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ESTIMATING THE PARAMETER OF THE POISSON DISTRIBUTION USING FIRST ORDER NEGATIVE MOMENTS

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ABSTRACT: A negative moment estimator of the parameter of the Poisson distribution is being proposed and the asymptotic variance of the negative moment estimator is derived in terms of hyper-geometric series function.

Keywords: Negative moments, Poisson distribution, hyper-geometric,series function.

1. INTRODUCTION

Grab and Savage (1954) discussed the use of negative moments in sampling problems, where the sample size is a random variable. Hence the variance of the sample mean depends on the first order negative moment of the positive binomial or the Poisson distributions. Tiku (1964) gave an expression for obtaining an approximate value of the first negative moment of the zero truncated Poisson distribution. He also prepared a table comparing the exact and approximate values of the first negative moment for different values of the parameter. Stancu (1968) obtained the rth negative moment of the zero truncated binomial and Poisson distributions. He also expressed the negative moments of discrete random variables in terms of factorial moments. Chao and Strawderman (1972) gave a technique based on the integration of the probability generating function for obtaining negative moments of the form $E\left[(X+A)^{-k}\right]$ where X+A > 0. They also gave a recurrence relation for the first negative moment of the Poisson distribution. Kumar and Consul (1979) derived a recurrence for the rth negative moment of the modified power series distribution and used it to obtain the bias of the maximum likelihood estimators of the parameters of the Lagrangian Binomial and Lagrangian Poisson distributions. Ahmad and Sheikh (1983) using Chao and Strawderman (1972) technique obtain the first negative moment of the hyper-Poisson distribution.

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In this paper the negative moment of the Poisson distribution is used to estimate the parameter of the distribution. An expression for the variance of the negative moment estimator has also been derived using hyper- geometric functions.

2. ESTIMATION OF THE PARAMETER USING NEGATIVE MOMENTS

The first order negative moment of the Poisson distribution with parameter θ, given by Chao and Strawderman (1972) is:

$$\mu'_{-[1]} = E\left(\frac{1}{X+1}\right) = \frac{1-e^{-\theta}}{\theta}, \quad \theta > 0. \tag{1}$$

For a sample of size n, the negative moment estimator of θ is given by the equation:

$$m'_{-[1]} = \frac{1-e^{-\hat{\theta}}}{\hat{\theta}}, \tag{2}$$

where $m'_{-[1]} = \frac{1}{n}\sum_{i=1}^{n}(x_i+1)^{-1}$. $\hat{\theta}$ can be obtained by iteration.

Differentiating (2) with respect to $\hat{\theta}$, squaring both sides and taking expectation we get:

$$\mathrm{Var}(\hat{\theta}) \doteq \left\{\frac{\theta e^{-\theta} - 1 + e^{-\theta}}{\theta^2}\right\}^{-2} \mathrm{Var}(m'_{-[1]}), \tag{3}$$

where $\mathrm{Var}(m'_{-[1]}) \doteq \frac{1}{n}\mathrm{Var}\left[(X+1)^{-1}\right]$ (4)

We may use Ahmad and Sheikh (1983) result for E $(X+A)^{-1}$ in terms of hypergeometric series function to estimate θ and Var (θ) for a given value of A.

Now $E\left(\frac{1}{X+A}\right) = e^{-\theta}A^{-1}\ {}_1F_1[A;\ A+1;\ \theta].$ (5)

Where

$${}_nF_m[a_1, a_2, \ldots a_n; b_1, b_2, \ldots b_n; x] = 1 + \frac{a_1 a_2 \ldots a_n}{b_1 b_2 \ldots b_n} x + \frac{a_1(a_1+1)a_2(a_2+1)\ldots a_n(a_n+1)}{b_1(b_1+1)b_2(b_2+1)\ldots b_n(b_n+1)} \frac{x^2}{2!} + LL$$

Suppose A = 1, we have:

$$E\left(\frac{1}{X+1}\right) = e^{-\theta} \; {}_1F_1[1; 2; \theta]$$

(6)

and

$$E\left[(X+1)^{-2}\right] = e^{-\theta} \; {}_2F_2[1,1; 2,2; \theta] \qquad \textbf{(7)}$$

Slater (1960) has developed comprehensive tables for $_nF_m[(a)_n; (b)_m; x]$. The tables can be used to estimate θ and $\mathrm{Var}(\hat{\theta})$ easily.

It has been shown by Ahmad and Sheikh (1983) that negative moment estimator is more efficient than the positive moment estimator in hyper-Poisson distributions of which Poisson distribution is a special case.

REFERENCES

1. Ahmad, M. and Sheikh, A. K.(1983). Some estimation problem of a class of inverted distributions. 44th session of the International Statistical Institute. Spain. 1-6.

2. Chao, M.T. and Strawderman, W.E. (1972). Negative moments of positive random variables. J. Amer. Statist. Assoc. 67, 429-431.

3. Grab, E.L. and Savage, I.R. (1954). Tables of the expected value of 1/X for positive Bernoulli and Poisson variables. J. Amer. Statist. Assoc. 49, 169-177.

4. Kumar A. and Consul, P.C. (1979). Negative moments of a modified power seriesdistribution and bias of the maximum likelihood estimator. Communications in Statistics. A. 8. 151 – 166.

5. Tiku, M.L. (1964). A note on the negative moments of a truncated Poisson variate. Journal of American Statistical Association. 59, 1220 – 1224.

6. Slater, L. J. (1960). Confluent hypergeometric functions. Cambridge University Press,N.Y.

7. Stancu, D.D. (1968). On the moments of negative order of the positive Bernoulli and Poisson variables. Studia Universitatis Babes Bolyai Series, Mathematics and Physics, 1, 29-31.