DEPARTMENT OF MATHEMATICS, GC UNIVERSITY, LAHORE

Admission Test for M.Phil (Mathematics)/Session 2009-11

Max. Time: **90 Minutes**

Max. Marks: **60**

Name of Candidate____________________ **Father / Guardian Name**____________________

Form No. ____________ **Signature of Candidate** ____________________

Note: ***Please put a tick (✓) mark on the correct answer in each question. Overwriting will not be evaluated.***

1. Let A be a z x z real matrix such that $A\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$, $A\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$ than A =

   (i) $\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$

   (ii) $\begin{bmatrix}1 & 1\\0 & 0\end{bmatrix}$

   (iii) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

   (iv) $\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}$

   (v) $\begin{bmatrix}0 & 0\\1 & 0\end{bmatrix}$

2. If A is a matrix with characteristic equation $x^2 - 4x + 1 = 0$ then det A =

   (i) 2 (ii) 0 (iii) -2 (iv) 1
   (v) 4

3. Let V and W be two proper subspaces of the space R^4. Then $V \cap W$ is a subspace of R^4 of dimension;

   (i) one only
   (ii) two only
   (iii) zero only
   (iv) 0, 1, 2 only
   (v) four only

4. In the complex plane, the equation $z^2 = |z|^2$ represents
   (i) a pair of points
   (ii) a circle
   (iii) half line
   (iv) a line
   (v) none of these

5. The function of $f : C \rightarrow R$, from the complex set C into the real set R defined by $f(z) = |z|^2$ is:
   (i) differentiable for all z
   (ii) discontinuous for all z
   (iii) differentiable only at $z = 0$
   (iv) continuous only at $z = 0$
   (v) none of these

(P.T.O)

6. The alternating group A_4 has,

(i) five elements of order 3
(ii) two elements of order 2
(iii) one element of order 4
(iv) three elements of order 2
(v) none of these

7. The Quaternion group $\{\langle a,b : a^4 = e = b^4 = (ab)^4, bab^{-1} = a^{-1}\}$ has
(i) one element of order 2
(ii) three elements of order 4
(iii) seven elements of order 4
(iv) no element of order 2
(v) four elements of order 4

8. Let a function f(z) be analytic in a simply connected domain D and C be a closed continuous curve in D. then

(i) $\int_C f(z)dz = 1$

(ii) $\int_C f(z)dz = 0$

(iii) $\int_C f(z)dz = \int_D f(z)dz$

(iv) $\int_D f(z)dz = 0$

(v) none of the above

9. If ∇ is the vector differential operator and φ is any scalar function then, $\nabla(\nabla\varphi) =$
(i) 0
(ii) 1
(iii) $\nabla^2\varphi$
(iv) $(\nabla\varphi).\nabla$
(v) none of the above.

10. If f(x, y, z) = xyz is a scalar function and $\nabla = \underline{i}\frac{\partial}{\partial x} + \underline{j}\frac{\partial}{\partial y} + \underline{k}\frac{\partial}{\partial z}$ *then* $\nabla f =$

(i) $x\underline{i} + y\underline{j} + z\underline{k}$
(ii) $xy\underline{i}$
(iii) $yz\underline{i}$
(iv) $xyz\underline{j}$
(v) none of these

11. An integrating factor for the differential equation $\frac{-2y}{x}dx + (x^2y \quad \cos y + 1)dy = 0$ is,

(i) 1
(ii) -2x
(iii) $\frac{-2}{x}$
(iv) x^2
(v) $\frac{1}{x^2}$

12. If $f(x) = e^x - e^{-x}$, then $[f'(x)]^2 - [f(x)]^2 =$
(i) 4
(ii) $4e^{2x}$
(iii) $2e^{-x}$
(iv) $2e^x$
(v) 2

13. For which value of k, x^k is a solution for the differential equation $x^2y'' - 3xy' + 4y = 0$?

(i) 4
(ii) 3
(iii) 2
(iv) 1
(v) none of these.

14. If $f(x) = \int_1^{x^2} \frac{dt}{1+t^3}$ then $f'(2) =$

(i) 4/65
(ii) $\frac{1}{9}$
(iii) $\ln(\frac{65}{2})$
(iv) $\ln(\frac{9}{2})$
(v) 0.23

15. The radius of curvature of $f(x) = x + \frac{1}{x}$ at the point P (1, 2) is,

(i) 1
(ii) $\sqrt{2}$
(iii) 4
(iv) 2
(v) $\frac{1}{2}$

16. For a pure radioactive model, the rate of decaying variation w.r.t. time t of mass M satisfies the equation $\frac{dM}{dt} = \frac{-M}{10}$. Find M in terms of its initial value M_0 after 20 units of time t as,

(i) $\frac{1}{2}M_0$
(ii) $\frac{1}{4}M_0$
(iii) $\frac{M_0}{2e}$
(iv) $\frac{M_0}{e}$
(v) $\frac{M_0}{e^2}$

17. For $x^2z - 2yz^2 + xy = 0$, find $\frac{\partial x}{\partial z}$ at (1, 1, 1) as;

(i) 0
(ii) 4/3
(iii) -1
(iv) 1
(v) none of these

18. The solution set on the number line $(-\infty, \infty)$ of the inequality $\frac{1}{x-2} < \frac{1}{x+3}$ is the set,

(i) (-3, -2)
(ii) (-3, 2)
(iii) (2, 3)
(iv) (-2, 2)
(v) (0, 2)

19. If A is a countable subset of the unit interval [0,1] then the labesque measure of A is,

(i) $\frac{1}{2}$
(ii) 0
(iii) 2/3
(iv) 3^{-1}
(v) none of these

(P.T.O)

20. The Inequality $\frac{x^2}{a^2}+\frac{y^2}{b^2}<2$ *in* $\mathbb{R}^2$ is,
   (i) a closed set
   (ii) an open set
   (iii) a compact set
   (iv) a dense set
   (v) none of these

21. A parabola is homeomorphic to:
   (a) an ellipse
   (b) a straight line
   (c) a hyperbola
   (d) a circle
   (e) none of these

22. Which of the following property is NOT true for the interval [0, 1] in $\mathbb{R}$?
   (i) it is compact
   (ii) it is disconnected
   (iii) it is connected
   (iv) it is closed
   (v) none of these

23. If X is any set, then the collection of all one point subsets of X is a basis for which topology on X:
   (i) confinite
   (ii) discrete
   (iii) indiscrete
   (iv) quotient
   (v) none of these

24. Let (X, τ) be a topological space, and $Y \subset X$. Then which of the following defines a subspace topology on Y:
   (i) $\{(Y \cup V)\} : V \in \tau\}$
   (ii) $\{(Y \cap V)\} : V \in \tau\}$
   (iii) $\{(Y-V)\} : V \in \tau\}$
   (iv) $\{(V-Y)\} : V \in \tau\}$
   (v) none of these

25. Which of the following is a connected subset of $\mathbb{R}$?
   (i) $\mathbb{Q}$
   (ii) $[0,1) \cup (1,2)$
   (iii) $[0,\frac{1}{2}) \cup [\frac{1}{2},1)$
   (iv) $\mathbb{Z}$
   (v) none of these

26. Which of the following is NOT a compact subset of $\mathbb{R}$?
   (i) $[0,1) \cup \{1\}$
   (ii) $[2,3) \cup (3,4]$
   (iii) $[2,3]$
   (iv) $[2,3) \cap (3,4]$
   (v) none of these

27. If x(y+z) > w and x, y, z, and w are all integers, which of the following must be true ?
   (i) $x(y+z)>0$
   (ii) $xy+z=w$
   (iii) $xy+xz=w$
   (iv) $x+y+z = w$
   (v) x,y,z and w are all positive

28. In the group Z of all integers, which of the subset is not its subgroup?
(i) $\{0\}$
(ii) $\{n \in \mathbb{Z} : n > 0\}$
(iii) $\{m \in \mathbb{Z} : m$ is divisible by 6 and 9$\}$
(iv) $\{n \in \mathbb{Z} : n$ is even$\}$.
(v) $\mathbb{Z}$

29. Let V_1 and V_2 ($\neq V_1$) be two 6 dimensional subspaces of a 10-dimensional space V. What is the dimension of $V_1 \cap V_2$?
(i) 0
(ii) 1
(iii) 2
(iv) 4
(v) 6

30. Let $\mathbb{R}[x]$ be the ring of polynomials in x. Which of the following subsets form its subring;
(i) all the polynomials with coefficient in $\mathbb{R}$ of x is zero
(ii) all the polynomials of even degree
(iii) all the polynomials of odd degree
(iv) All the polynomials of degree 2
(v) none of these

31. If f is a function defined on the interval (2,3) on the real line $R = (-\infty, \infty)$ such that $2 < f(x) < 3$, x in (2,3) then;
(i) f is bounded
(ii) f is negative
(iii) f is strictly increasing
(iv) f is polynomial function of degree 1
(v) f is non-constant

32. For what value / values of k, the vector (1, 2, k, 5) of the space R^4 is a linear combination of the vectors (0,1,1,1), (0,0,01) and (1,1,2,3) of R^4;
(i) no value
(ii) -1 only
(iii) 1 only
(iv) 3 only
(v) infinitely many

33. The harmonic conjugate V (x,y) for the harmonic function $u(x,y) = y+3xy^2-x^3$ is,
(i) $-y+3xy^2-x^3$
(ii) $x+3xy^2-x^2$
(iii) y^3-3x^2y-x
(iv) y^3-3x^2y+x
(v) $y^3-3x^2y+x^2$

34. If A is a 2 X 2 real matrix then
(i) all the entries of A^2 are none-negative
(ii) det A^2 is non-negative
(iii) if A has two distinct eigen values then so has A^2
(iv) A^2 is a scalar matrix
(v) none of these

(P.T.O)

35. If S is a non-empty set such that $|S| = k$ and if there are n one-to-one functions on S then n = ?

(i) k^2
(ii) k^k
(iii) 2^k
(iv) $\lfloor K$
(v) none of these

36. If n stands for number of non-isomorphic group of order 4 then n =

(i) 1
(ii) 0
(iii) 2
(iv) 3
(v) 5

37. In the symmetric group S_3 if m stands for the number of subgroup of S_3 then m = ...?

(i) 1
(ii) 2
(iii) 3
(iv) 4
(v) 6

38. A group of order 144 has,

(i) 2-Sylow subgroup of order 4
(ii) 5-Sylow subgroup of order 5
(iii) 3-Sylow subgroup as abelian group
(iv) 3-Sylow is cyclic of 3
(v) none of these

39. $\int_0^a \int_0^b dxdy = ...?$

(i) a
(ii) b
(iii) $\frac{a}{b}$
(iv) a-b
(v) ab

40. $\int_0^a \int_0^b \int_0^c dxdydz = ...?$

(i) $\frac{ab}{c}$
(ii) $\frac{a}{bc}$
(iii) ab+c
(iv) a+b+c
(v) abc

41. A cyclic group of order 8 has n, conjugate classes where n =?

(i) 2
(ii) 4
(iii) 6
(iv) 8
(v) none of these

42. Which of the following groups you find is cyclic ?

(i) $C_2 \times C_4$
(ii) $C_2 \times C_6$
(iii) $C_3 \times C_6$
(iv) $C_3 \times C_4$
(v) $C_4 \times C_6$

43. Let (R, +, .) be a ring. Then
   (i) (R, +) is a simple group
   (ii) (R, +) is a cyclic group
   (iii) (R, .) is a group
   (iv) (R, .) is a symmetric group
   (v) (R, .) is a semi-group

44. Let $h : G \to \overline{G}$ be a group homomorphism from group G into $\overline{G}$ if g in G is of order 10 then
   (i) h(g) is of order 3
   (ii) h(g) is of order 7
   (iii) h(g) is of order 12
   (iv) h(g) is of order 4
   (v) none of these

45. Let $T : R^5 \to R^3$ be a linear transformation from R^5 into R^3 such that KerT is a subspace of
R^5 of dimension 3. Then $T(R^5)$ is a
   (i) trivial subspace of R^5
   (ii) subspace of degree 1
   (iii) subspace of dimension 2
   (iv) subspace of degree 3
   (v) none of these

46. If $x < 0$ then $|x| = \ldots\ldots..$, for a real number x,
   (i) $\pm x$
   (ii) x
   (iii) $- x$
   (iv) 0
   (v) none of these

47. The inequality $\frac{x}{x-1} \geq 0$ holds true for,
   (i) $x > 1$
   (ii) $0 < x < 1$
   (iii) $0 < x < \frac{1}{2}$
   (iv) $0 < x < \frac{1}{4}$
   (v) none of these.

48. A topological space X is a T_1-space iff each singleton in X is,
   (i) open
   (ii) closed
   (iii) half open
   (iv) half closed
   (v) none of these

49. The topology defined on the real line is called;
   (i) left ray topology
   (ii) right ray topology
   (iii) cofinite topology
   (iv) usual topology
   (v) none of these

50. The real space R is a ....
   (i) first category space
   (ii) second category space
   (iii) closed space
   (iv) finite Space
   (v) open space

51. The smallest field should have,
   (i) one element
   (ii) two elements
   (iii) three elements
   (iv) four elements
   (v) ten elements

(P.T.O)

52. Let $\varphi : G \to \overline{G}$ be an injective homomorphism from a group G into $\overline{G}$. The Ker φ = ...
   - (i) G
   - (ii) {e}
   - (iii) $\overline{G}$
   - (iv) φ
   - (v) none of these

53. For a ring R, a group M forms an R-module. Then
   - (i) M is a multiplicative group
   - (ii) M is necessarily cyclic
   - (iii) M is a additive abelian group
   - (iv) M is a simple group
   - (v) none of these

54. Let G be an abelian group. Then
   - (i) the centre of G = {e}
   - (ii) the centre of G = φ
   - (iii) the centre of G is finite
   - (iv) the centre of G is infinite
   - (v) the centre of G = G

55. Let G be a cyclic group of order 6. Then
   - (i) G is simple group
   - (ii) G has one non abelian sub-group
   - (iii) $G \cong C_2 \times C_3$
   - (iv) G is non-abelian
   - (v) none of these

56. If X = {a, b, c} and τ ={φ, {x}, {a, b}, X} is a topology on X. Then the neighbourhood system N (a) of a in X is
   - (i) {{a, b}, X}
   - (ii) {X}
   - (iii) {{a}, X}
   - (iv) N {{a}, {a, b}, {a, c}, X}
   - (v) none of these

57. For a, b in $\mathbb{R}$, the property; "a > b or a = b or a < b" is called as;
   - (i) cancellation property
   - (ii) left distributive property
   - (iii) right distributive property
   - (iv) inverse property
   - (v) none of these

58. Let G = {1, w, w^2} be the group of all cube roots of unity and F = {0, 1} be a field. Then FG-algebra has
   - (i) two elements
   - (ii) three elements
   - (iii) four elements
   - (iv) nine elements
   - (v) none of these

59. The alternating group A_4 is
   - (i) abelian
   - (ii) cyclic
   - (iii) simple
   - (iv) non-abelian
   - (v) none of these

60. The general linear group $GL_2(F_q)$ has order,
   - (i) q^2-1
   - (ii) q^2+1
   - (iii) q^2+q+1
   - (iv) (q^2-1) (q^2-q)
   - (v) none of these

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