation of the matrix parameters and covariance matrix is complicated and very difficult without computer software. The estimation of the Moving Average matrices is especially an ordeal. If we opt to ignore the MA component(s) we are left with the VAR(p) model given by:

$$\mathbf{x}_i = \phi_1 \mathbf{x}_{i-1} + \phi_2 \mathbf{x}_{i-2} + \dots + \phi_p \mathbf{x}_{i-p} + \mathbf{a}_i \quad (2.2)$$

A model with  $p$  autoregressive matrix parameters is an VAR(p) model or a vector AR model.

The parameter matrices may be estimated by multivariate least squares, but there are other methods such as maximum likelihood estimation.

If the VAR(p) model is written in structural form as

$$\mathbf{Y} = \mathbf{BZ} + \mathbf{E} \quad (2.3)$$

then the following LS estimator can be obtained

$$\hat{\mathbf{B}} = \mathbf{YZ}'(\mathbf{ZZ}')^{-1} \quad (2.4)$$

An alternate estimator can be shown to be

$$\hat{\boldsymbol{\beta}} = \text{vec}(\hat{\mathbf{B}}') = \left( \mathbf{I}_k \otimes (\mathbf{ZZ}')^{-1} \mathbf{Z} \right) \text{vec}(\mathbf{Y}') \quad (2.5)$$

It can be shown that the LS estimator is normal, and consistent

$$\sqrt{T} \left( \hat{\boldsymbol{\beta}} - \boldsymbol{\beta} \right) \xrightarrow{\text{asy}} \mathbf{N} \left( \mathbf{0}, \boldsymbol{\Sigma}_E \otimes \left( \text{plim} \frac{\mathbf{ZZ}'}{T} \right)^{-1} \right) \quad (2.6)$$

whereas the LS estimator for the covariance matrix of the errors  $\boldsymbol{\Sigma}_E$  is

consistent as well.

### 3. ANALYSIS

In this section the analysis of data has been carried out. The analysis has been carried out by using EVIEW package. We have first obtained the autocorrelations and partial autocorrelations of imports and exports. These are given for various categories as under:

Figure 1: ACF and PACF of Total Exports

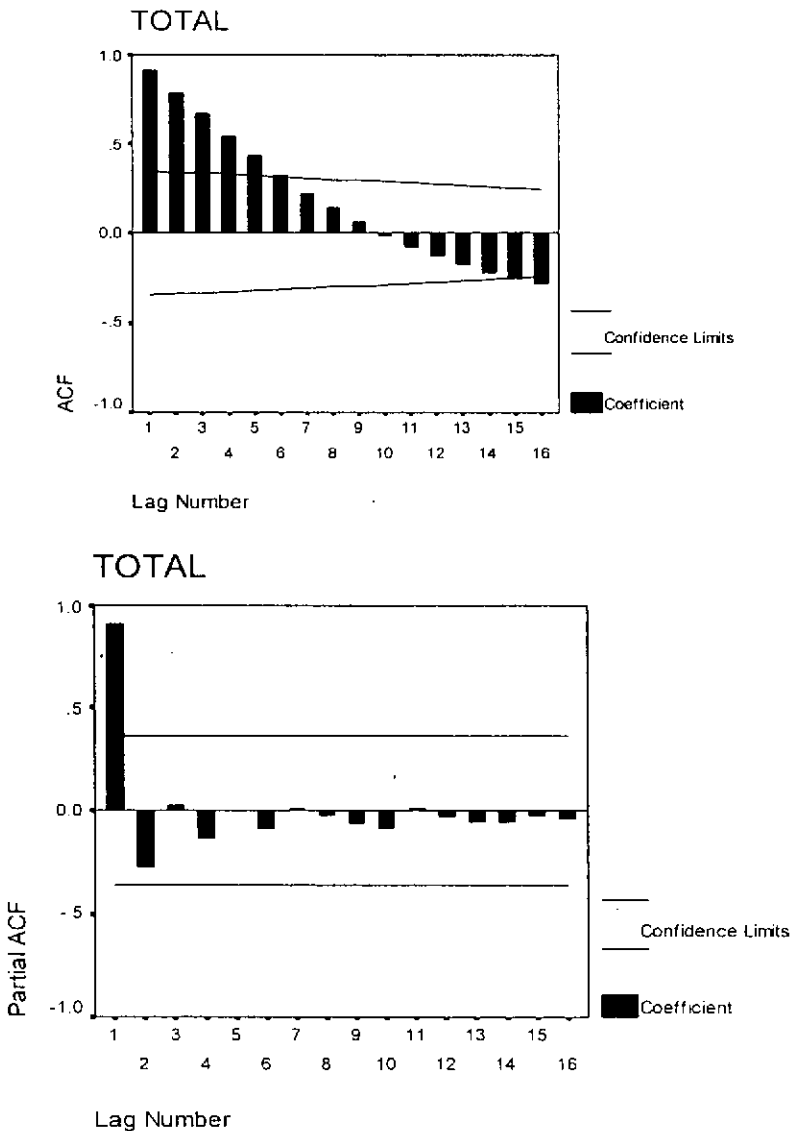


Figure 2: ACF and PACF of Primary Commodities

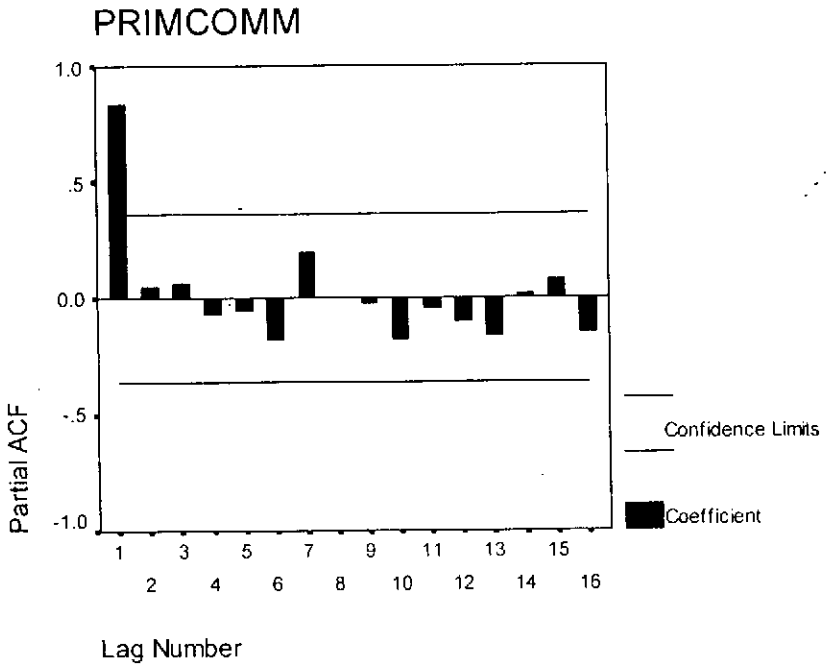
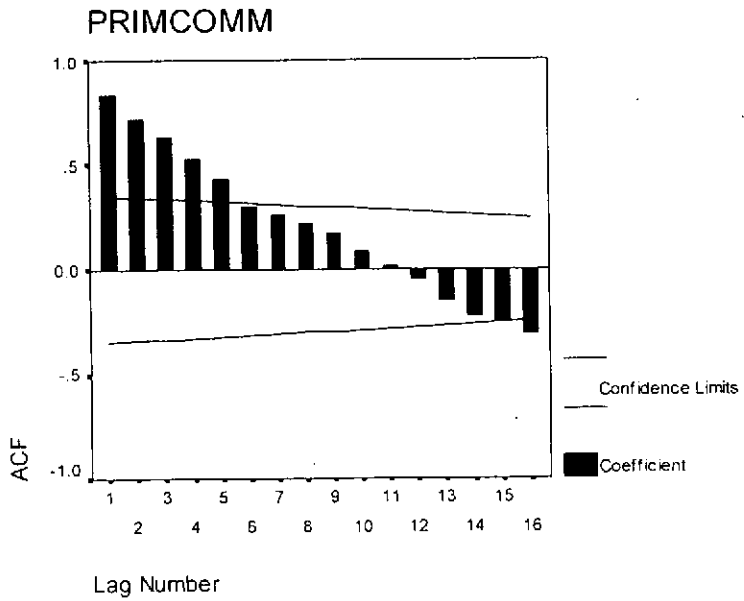


Figure 3: ACF and PACF of Semi-manufactured Commodities

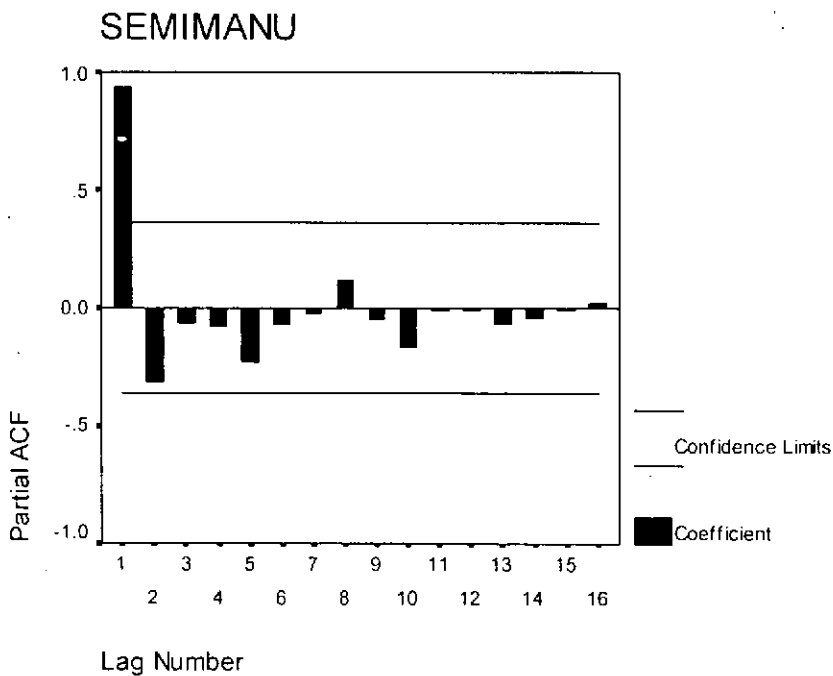
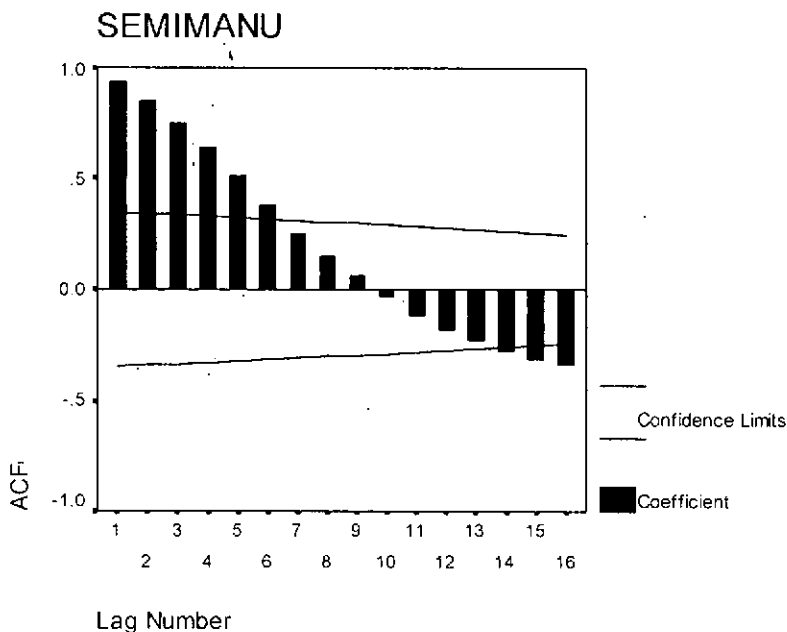


Figure 4: ACF and PACF of Total Imports

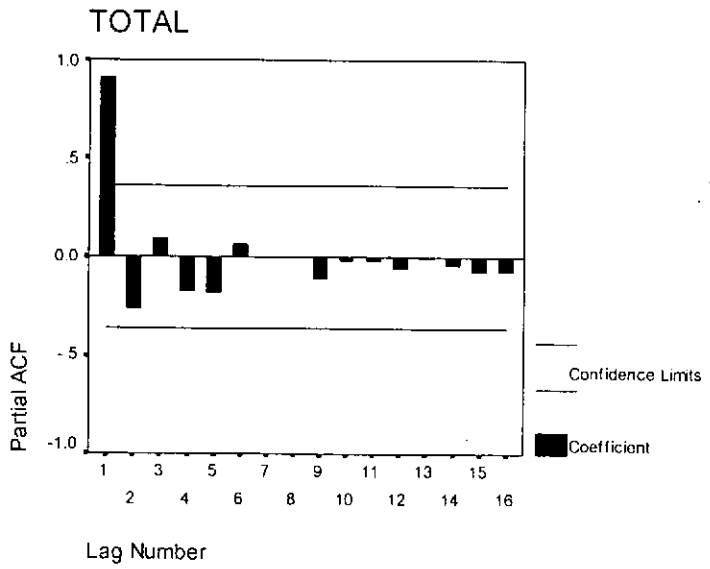
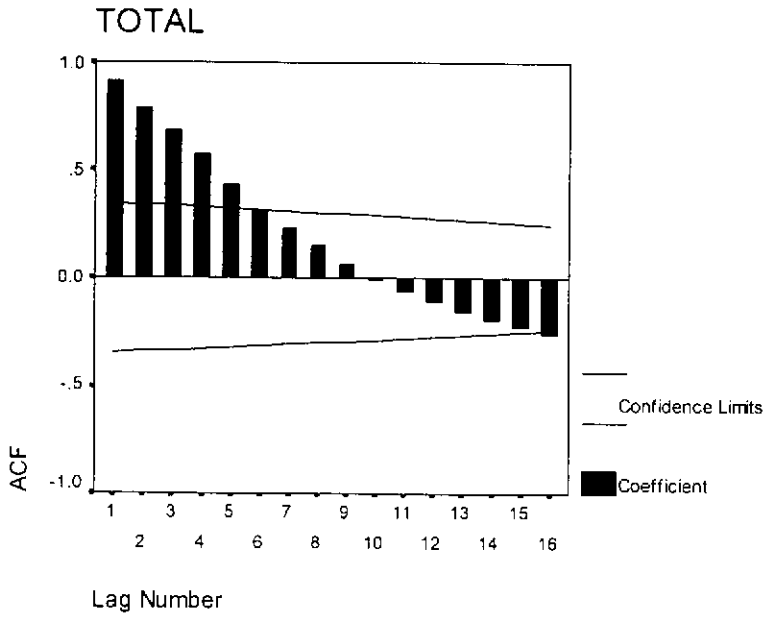


Figure 5: ACF and PACF of Consumer Goods

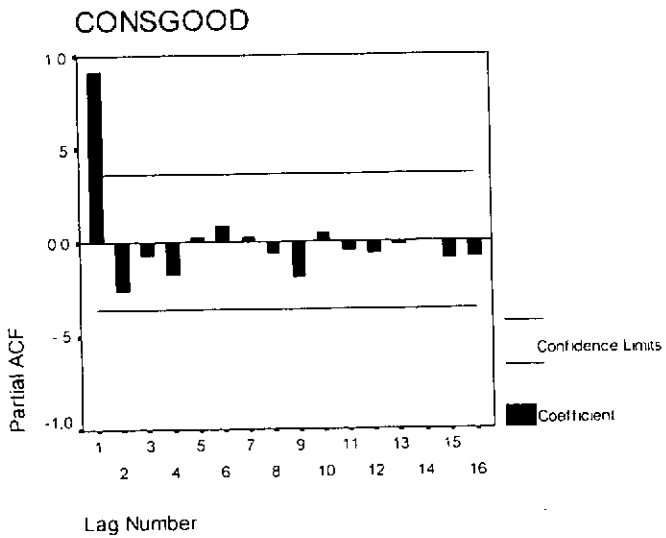
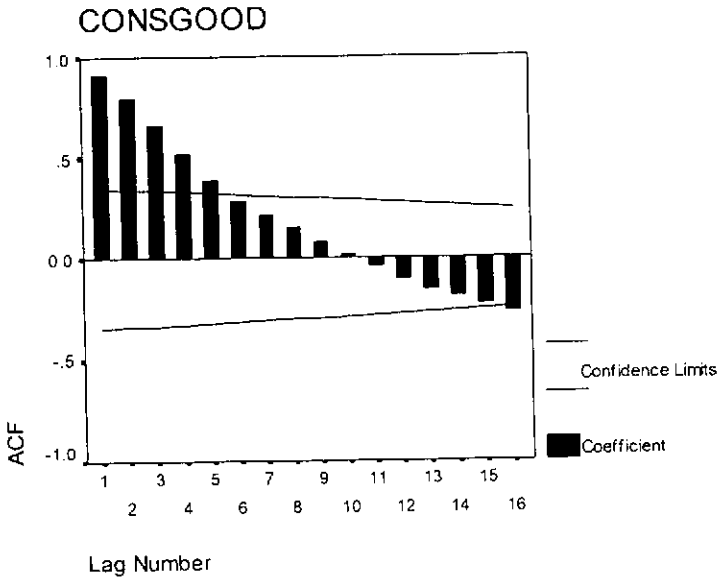
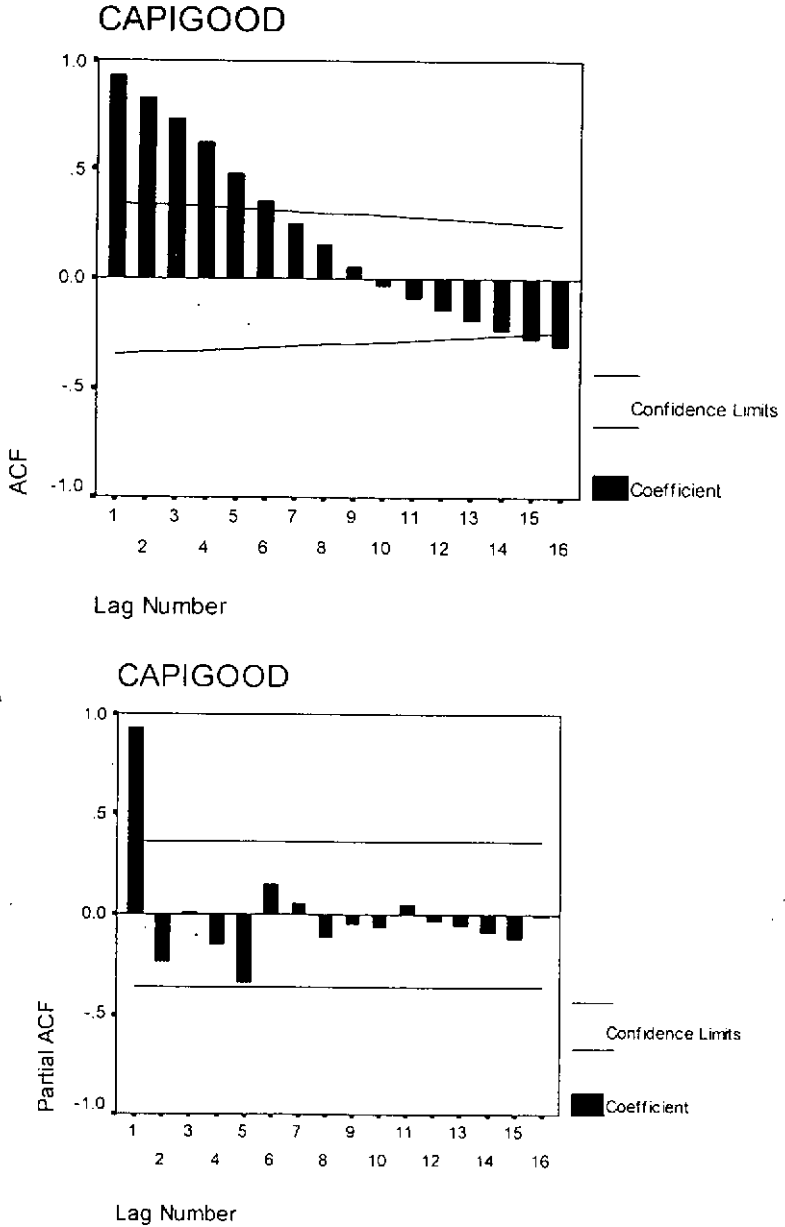


Figure 6: ACF and PACF of Capital Goods



Now, looking at these graphs we can see that the AR(1) model is appropriate and since the categories are inter-related the VAR(1) model has been used. The results of these analyses are given in the following tables:



**Table 1: Coefficients for Vector Auto Regressive Models for Exports**

	Y <sub>1</sub>	Y <sub>2</sub>	Y <sub>3</sub>
Y <sub>1(t-1)</sub>	0.875584 **	-0.008445 **	-0.060491 **
Y <sub>2(t-1)</sub>	-0.287653 *	0.356778 **	0.177288 **
Y <sub>3(t-1)</sub>	1.338589 *	0.435342 **	1.234399 **
C	5339.948 **	5192.318 **	525.2650 **

\*\* : Significant at 1%

\* : Significant at 5%

Y<sub>1</sub>: Total Exports

Y<sub>2</sub>: Primary Commodities

Y<sub>3</sub>: Semi-Manufactured Commodities

From above table we have obtained following forecasting models for exports:

$$Y_{1t} = 5339.948 + 0.875584 Y_{1(t-1)} - 0.287653 Y_{2(t-1)} + 1.338589 Y_{3(t-1)} \quad (3.1)$$

$$Y_{2t} = 5192.318 - 0.008445 Y_{1(t-1)} + 0.356778 Y_{2(t-1)} + 0.435342 Y_{3(t-1)} \quad (3.2)$$

$$Y_{3t} = 525.2650 - 0.060491 Y_{1(t-1)} + 0.177288 Y_{2(t-1)} + 1.234399 Y_{3(t-1)} \quad (3.3)$$

**Table 2: Summary Measures for Exports Models**

	Y <sub>1</sub>	Y <sub>2</sub>	Y <sub>3</sub>
R-squared	0.996727	0.903249	0.979093
S.E. equation	8137.929	4995.316	3939.820
F-statistic	2436.522	74.68659	374.6546

**Table 3: Coefficients for Vector Auto Regressive Models for Imports**

	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>
X <sub>1(t-1)</sub>	2.528028 **	0.602635 **	0.217747 **
X <sub>2(t-1)</sub>	-2.236726 **	0.127704 **	-0.137670 **
X <sub>3(t-1)</sub>	-4.414940	-1.975461 **	-0.048805 **
C	9583.662 **	6088.962 **	1532.756 **

- $^{***}$ : Significant at 1%  
 $^{**}$ : Significant at 5%  
 $X_1$ : Total Imports  
 $X_2$ : Capital Goods  
 $X_3$ : Consumer Goods

From above table the forecasting models have been obtained as under;

$$X_{1t} = 9583.662 + 2.528028X_{1(t-1)} - 2.236726X_{2(t-1)} - 4.414940X_{3(t-1)} \quad (3.4)$$

$$X_{2t} = 6088.962 + 0.602635X_{1(t-1)} + 0.127704X_{2(t-1)} - 1.975461X_{3(t-1)} \quad (3.5)$$

$$X_{3t} = 1532.756 + 0.217747X_{1(t-1)} - 0.137670X_{2(t-1)} - 0.048805X_{3(t-1)} \quad (3.6)$$

Table 4: Summary Measures for Imports Models

	$X_1$	$X_2$	$X_3$
R-squared	0.988630	0.954460	0.990895
S.E. equation	18338.09	12127.59	2446.690
F-statistic	695.6153	167.6710	870.6189

Now on the basis of tables 1 to 4 we have drawn some conclusions. These conclusions and recommendations have been given in the following section.

#### 4. CONCLUSIONS

The imports and exports of Pakistan have been under fluctuations from one time to another. In this paper we have used the multivariate version of powerful Box and Jenkins (1970) ARIMA models. The result of the analysis has been given in section 3 of the paper. From these results we have drawn following conclusions.

The coefficients of the models for exports have been given in table 1. Using these coefficients the forecast models for exports has been given in equations (3.1) to (3.3). We have also found that most of these coefficients are significant at 1% indicating that we can forecast the exports of a particular year on the basis of information of previous year. Further, the  $R^2$  values are very high and the associated F-statistic is significant. This indicates that the forecasts obtained on the basis of these models will be highly reliable. Also from table 2 we can see that the standard error of these equations is relatively low. This indicates that the prediction error from these models will not be very high.

The coefficients of the models for imports have been given in table 3. Using these coefficients the forecast models for imports has been given in equations (3.4) to (3.6). We have also found that most of these coefficients are significant at 1% indicating that we can forecast the imports of a particular year on the basis of information of previous year. Further, the  $R^2$  values are very high and the associated F-statistic is significant. This indicates that the forecasts obtained on the basis of these models will be highly reliable. Also from table 4 we can see that the standard error of these equations is relatively low. This indicates that the prediction error from these models will not be very high.

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