A MONTE-CARLO COMPARISON OF MULTIVARIATE HOMOGENEITY TESTS FOR DEPENDENT CLUSTER SAMPLES

By

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ABSTRACT

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Some key words: *Robust, superpopulation, consistent, null distribution.*

1. INTRODUCTION

Pervaiz (1988a) obtained asymptotically robust tests for testing the equality of two covariance matrices under cluster sampling design. The cluster samples were supposed to be independent. But the cluster samples may or may not be independent. For example consider the male and female population of U.K. for different area clusters. Here the clusters consist of units from both finite populations. To obtain cluster samples from finite populations clusters are selected from union of populations and partitioned for the respective finite population. Therefore cluster samples are no more independent. Pervaiz (1988b) considered the case of dependent cluster

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$$\underline{S}_i = \frac{1}{n_{oi}} \sum_{c=1}^{n} \sum_{e=1}^{m_{ic}} (\underline{x}_{ice} - \overline{\underline{x}}_{i..})(\underline{x}_{ice} - \overline{\underline{x}}_{i..})^T$$

Sample covariance matrix.

$$\underline{S}_i = \begin{bmatrix} s_{i,20} & s_{i,11} \\ s_{i,11} & s_{i,02} \end{bmatrix}$$

2.2. Sampling design

We assume that n clusters are selected from the N clusters of U by simple random sampling. The cluster samples from finite populations are obtained by respective partition of selected clusters from U. Within each selected cluster all subunits are included in a sample.

2.3. Null hypothesis of interest

We adopted model/superpopulation approach with unrestrictive assumptions - (cf. Pervaiz, 1986).

We assume that $\underline{x}_{ice}$ are random variables which implies that $\underline{C}_i$'s are also random variables. We also assume that $\underline{C}_i$ converges to $\underline{\Sigma}_i$ as N_i increases. Then the finite population with covariance matrices $\underline{C}_i$'s may be viewed as samples from infinite populations called superpopulation with covariance matrices $\underline{\Sigma}_i$. The hypothesis of interest is:

$$H_o : \underline{\Sigma}_1 = \underline{\Sigma}_2 \quad \text{vs} \quad H_1 : \underline{\Sigma}_1 \neq \underline{\Sigma}_2.$$

2.4. Description of test statistics

(a) Standard Error Test

Let $\underline{S}_i^v = (s_{i,20}, s_{i,02}, s_{i,11})^T$. From Fuller (1975) and Skinner (1986) we say $\underline{S}_i^v$ obey the central limit laws as $n \longrightarrow \infty$, i.e.

$$n^{1/2} \left[\begin{pmatrix} \underline{S}_1^v \\ \underline{S}_2^v \end{pmatrix} - \begin{pmatrix} \underline{\Sigma}_1^v \\ \underline{\Sigma}_2^v \end{pmatrix} \right] \xrightarrow{dist.} N_6 \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \underline{\Gamma}_1 & \underline{\Gamma}_{12} \\ \underline{\Gamma}_{21} & \underline{\Gamma}_2 \end{pmatrix} \right]$$

Therefore $\underline{V}(\underline{S}_1^v - \underline{S}_2^v) = \frac{1}{n} (\underline{\Gamma}_1 + \underline{\Gamma}_2 - 2\underline{\Gamma}_{12})$ because of $\underline{\Gamma}_{12} = \underline{\Gamma}_{21}$.

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$$\bar{\underline{S}}_i^v = \frac{1}{n'} \sum_{g=1}^{n} \underline{S}_{i,g}^v.$$

The $\hat{\underline{V}}^+(\underline{S}_1^v - \underline{S}_2^v)$ can be determined by using

$$\hat{\underline{\Gamma}}_i^+ = \frac{1}{n'-1} \sum_{g=1}^{n} (\underline{S}_{i,g}^v - \bar{\underline{S}}_i^v)(\underline{S}_{i,g}^v - \bar{\underline{S}}_i^v)^T$$

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$$\hat{\Gamma}_{12}^+ = \frac{1}{n'-1} \sum_{g=1}^{n'} (S_{1,g}^v - \bar{\underline{S}}_1^v)(\underline{S}_{2,g}^v - \bar{\underline{S}}_2^v)^T.$$

(c) Jackknife Test

Let $\underline{S}_{i,c}^v = n\, \underline{S}_i^v - (n-1)\, \underline{S}^v_{i-c}$. The elements of $\underline{S}^v_{i-c}$ are second order sample moments by using $(n-1)$ clusters with c-th cluster omitted. The jackknife estimators $\underline{S}_i^{*v}$ are the average of $\underline{S}^v_{i,c}$. The $\underline{S}^v_{i,c}$ are approximately independent and have asymptotically equal mean vectors and covariance matrices. Thus the null distribution of the test statistic:

$$(\underline{S}_1^{*v} - \underline{S}_2^{*v})^T [\hat{V}^*(\underline{S}_1^v - \underline{S}_2^v)]^{-1} (\underline{S}_1^{*v} - \underline{S}_2^{*v})$$

is approximately Hotelling's T^2 with 3 and $(n-1)$ degrees of freedom.

The $\hat{V}^*(\underline{S}_1^v - \underline{S}_2^v)$ can be determined by using

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and

$$\hat{\underline{\Gamma}}_{12}^* = \frac{1}{n-1} \sum_{c=1}^{n} (\underline{S}_{1,c}^v - \underline{S}_1^{*v})(\underline{S}_{2,c}^v - \underline{S}_2^{*v})^T.$$

3. SAMPLING EXPERIMENTS

The population used in empirical investigation was collected by the U.S. Bureau of the census in March, 1967 Current Population Survey. There are 3240 clusters or primary units. The clusters consist of units from both finite populations. We deleted clusters having less than 4 units for each population. The variables

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The observed significance levels for the test-sample size-matrix combinations are recorded in the Table. The observed sizes are ranging from a minimum of 3% to a maximum of 6.6% for the standard error test, from a minimum of 1.8% to a maximum of 5.2% for the grouping test and from a minimum of 4.4% to a maximum of 8.8% for the jackknife test. There is no affect of size of samples on these tests.

The standard error test maintained very good nominal levels for all situations. Its observed significance levels performance is better than the independent cluster samples from the natural populations (cf. Pervaiz 1986). The observed significance level performance of the grouping test is very good for (p). It rejected the null hypothesis too infrequently for the sample size - matrix combinations (55 & 70-q). The sizes for these combinations are (1.8 & 2.8)%, respectively. An interesting point is that the test produced such a low observed size i.e. 1.8%. The jackknife test produced high sizes for combinations (50,55,60 & 75-p) and (65 & 70-q). The observed sizes for these combinations are (7.8,8.4,8.8 & 7.2)% and (7.6 & 7.4)%, respectively. As a whole its observed significance level performance is not as good as that for the independent samples from the natural poplulations (cf. Pervaiz 1986).

The grouping test is less powerful than the jackknife test for all situations but it has comparable power with the standard error test for large samples. However it has very low power with samples of size $n_1=n_2\leq 35$. The standard error test is little less powerful than the jackknife test.

CONCLUSIONS

The standard error test performs better in maintaining nominal levels than the others. The test is more powerful than the grouping test for moderate size of samples. Therefore it may be a better choice. The reason for not very good performance of the grouping and the jackknife tests may be small group size (L=5) and statistics computed, are compared with the 5% and 1% points of the approximate null distributions. The results for the 1% case, essentially corroborate those of the 5% case, so we are not reporting here. There are five sampling units in each group for the grouping test, (L=5).

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50	0.078	0.60	0.800	0.752	0.960
55	0.084	0.050	0.832	0.778	0.970
60	0.088	0.058	0.822	0.778	0.978
65	0.062	0.076	0.828	0.830	0.988
70	0.056	0.074	0.854	0.776	0.988
75	0.072	0.066	0.882	0.848	1.000

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