# SHAPE OF A DIE AND A TRICK 

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#### Abstract

in this paper the shape of the contemporary die is described. An illustration is included about the exploitation of the die. The conclusion is that the cubic shape is the most suitable shape.


## 1. WHY CUBIC? .

A homogeneous cube with faces labeled one to six is called a die (Piscunov, 1974)[1]. The origin of playing card and dice is not precisely known. But we can well think about the reasons why a die is cubic in shape. In ancient Greece, Egypt and the orient the dice were constructed in the same manner as we see them today. The numbers on a die are always in a magical way to have a total of seven on the opposite sides.

A die is supposed to be a regular polyhedron so that the chance of getting each face is equal. Leonhard Euler, worked on this problem. He concluded that there could be only five regular polyheda, namely, tetrahedron (four faces), hexahedron-cubic (six faces), octrahedron (eight faces), dodecahedron (twelve faces) and icosahedron (twenty faces). Out of these five forms the cubic form is easy to construct and rolls in a very smooth and suitable way. The tetrahedron and octahedron do not roll properly. The dodecahedron and icosahedron roll out very quickly. Logically the cubic is the only choice of the five regular polyhedrons described above.

As mentioned earlier the sides of the cube are numbered from one to six and in principle the spots are arranged in such a manner to have a total of seven for all pairs of opposite sides. Many tricks are made with dice on the base of this principle. One of them is described here.

## 2. AN OLD TRICK

In olden times the magicians used a tr $^{1}$ ick with dice. The magician turns back form the observers and asks one of the observers to throw three dice on a table. The observer is then asked to note the numbers, which appear on the upper sides of the dice and sum them. The observer is further asked to select any one of the rolled dice and add the number of the lower side of the dice to the total previously got. The dice is to be rolled again and the number on the top is further added to the total in the second phase. The magician thereafter turns his face towards the

[^0]observers and pretends to think deeply for some time and tells the correct total (Vergara, 1962)[2].

For example, suppose the three dice rolled have numbers 1,2 and 3 on the top. The total is 6 . We select second die. The number on the bottom of this die is 5 . Adding 5 to the previous total of 6 we get 11 . We roll this die again and get 6, say. The final total is now 17. The magician sums the numbers 1,3,6 and 7 , which sum up to 17 .

## 3. CONCLUSION

A die is supposed to be a regular polyhedron so that the chance of getting each face is equal. Leonhard Euler, concluded that there can be only five regular polyheda, namely, tetrahedron, hexahedron (cubic), octrahedron, dodecahedron and icosahedron. Out of these, the cubic form is easy to construct and rolls in a very smooth and suitable way. The sides of the cube are numbered to have a total of seven for all pairs of opposite sides to make tricks

## REFERENCES

[1] Piscunov, N. (1974), "Differential and Integral Calculus, vol. II", (translated form Russian by George Yandovsky), Mir Publishers, Moscow.
[2] Vergara, William C. (1962), "Mathematics in everyday things", A SIGNET SCIENCE LIBRARY BOOK, The New American Library, New York.

# RELATIVE EFFICIENCIES OF MAXIMUM LIKELIHOOD, MINIMUM CHI-SQUARE, AND MINIMUM B ESTIMATORS. 

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#### Abstract

Debate on the choice between maximum likelihood estimator and minimum chi-square estimator is still on. Rao's school of thought advocates for the maximum likelihood estimator and Berkson's school of thought for the minimum chi-square estimator. In this paper a numerical study has been made by computing the relative efficiency of these estimators together with that of minimum B estimator for the Exponential parameter. The study reveals that for few choices of ' $A$ ' and ' $B$ ', the constants in Statistic B introduced by Khan and Baily (1987), the minimum B estimator is more efficient than the other two. Also maximum likelihood estimator is more efficient than the minimum chi-square estimator.


## 1. INTRODUCTION

Khan and Baily(1987) have shown that if $X$ is Poisson random variable with mean $\mu$ then $X^{2 / 3}$ is symmetric about $\mu$ and $E(X+A)^{2 / 3}=\mu^{2 / 3}+O$ $\left(\mu^{-1 / 3}\right)$ and $\operatorname{Var}(X+A)^{2 / 3}=4 / 9\left(\mu^{1 / 3}\right)+O\left(\mu^{-2 / 3}\right)$, where $A$ is some constant. Following Fisher's (1922) approach they proposed a new $X^{2}$-type test statistic of goodness of fit as:

$$
\begin{equation*}
B=\frac{9}{4} \sum_{r=1}^{K} \frac{\left(\left(o_{r}+A\right)^{\frac{2}{3}}-\left(N f_{r}[\theta]+B\right)^{\frac{2}{3}}\right)^{2}}{\left(N f_{r}[\theta]\right)^{\frac{1}{2}}} \tag{1.1}
\end{equation*}
$$

Where $o_{r}$ is assumed to be distributed as multinomial with $E\left(o_{r}\right)=N f_{r}, f_{r}[\theta]$ is the probability that the observation fall in the rth cell, and $A, B$ are some constants.

Khan (1988a) has shown that the estimator of $\theta$, say $\hat{\theta}_{B}$ obtained by minimizing statistic $B$ has variance as:
$\operatorname{Var}\left(\hat{\theta}_{B}\right)=\operatorname{Var}(\mathrm{mle})+\mathrm{O}\left(\mathrm{N}^{-2}\right)$ which reduces to $\operatorname{Var}$ (mle) for the choices of $A=B=0.0$ and $A=(3 / 2) B$, where $\operatorname{Var}$ (mle) is the variance of maximum likelihood estimator of $\theta$ obtained by Rao (1962).

In the estimation theory there are differences of opinions on the efficiencies of minimum chi-square estimator and the maximum likelihood estimator. Berkson (1980) went for the minimum chi-square estimate while

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